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구조물의 동특성 추정을 위한 순차적 기법

SEQUENTIAL ALGORITHMS FOR DYNAMIC STRUCTURAL IDENTIFICATION

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요 약

구조물의 동적실험을 통하여 얻은 하중과 거동에 대한 시간기록을 분석하여, 구조계의 동특성계수들을 추정하는 기법에 대하여 연구하였다. 실험과정 및 해석모형과정의 오차를 고려하기위하여, 하중기록과 구조거동기록간의 관계를 추계론적 자동회기 및 이동평균모형(Stochastic Auto-Regressive and Moving-Average (ARMAX) Model)을 사용하여 모형화하였다. 미지의 ARMAX 계수행렬들은 순차적 예측오차기법을 사용하여 추정하였으며, 계수추정기법의 효율성을 증진시키기 위하여, Exponential Data Weighting, Global Data Weighting 및 Square Root Estimation 기법을 활용하였다. 다중거동측정계의 예제해석을 통하여 이의 효율성을 분석하였다.

INTRODUCTION

In this paper, a time domain method for the identification of linear structural dynamic systems is presented. The study focuses on a sequential prediction error method [1-4] incorporating several techniques for improving the convergence. They are the exponential data weighting, global data weighting and square root estimation techniques [1-3]. The stochastic auto-regressive and moving average model (ARMAX model) [1-4] is adapted to process the measured excitation and response records contaminated by noises. The effectiveness of those techniques is investigated through an analytical simulation study on an idealized system with two degrees of freedom.

SEQUENTIAL PREDICTION ERROR METHOD

The relationship between the measurement records for the excitation and response of a structural system can be obtained as a stochastic auto-regressive and moving average model (ARMAX model [1-4]) as

$$P(z)y(k) = Q(z)u(k) + H(z)e(k) \qquad (1)$$

where $u(k)$ and $y(k)$ are the observation vectors for the excitation and response records at $t = k\Delta t$; $e(k)$ is the prediction error (noise) vector; and $P(z), Q(z)$ and $H(z)$ are the transfer operators defined as

$$P(z) = Iz^r + P_1 z^{r-1} + \cdots + P_r \qquad (2)$$
$$Q(z) = Q_o z^r + Q_1 z^{r-1} + \cdots + Q_r \qquad (3)$$
$$H(z) = Iz^r + H_1 z^{r-1} + \cdots + H_r \qquad (4)$$

where z is the forward shift operator ($zy(k) = y(k+1)$) ; P_i's and Q_i's are the coefficient matrices associated with the structural parameters (mass, damping, stiffness matrices) or the modal parameters of the structural system ; and H_i's are the matrices related to the prediction error (noise) terms.

The unknown ARMAX parameters, P_i's, Q_i's and H_i's, can be evaluated by minimizing the norm of the prediction error as

$$V_N(\theta) = \frac{1}{2}\sum_{k=1}^{N} e(k,\theta)^T e(k,\theta) \qquad (5)$$

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where N is the number of data points. The minimization may be carried out efficiently by a sequential algorithm such as the sequential prediction error method as

$$\hat{\theta}(k+1) = \hat{\theta}(k) + F(k+1)\Psi(k)(y(k+1) - \hat{W}(k)^T\hat{\theta}(k)) \quad (6)$$

$$F^{-1}(k+1) = F^{-1}(k) + \Psi(k)\Psi(k)^T \quad (7)$$

$$\Psi(k)^T = \hat{H}^{-1}(z;k-1)\hat{W}(k)^T \quad (8)$$

where $\hat{\theta}(k+1)$ denotes the estimate of the unknown parameter vector θ, which consists of the ARMAX parameters , at time $k+1$; $\hat{W}(k)$ is the matrix of the regression vectors in terms of the measurements for the excitation and response and the estimate for the prediction error ($\hat{e}(k) = y(k) - \hat{W}(k-1)^T\hat{\theta}(k-1)$) ; $\Psi(k)$ is the matrix of the negative gradient vectors of the prediction error $e(k,\theta)$; $F(k)$ is the adaptation gain matrix which is the inverse of the second derivative matrix of the criterion function $V_k(\theta)$; and $\hat{H}(z;k-1)$ is the transfer operator for the prediction error based on $\hat{\theta}(k-1)$. The adaptation gain matrix $F(k)$ in Eq. 7 can be easily evaluated by using the "matrix inversion lemma" as follows:

$$F(k+1) = F(k) - F(k)\Psi(k)[I + \Psi(k)^T F(k)\Psi(k)]^{-1}\Psi(k)^T F(k)^T \quad (9)$$

TECHNIQUES FOR IMPROVING CONVERGENCE

Exponential Data Weighting [1-3]

As the sequential prediction proceeds, it is apparent that the recent estimated prediction errors are more informative than the past ones. Hence it may be reasonable to gradually disregard the prediction errors at the past time instances. These concepts can be incorporated into the algorithm by modifying the criterion function given in Eq. 5 as follows.

$$V_{k+1}(\theta) = a(k)V_k(\theta) + \frac{1}{2}e(k+1,\theta)^T e(k+1,\theta) \quad (10)$$

where $a(k) = 1 - \alpha(1 - a(k-1))$ with typical values of $a(0) = 0.95 \sim 1.0$ and $\alpha = 0.99$.

From the above recursive relationship of $a(k)$, it can be shown that the value of $a(k)$ converges to unit as $k \to \infty$, and

$$\prod_{i=1}^{k} a(i) = a(k)a(k-1)\cdots a(1)a(0) \cong 0 \quad (11)$$

Consequently, the criterion function $V_{k+1}(\theta)$ can be rewritten as

$$V_{k+1}(\theta) = \frac{1}{2}e(k+1,\theta)^T e(k+1,\theta) + a(k)\frac{1}{2}e(k,\theta)^T e(k,\theta) + \cdots + a(k)\cdots a(1)a(0)\frac{1}{2}e(0,\theta)^T e(0,\theta) \quad (12)$$

Therefore, as the time proceeds, the effect of the uncertainty of the initial value becomes negligible. It can be easily shown that the above modification of the criterion function equivalently implies the modification of the adaptation gain matrix as

$$F(k+1)^{-1} = a(k)F(k)^{-1} + \Psi(k)\Psi(k)^T \quad (13)$$

Global Data Weighting [1-3]

The length of the measured data sequence is usually limited in the actual experiments. Hence, the estimates by the sequential algorithm may not accurate enough. Such a problem may be overcomed to some extent by using the global iterations which are frequently used in the batch algorithms. For the present purpose, the performance functions for the ith global iteration is defined as

$$V_k^{(i)}(\theta) = \frac{1}{w}V_N^{(i-1)}(\theta) + \frac{1}{2}\sum_{j=1}^{k} e(j,\theta)^T e(j,\theta) \quad (14)$$

where

$$V_N^{(1)}(\theta) = \frac{1}{2}\sum_{j=1}^{N} e(j,\theta)^T e(j,\theta) \quad (15)$$

and $w \geq 1$.

The above criterion function means that the prediction errors are weighted by w times after the completion of the previous global iteration. It may be expected that the estimates of the parameters improve as the iteration proceeds, since the prediction errors obtained by using recent estimates are more informative. The above modification of the criterion function

can be incorporated into the adaptation gain matrix as

$$F(k+1)^{-1} = \overline{F}(k)^{-1} + \Psi(k)\Psi(k)^T \qquad (16)$$

where $\overline{F}(1) = wF(N)$ of the previous iteration; and $\overline{F}(k) = F(k)$ for $k \neq 1$.

Square Root Estimation [1-3]

With the sequential algorithm, inversions of the matrices with the same dimension (n) of the output are required as in Eq. 9. Series of the matrix inversions may cause the adaptation gain matrix to lose the positive definiteness, especially when the quality of the measurement data and/or the concurrent estimates for the unknown parameters is poor. If the positive definiteness of the matrix is lost, the sequential prediction error algorithm could converge to the direction of increasing the prediction error. In this study, the square root estimation algorithm is employed to overcome the problem. Then, the positive definiteness of the adaptation gain matrix can be insured, since the diagonal elements of the triangular decomposition of the matrix are estimated by taking square roots of positive quantities. Details of the technique are described below.

For Single Regression [1-3]

Consider a case with a single response measurement and a $p-$dimensional unknown parameters. Then, the adaptation gain matrix $F(k+1)$ and the negative gradient vector $\psi(k)$ can be evaluated from the following relationship.

$$F(k+1)^{-1} = F(k)^{-1} + \psi(k)\psi(k)^T \qquad (17)$$

where $F^{-1}(0) = \beta I_p \quad (\beta > 0)$.

It is well known that the matrix, $F(k+1)$, is symmetric and positive definite at every time. Base on the modified Choleski decomposition, the matrix can be rewritten as

$$F(k+1) = G(k+1)^T G(k+1) \qquad (18)$$

where $G(k+1)$ is a $p \times p$ lower triangular matrix. By inverting both sides, Eq. 17 can be rewritten as

$$G(k+1)^T G(k+1) = G(k)^T[I_p + G(k)\psi(k)\psi(k)^T G(k)^T]^{-1} G(k) \qquad (19)$$

By defining a $p-$dimensional vector h as

$$h = G(k)\psi(k) = \{h_1, h_2, \cdots, h_p\}^T \qquad (20)$$

and by using the matrix inversion lemma, the inversion part of Eq. 19 can be rewritten as

$$(I_p + hh^T)^{-1} = (I_p - \frac{hh^T}{1+h^Th}) = H^T H \qquad (21)$$

where H is also a lower triangular matrix. It is noted that in Eq. 21 an inversion of a scalar quantity rather than a $p \times p$ dimensional matrix is required.

By defining a scalar quantity, σ_i, as

$$\sigma_{i+1}^2 = \sigma_i^2 + h_i^2, \quad \text{for } i = 1, \cdots, p \qquad (22)$$

with $\sigma_0^2 = 1$, the matrix H can be efficiently evaluated as

$$H_{ii} = \frac{\sigma_i}{\sigma_{i+1}} \quad \text{with } \sigma_0 = 1 \qquad (23)$$

$$H_{ij} = -\frac{h_i h_j}{\sigma_{i+1}\sigma_i} \quad \text{for } j < i \qquad (24)$$

Then, from Eqs. 19 and 21, the matrix $G(k+1)$ can be obtained as

$$\begin{aligned} G_{ij}(m+1) &= \sum_{s=j}^{i} H_{is} G_{sj}(k) \\ &= \frac{\sigma_i}{\sigma_{i+1}} G_{ij}(k) - \frac{h_i}{\sigma_i \sigma_{i+1}} \sum_{s=j}^{i-1} h_s G_{sj}(k) \end{aligned} \qquad (25)$$

For the effective computation of Eq. 25, define a column vector $g^{(i)}$ of which the jth element is

$$g_j^{(i)} = \sum_{s=j}^{i} h_s G_{sj}(k) = g_j^{(i-1)} + h_i G_{ij}(k) \qquad (26)$$

Then, Eq. 25 can be rewritten as

$$G_{ij}(m+1) = \frac{\sigma_i}{\sigma_{i+1}} G_{ij}(k) - \frac{h_i}{\sigma_i \sigma_{i+1}} g_j^{(i-1)} \qquad (27)$$

Moreover, the last value of $g^{(p)}$ implies that

$$\begin{aligned} g^{(p)} &= G^T(k)h = G^T(k)G(k)\psi(k) \\ &= F(k)\psi(k) \end{aligned} \qquad (28)$$

Hence, the quantity $F(k+1)\psi(k)$ used in the sequential prediction error algorithm as shown in Eqs. 6, 7, 8 can be easily obtained as

$$\begin{aligned} \frac{1}{\sigma_p^2} g^{(p)} &= \frac{F(k)\psi(k)}{1 + \psi(k)^T F(k)\psi(k)} \\ &= F(k+1)\psi(k) \end{aligned} \qquad (29)$$

For Multi-Regression [1-3]

Let us consider a case with an $n-$dimensional response measurement vector and a $p-$dimensional parameter vector. Then, we can define a $p \times n$ negative gradient matrix $\Psi(k)$ and a $p \times p$ adaptation gain matrix $F(k)$. As the case of a single regression vector, the adaptation matrix can be written as

$$\begin{aligned} F(k+1) &= G(k+1)^T G(k+1) \\ &= G(k)^T H^T H G(k) \end{aligned} \quad (30)$$

where $G(k)$ and H are both lower triangular matrices and H is defined as

$$H^T H = [I_p + G(k)\Psi(k)\Psi(k)^T G(k)^T]^{-1} \quad (31)$$

By defining a $p \times n$ dimensional matrix h as

$$h = G(k)\Psi(k) = [h_1, h_2, \cdots, h_p]^T \quad (32)$$

and by using the matrix inversion lemma, $H^T H$ can be written as

$$H^T H = [I_p - h(I_n + h^T h)^{-1} h^T] \quad (33)$$

where an inversion of an $n \times n$ matrix is required instead of an inversion of a $p \times p$ matrix ($p \gg n$). The inversion part in Eq. 33 can be rewritten as

$$\begin{aligned} I_n + h^T h &= I_n + h_1 h_1^T + \cdots + h_p h_p^T \\ &\equiv S_{p+1}^{-1} \end{aligned} \quad (34)$$

Hence, the following recursive relationship for S_{m+1}^{-1} can be obtained as

$$S_{m+1}^{-1} = S_m^{-1} + h_m h_m^T, \quad m = 1, \cdots, p \quad (35)$$

where $S_o^{-1} = I_n$.

It is noted that Eq. 35 is in the similar expression to Eq. 17. Therefore, by applying the single regression square root algorithm, S_{m+1} can be efficiently evaluated by performing inversions of scalar quantities as

$$S_{m+1} = S_m - \frac{S_m h_m h_m^T S_m}{1 + h_m^T S_m h_m}, \quad m = 1, \cdots, p \quad (36)$$

Then, Eq. 33 can be evaluated as

$$H^T H = I_p - h S_{p+1} h^T \quad (37)$$

EXAMPLE ANALYSIS AND DISCUSSIONS

In order to investigate the effectiveness of the techniques presented in this study, example studies are carried out for an idealized case with 2 degrees of freedom as in Fig. 1. Artificial time histories of two excitations and two acceleration responses are generated based on the assumed exact ARMAX parameters listed in Tables 1-3, and the simulated records are shown in Fig. 2. Output noises with intensities of 10 % of the response components in the RMS values are added onto the responses duration the simulation process. The parameters are estimated by the sequential algorithm incorporating the techniques described in the previous sections, and the results are compared with the assumed exact values in Tables 1 - 3.

From Table 1, it can be observed that, by using the exponential data weighting technique ($a(0) = 0.97, \alpha = 0.99$), the better estimates can be obtained. The square root estimation technique has been used in this case. In Table 2, the effectiveness of the weighted global iteration technique has been examined. In this case, the exponential data weight technique is not utilized, while the square root estimation algorithm is used. Improvement of the estimated results can be also found, if the global data weighting technique is employed. In Table 3, the effectiveness of the square root estimation is shown. The exponential and the global data weighting techniques are utilized in this case. It is noted that divergence problems have been experienced at the third global iteration, if the square root estimation algorithm is not employed. But the estimates obtained by using the square root algorithm converge to the exact values without such problems.

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Table 1. Effect of Exponential Data Weighting

Parameters	Exact Values		Estimated Values			
			Not Used		Used	
P_1	1.819	0.165	1.826	0.172	1.820	0.165
	0.033	1.871	0.034	1.857	0.033	1.868
P_2	-0.963	-0.022	-0.973	-0.032	-0.965	-0.022
	-0.004	-0.971	-0.005	-0.961	-0.004	-0.968
Q_0	0.400	0.000	0.404	0.004	0.399	-0.001
	0.000	0.500	0.002	0.507	-0.002	0.499
Q_1	-0.771	-0.036	-0.780	-0.044	-0.769	-0.035
	-0.006	-0.976	-0.019	-0.978	-0.003	-0.973
Q_2	0.371	0.036	0.381	0.044	0.370	0.036
	0.006	0.476	0.016	0.471	0.004	0.472
H_1	-1.819	-0.165	-1.268	-0.070	-1.751	-0.181
	-0.033	-1.871	-0.146	-1.225	-0.072	-1.786
H_2	0.963	0.022	0.565	0.263	0.897	0.048
	0.004	0.971	0.066	0.456	0.044	0.885

Note : Number of data points = 1000 ; No global iteration.

Table 2. Effect of Global Data Weighting

Parameters	Exact Values		Estimated Values			
			First Iteration		Fifth Iteration	
P_1	1.819	0.165	1.793	0.206	1.819	0.161
	0.033	1.871	0.013	1.975	0.033	1.869
P_2	-0.963	-0.022	-0.942	-0.053	-0.964	-0.018
	-0.004	-0.971	-0.006	-1.049	-0.005	-0.967
Q_0	0.400	0.000	0.468	-0.121	0.401	-0.002
	0.000	0.500	0.015	0.538	-0.005	0.504
Q_1	-0.771	-0.036	-0.849	0.132	-0.772	-0.032
	-0.006	-0.976	0.024	-1.146	0.000	-0.979
Q_2	0.371	0.036	0.398	0.008	0.372	0.032
	0.006	0.476	-0.002	0.692	0.005	0.475
H_1	-1.819	-0.165	-1.398	0.108	-1.848	-0.161
	-0.033	-1.871	0.251	-1.327	-0.005	-1.877
H_2	0.963	0.022	0.573	-0.097	0.985	0.019
	0.004	0.971	-0.220	0.726	-0.016	0.973

Note : Number of data points = 300

Table 3. Effect of Square-Root Estimation

Parameters	Exact Values		Estimated Values			
			Not Used		Used	
P_1	1.819	0.165	1.849	0.247	1.819	0.164
	0.033	1.871	-0.041	1.639	0.033	1.875
P_2	-0.963	-0.022	-0.975	-0.089	-0.964	-0.019
	-0.004	-0.971	0.100	-0.637	-0.005	-0.971
Q_0	0.400	0.000	0.414	-0.093	0.399	-0.001
	0.000	0.500	0.009	0.513	-0.005	0.505
Q_1	-0.771	-0.036	-0.792	-0.057	-0.771	-0.035
	-0.006	-0.976	-0.005	-0.893	0.001	-0.978
Q_2	0.371	0.036	0.398	0.067	0.373	0.035
	0.006	0.476	0.006	0.309	0.002	0.483
H_1	-1.819	-0.165	-1.287	-0.072	-1.812	-0.181
	-0.033	-1.871	0.538	-1.074	-0.031	-1.878
H_2	0.963	0.022	0.502	-0.093	0.961	0.035
	0.004	0.971	-0.192	0.639	0.002	0.972

Note : Number of data points = 300 ; 10 global iterations

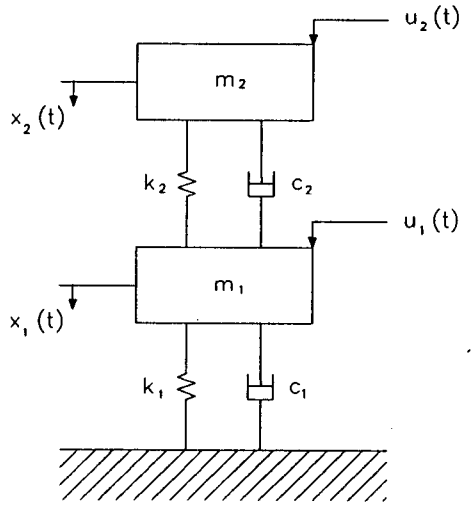

Fig. 1. Example Case with 2-DOF

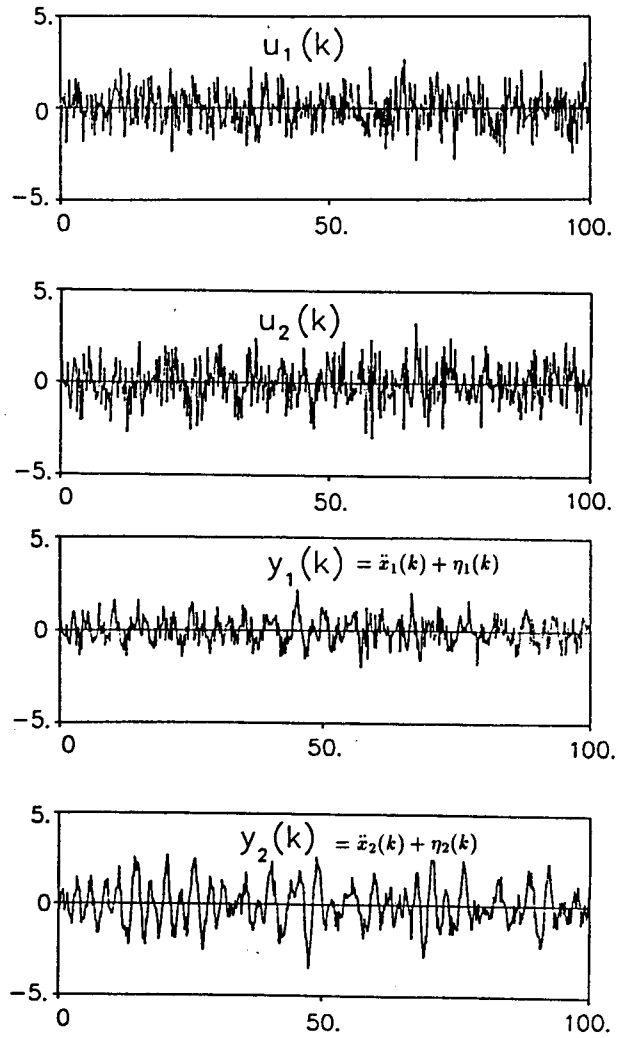

Fig. 2. Simulated Time Histories for the excitation and the acceleration response