Inertia Space에서 우주 로봇의 적응제어

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Adaptive Control of Space Robot in Inertia Space

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Abstract

In this paper, dynamic modeling and adaptive control problems for a space robot system are discussed. The space robot consists of a robot manipulator mounted on a free-floating base where no attitude control is applied. Using an extended robot model, the entire space robot can be viewed as an under-actuated robot system. Based on nonlinear control theory, the extended space robot model can then be decomposed into two subsystems: one is input-output exactly linearizable, and the other is unlinearizable and represents an internal dynamics. With this decomposition, a normal form-augmentation approach and an augmented state-feedback control are proposed to facilitate the design of adaptive control for the space robot system against parameter uncertainty, unknown dynamics and unmodeled payload in space applications. We demonstrate that under certain conditions, the entire space robot can be represented as a full-actuated robot system to avoid the inclusion of internal dynamics. Based on the dynamic model, we propose an adaptive control scheme using Cartesian space representation and demonstrate its validity and design procedure by a simulation study.

1 Introduction

An increasing research interest has been directed to space robotics in both research and experiments, such as space robot dynamics, modeling, control, human-machine interface, high-level planning and sensing systems. This is because robotic technology is beneficial for space exploration in various aspects. First, due to inhospitable environment in space, the use of robots can minimize the risk that astronants may face. Second, the use of robots can greatly reduce the extra-vehicular activity of astronants and thus increase the productivity of the mission. Third, the use of robots offers high playload capacity and operating dexterity that astronants cannot provide, and possibilities to work on the experiments that are sensitive to human contaminations. Last, the use of robots provides an economical solution by eliminating various needs of human facility and costly insurance.

The use of robots in space, however, is a challenge problem in controlling both the robot and space vehicle which could be spacecraft, space station, or satellite, and is referred to as the "base" in this paper. Due to the dynamic interaction of the robot and the base where the robot arm is mounted, the motion of the robot may alter the base trajectory. On the other hand, the robot end-effector may miss the desired target due to the motion of the base, especially when the mass and inertia moment of the robot arm are not negligible in comparison to the base.

Various research has been directed to the dynamics and modeling problems of the space robot system. Longman [1] discussed the kinematic relationship in joint and inertia space and workspace of a space robot. Vafa and Dubowsky [2], and Papadojoulous and Dubowsky [3] introduced the concept of virtual manipulator to represent the dynamics of a space robot and made it possible to reproduce the kinematic behavior of a space robot by the kinematics of a modified fixed-base robot.

Relatively little attention has been made to the control of the space robot system. Xu et al. [4] discussed the dynamic properties of the space robot system and found that dynamics of the space robot system is nonlinearly parameterized. This causes fundamental difference between the space robot and the fixed-base robot, and results in infeasibility of most adaptive control and nonlinear control schemes currently used in robot control. In their other paper [5], they proposed an adaptive control scheme for a space robot system when the base is attitude-controlled. For the case of no attitude-controlled base, nonlinear parameterization problem must be considered and feasible control scheme is demanded against unmodeled and unknown dynamics which is highly possible in space applications.

This paper will focus on the dynamic modeling and adaptive control problems of the space robot system with a free-floating base. We use input-output exact linearization theory to model and decouple the entire space robot system, and conclude that the fundamental difference between the space robot system and the fixed-base robot lies on the existence of internal dynamics. Based on the model, we investigate the state-feedback control and adaptive control problems for the space robot system in a great detail. Starting from a general robot system analysis, we propose an extended robot model including the base as a pseudo-robot and the real robot arm mounted on the base. Using this model, the entire robot system can be viewed as an under-actuated robot system with respect to the virtual fixed frame on orbit. Then, the model is decomposed into two subsystems, one is input-output linearizable, and the other is unlinearizable and represents an internal dynamics of the system. Based on the existence of non-trivial internal dynamics in the space robot system, we demonstrate the nonlinear parameterization of the robot dynamics.

To overcome the nonlinear parameterization problem, a normal form-augmentation approach to the space robot control system is proposed. Using this approach, we further develop an augmented state-feedback control method to facilitate the adaptive control design for the space robot system subject to parameter uncertainty. We propose an adaptive control scheme using Cartesian space representation to comply with customary task specification and planning in most space applications. To show the feasibility of the proposed approach, a simulation study is presented and the design procedure of the adaptive controller is illustrated.

2 Dynamic Equation for General Robot Systems

A robot manipulator is a typical high-dimensional nonlinear dynamic system. To derive its explicit dynamic equation, we need first to determine the total kinetic energy K of the robot system along a motion trajectory. If $q = (q_1 \cdots q_n)^T \in \Re^n$ represents the joint position for an n-joint robot manipulator, then the kinetic energy is a quadratic form of joint velocities contained in $\dot{q}$, i.e.,

$$K = \frac{1}{2}\dot{q}^T W \dot{q}. \tag{1}$$

The n by n matrix W in (1) is called the *inertial matrix* of the robot system in joint space representation, and is always positive-definite and symmetric.

It is well known that motion of a closed dynamic system obeys the law of searching for the shortest path between two distinct terminal points on the system constraint surface. Mathematically, this follows *geodesic equation* [6,7]. In dynamics, the motion of the dynamic system is governed by Lagrange equation. Lagrange equation is based on the *principle of the minimum action* which is defined by an integral of the scalar Lagrangian function $L(q(t), \dot{q}(t)) = K - P$, the difference between the kinetic energy and potential energy of the dynamic system.

Since space robot systems work under zero-gravity environment, and if no elastic structure is involved, the potential energy $P = 0$, and $L = K$. In this case, the Lagrange equation can be written as

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{q}}\right) - \frac{\partial K}{\partial q} = \tau, \tag{2}$$

where $\tau = (\tau_1, \cdots, \tau_n)^T$ is the external torque/force vector. Substituting (1) into (2), we obtain

$$W\ddot{q} + \dot{W}\dot{q} - \frac{1}{2}W_d\dot{q} = \tau \tag{3}$$

with an n by n matrix W_d defined by

$$W_d = (I \otimes \dot{q}^T)\frac{\partial W}{\partial q} = \begin{pmatrix} \dot{q}^T \frac{\partial W}{\partial q_1} \\ \vdots \\ \dot{q}^T \frac{\partial W}{\partial q_n} \end{pmatrix}. \tag{4}$$

In (4), I is the n by n identity matrix, and $A \otimes B$ is Kronecker product of two arbitrary matrices A and B. Namely, if A is a k by l matrix $\{a_{ij}\}$ and B is an m by n matrix, then

$$A \otimes B = \begin{pmatrix} a_{11}B & \cdots & a_{1l}B \\ \vdots & \cdots & \vdots \\ a_{k1}B & \cdots & a_{kl}B \end{pmatrix}$$

which is a km by ln matrix. It can also be seen that $(A \otimes B)^T = A^T \otimes B^T$.

Since the inertial matrix W of the robot system depends only on the position q, its time-derivative must be

$$\dot{W} = \sum_{i=1}^{n} \frac{\partial W}{\partial q_i}\dot{q}_i = \left(\frac{\partial W}{\partial q}\right)^T (\dot{q} \otimes I). \tag{5}$$

Comparing (5) with the transpose of W_d, i.e.,

$$W_d^T = \left(\frac{\partial W}{\partial q}\right)^T (I \otimes \dot{q}),$$

it is clear that $\dot{W} \neq W_d^T$ because $I \otimes \dot{q} \neq \dot{q} \otimes I$. However, we can show by some manipulation that

$$(I \otimes \dot{q})\dot{q} \equiv (\dot{q} \otimes I)\dot{q}.$$

Therefore,

$$\dot{W}\dot{q} = W_d^T \dot{q}. \tag{6}$$

Using the above result, the Lagrange equation (3) can be reduced to

$$W\ddot{q} + (W_d^T - \frac{1}{2}W_d)\dot{q} = \tau, \tag{7}$$

This version allows one to determine the entire dynamics by computing only two matrices W and W_d, and reduce the considerable computation time of $\dot{W}$.

By (6), we can rewrite $W_d^T\dot{q} = \frac{1}{2}\dot{W}\dot{q} + \frac{1}{2}W_d^T\dot{q}$. Substituting it into (7) yields

$$W\ddot{q} + \frac{1}{2}\dot{W}\dot{q} + \frac{1}{2}(W_d^T - W_d)\dot{q} = W\ddot{q} + C\dot{q} = \tau, \tag{8}$$

where we define

$$C = \frac{1}{2}\dot{W} + \frac{1}{2}(W_d^T - W_d). \tag{9}$$

It is clearly seen that the first term $\dot{W}$ in (9) is symmetric, and the second term $\frac{1}{2}(W_d^T - W_d)$ is skew-symmetric. Due to the property of skew-symmetry, it is true that for an arbitrary vector $z \in \Re^n$,

$$z^T(W_d^T - W_d)z \equiv 0.$$

Using this identity, we can show that for any $z \in \Re^n$,

$$z^T C z = \frac{1}{2} z^T \dot{W} z. \tag{10}$$

From equation (7), it is also important that the dynamic equation is linear with respect to the inertial matrix W, i.e., the robot dynamics is linearly parameterized. This important property enables us to design adaptive control against parameter uncertainty based on linear structure. For the space robot system, however, this property is no longer valid, as will be addressed in later sections.

To determine the inertial matrix W of the robot system in joint space with a certain reference frame, we may first derive the system kinetic energy K which, based on the definition in (1), must be a quadratic form of all components of $\dot{q}$. Then, combining all coefficients of the quadratic formula, we can find the inertial matrix W. An alternative way that is more efficient for computer programming utilizes the following compact formula [9] and [10]:

$$W = \sum_{j=1}^{n} J_j^T U_j J_j. \tag{11}$$

In (11), U_j is the 6 by 6 *generalized mass matrix* for link j of an n-joint robot and is defined by

$$U_j = \begin{pmatrix} m_j I_3 & m_j C_j^T \\ m_j C_j & \Phi_j \end{pmatrix}, \quad j = 1, \cdots, n, \tag{12}$$

where m_j is the mass of link j, Φ_j is the inertia tensor of link j with respect to the origin of the jth coordinate frame, I_3 is the 3 by 3 identity, and $C_j = (c_j \times)$ is a cross-product operator of the centroid position vector $c_j = (c_j^x \; c_j^y \; c_j^z)^T$ referred to the jth frame and is defined by

$$C_j = (c_j \times) = \begin{pmatrix} 0 & -c_j^z & c_j^y \\ c_j^z & 0 & -c_j^x \\ -c_j^y & c_j^x & 0 \end{pmatrix}$$

The matrix J_j in (11) is a 6 by n Jacobian matrix of link j, referred to as *subjacobian* J_j [9]. The subjacobian matrix can be determined via the following procedure:

1. For the first j links out from the n-link robot, we form the jth *submanipulator* with j joints from joint number 1 through joint number j, e.g., the nth submanipulator will be the robot itself;

2. Using Cartesian velocity defined by $\begin{pmatrix} v_j \\ \omega_j \end{pmatrix}$, the linear velocity v_j and the angular velocity ω_j of frame j, determine the 6 by j Jacobian matrix of the jth submanipulator which must be projected onto frame j;

3. Augment the resultant Jacobain matrix by post-adding a 6 by $(n-j)$ zero matrix to form a 6 by n matrix that is called the subjacobian J_j.

3 Extended Robot Model for the Space Robot System

We discussed the inertial matrix and derived the Lagrange formulation of dynamic equation for a general robot system in the last section. In this section, we will model a space robot system as an **extended robot** model to facilitate the complexity of space robot system modeling as well as revealing intrinsic properties of internal dynamics.

Consider a space robot system that is composed of a base which could be a space station or a space shuttle floating in space and an m-joint robot arm mounted on the base. If the reference orbit of the space station or the shuttle is considered as a *virtual fixed base*, then the space station or shuttle has six degrees of freedom: three for translation of its centroid and three for rotation about the reference frame. Therefore, we can view the base as the end-effector of a 6-joint serial chain mechanism, or "6-joint pseudo robot". In this way, the floating base possesses six independent displacements with respect to the fixed base. Incorporating this 6-joint pseudo-robot with the connected m-joint real robot arm, we constitute an extended robot model having totally $m + 6 = n$ joints for the space robot system. This extended robot model has its *fixed* base on the space orbit and its end-effector that performs desired tasks referred to the fixed base. Therefore, the extended robot model of the space robot system is identical to the regular earth-based robot except that the number of actuated joints is less than the number of its total joints. We can therefore categorize the space robot to be an **under-actuated robot system**.

3.1 The Inertial Matrix of the Space Robot System

With the extended robot model, the inertial matrix W of the space robot system can readily be determined by using (11). Since the extended robot system consists of 6 degrees of freedom (d.o.f.) floating base and m ($m = n - 6$) joint real robot arm, the n by n inertial matrix W can accordingly be partitioned into four blocks,

$$W = \begin{pmatrix} W_{bb} & W_{br} \\ W_{rb} & W_{rr} \end{pmatrix}, \tag{13}$$

where W_{bb} is the 6 by 6 symmetric submatrix attributed by the floating base, W_{rr} is the m by m symmetric block as an inertial matrix of the real robot arm with respect to the fixed base, and $W_{br} = W_{rb}^T$ is the 6 by m submatrix representing the interaction between the floating base and the robot arm.

Using the partitioned form (13), we can write the inversion of the matrix W by [11,12]

$$W^{-1} = \begin{pmatrix} W_{bb}^{-1} + W_{bb}^{-1}W_{br}\tilde{W}_{rr}^{-1}W_{br}^T W_{bb}^{-1} & -W_{bb}^{-1}W_{br}\tilde{W}_{rr}^{-1} \\ -\tilde{W}_{rr}^{-1}W_{br}^T W_{bb}^{-1} & \tilde{W}_{rr}^{-1} \end{pmatrix}, \tag{14}$$

where

$$\tilde{W}_{rr} = W_{rr} - W_{br}^T W_{bb}^{-1} W_{br}. \tag{15}$$

It can be seen that W is invertible if both W_{bb} and $\tilde{W}_{rr}$ are nonsingular. W_{bb}, as an upper-left block of (11), is positive-definite, symmetric, and thus invertible. Whereas for the matrix $\tilde{W}_{rr}$ in (15) which is referred to as the **effective inertial matrix** of the robot arm, we can show that it is also a positive-definite symmetric matrix. To this end, first let

$$T_{br} = \begin{pmatrix} -W_{bb}^{-1}W_{br} \\ I \end{pmatrix}. \tag{16}$$

The product of W and T_{br} becomes

$$WT_{br} = \begin{pmatrix} O \\ \tilde{W}_{rr} \end{pmatrix}. \tag{17}$$

Thus, the matrix T_{br} defined in (10) is virtually a right-transformation operator mapping W to $\tilde{W}_{rr}$. Then, premultiplying (17) by T_{br}^T, we immediately have

$$T_{br}^T W T_{br} = \tilde{W}_{rr}.$$

Since T_{br} contains an m by m identity matrix located at the bottom position of (16), $\text{rank}(T_{br}) = m$, i.e., T_{br} is always full-ranked. Now, for an arbitrary non-zero vector $\zeta \in \Re^m$, let $z = T_{br}\zeta \in \Re^n$. Obviously, $z \neq 0$ also. Because $W > 0$ is always true, $z^T W z = \zeta^T \tilde{W}_{rr} \zeta > 0$. Therefore, we have shown that the effective inertial matrix $\tilde{W}_{rr}$ is also positive-definite, symmetric, and is thus always invertible.

3.2 The Jacobian Matrix of the Space Robot System

The kinematic relationship of the space robot system can also be developed based on the extended robot model. Suppose the m-dimensional Cartesian displacement of the robot end-effector with respect to the fixed base is chosen as an output vector which is a differentiable function of the joint position q, and is denoted by $y = h(q) \in \Re^m$. The Jacobian matrix of y becomes

$$J = \frac{\partial h}{\partial q} = (J_b \; J_r), \tag{18}$$

where $J_b = \partial h / \partial q_b$ is of m by 6 and $J_r = \partial h / \partial q_r$ is of m by m. Likewise, we can also define an **effective Jacobian** matrix by

$$\tilde{J}_r = JT_{br} = J_r - J_b W_{bb}^{-1} W_{br}. \tag{19}$$

The definitions of the effective Jacobian matrix $\tilde{J}_r$ and the effective inertial matrix $\tilde{W}_{rr}$ show that the motion of the space robot arm mounted on the base, unlike the fixed-base robot, is determined by not only the motion itself, but also the interaction of the motion of the floating base.

Since $\dot{y} = J\dot{q}$ and $\ddot{y} = J\ddot{q} + \dot{J}\dot{q}$, we have

$$J\ddot{q} = (J_b \; J_r)\ddot{q} = \ddot{y} - \dot{J}\dot{q}. \tag{20}$$

This equation, however, cannot be uniquely solved for $\ddot{q}$, because J is now an m by $n = m + 6$ matrix. It looks like a redundant robot kinematic problem, and $\ddot{q}$ could have an infinite number of solutions. But, in the space robot case, the joint acceleration $\ddot{q}$ is also restricted by its dynamic equation. According to (8), the dynamic equation for a space robot system can be written by

$$W\ddot{q} + C\dot{q} = \begin{pmatrix} 0 \\ \tau \end{pmatrix}, \tag{21}$$

where $\tau \in \Re^m$ are joint torques only for the m-joint robot arm mounted on the floating base. Once we take the first six rows for both sides of (21), it can be observed that the interaction between the robot arm and the base follows the principle of momentum conservation, namely,

$$(W_{bb} \; W_{br})\ddot{q} = -(I \; O)C\dot{q}. \tag{22}$$

This equation just represents the dynamic constraint for $\ddot{q}$. Combining (22) with the kinematic equation (20), we can solve for $\ddot{q}$ in terms of the Cartesian output acceleration $\ddot{y}$ as long as the entire coefficient matrix of $\ddot{q}$ is nonsingular. Therefore, this n by n coefficient matrix, denoted by Q, can be suggested as an index to monitor control quality of the space robot system,

$$Q = \begin{pmatrix} W_{bb} & W_{br} \\ J_b & J_r \end{pmatrix}. \tag{23}$$

In fact, this *control quality matrix* Q is a combination of space robot dynamics and kinematics.

By calculating the determinant of Q defined in (23), we obtain [11,12]

$$\det(Q) = \det(W_{bb}) \cdot \det(J_r - J_b W_{bb}^{-1} W_{br}) = \det(W_{bb}) \det(\tilde{J}_r).$$

Since W_{bb} is always nonsingular, the invertibility of $\tilde{J}_r$ is evidently equivalent to the invertibility of Q. As will be seen in the next section, the invertibility of $\tilde{J}_r$ is required for control design. Therefore, the control quality matrix Q determines whether the control program could blow up. It has been shown by simulation study that at the kinematic singularity configuration of the robot arm, i.e., at points of $\det(J_r) = 0$, the control program may not necessarily break, since $\hat{J}_r$ can still be nonsingular at those singular points. However, considering the extreme case, in which the base is so heavy that W_{br} tends to be zero matrix and $\det(\hat{J}_r) \approx \det(J_r)$, the control program will break at the kinematic singularity. Therefore, the kinematic singularity should at least be avoided for a "safe" control process.

4 Exact Linearization and Internal Dynamics

A nonlinear autonomous dynamic system, such as a general robot system can always be modeled by the following state equation and output equation:

$$\begin{aligned} \dot{x} &= f(x) + \sum_{i=1}^{m} g_i(x)u_i = f(x) + G(x)u \\ y &= h(x), \end{aligned} \tag{24}$$

where $G(x) = (g_1(x) \cdots g_m(x))$. u and y are the system input and output, respectively, with the same dimension, i.e., $u, y \in \Re^m$. The extended robot model developed for the space robot system defines $n = 6 + m$ joint variables. The state vector x is thus a $2n$-dimensional vector $x = \binom{q}{\dot{q}}$. While the system input $u = (\tau_1 \cdots \tau_m)^T$ contains all m joint torques/forces of the robot arm if no reaction wheel is considered. Since Cartesian displacement (position and orientation) provides convenience of task specifications in most space applications, it is often adopted for definition of the output function $y = h(q)$.

With the Cartesian displacement as the output function, two possible cases must be remarked:

1. If the number of robot joints $m \le 6$, then the output y has m independent components representing m degrees of freedom of the robot end-effector with respect to its fixed base.

2. If $m > 6$, the robot is of kinematic redundancy. In this case, we may still define the output y to be an m-dimensional vector by appending $m - 6$ additional independent variables. As a matter of fact, $(m - 6)$ is the degree of redundancy.

As the output is so defined to have the same dimension as the input, based on (19), the effective Jacobian matrix $\hat{J}_r$ becomes an m by m square matrix, and is invertible if and only if the control quality matrix Q is nonsingular.

Based on the space robot dynamic equation given in (21), we can now deduce that

$$\dot{x} = \begin{pmatrix} \dot{q} \\ \ddot{q} \end{pmatrix} = \begin{pmatrix} \dot{q} \\ -W^{-1}C\dot{q} \end{pmatrix} + \begin{pmatrix} O \\ W^{-1} \end{pmatrix}\begin{pmatrix} 0 \\ u \end{pmatrix}. \tag{25}$$

This exhibits that $f(x)$ is just the first term on the right-hand side of (25), and $G(x)$ is a $2n$ by m matrix formed by the last m columns of the matrix $\begin{pmatrix} O \\ W^{-1} \end{pmatrix}$, where O is the n by n zero matrix. Based on (14) and (16), we obtain

$$G(x) = (g_1(x) \cdots g_m(x)) = \begin{pmatrix} O \\ T_{br}\hat{W}_{rr}^{-1} \end{pmatrix}.$$

In order to study a multi-input/multi-output (MIMO) nonlinear system, first thing is to find a *relative degree*, r, of the system at a point x^0. The relative degree reveals how high order of output time-derivatives is at least required for uniquely inverting the system. In general, each output channel has its individual value of the relative degree so that $r = \{r_1, \cdots, r_m\}$ if the system has totally m output channels. The formal definition of the relative degree for a MIMO system of the state equation (24) is as follows [13,14]:

Definition 1 *$r = \{r_1, \cdots, r_m\}$ is a relative degree (vector) of the system if*

1. for all $1 \le j \le m$, $1 \le i \le m$ and all $k < r_i - 1$, and for all x in a neighborhood of x^0, Lie derivatives

$$L_{g_j}L_f^k h_i(x) = 0,$$

2. and the m by m matrix

$$D(x) = \begin{pmatrix} L_{g_1}L_f^{r_1-1}h_1(x) & \cdots & L_{g_m}L_f^{r_1-1}h_1(x) \\ \cdots & \cdots & \cdots \\ L_{g_1}L_f^{r_m-1}h_m(x) & \cdots & L_{g_m}L_f^{r_m-1}h_m(x) \end{pmatrix}$$

is nonsingular at $x = x^0$.

In the above definition, *Lie derivative* for a scalar function $h_i(x)$ along a vector field η is defined via a dual product between the gradient of h_i and the vector η, i.e., $L_\eta h_i(x) = \frac{\partial h_i}{\partial x}\eta$. The matrix $D(x)$ is often called the *decoupling matrix* [14,15].

For a space robot system, if Cartesian displacement of the robot end-effector is chosen as an output vector, intuitively each output channel reflects an independent degree of freedom and all channels are at equal levels. We may therefore claim that $r_1 = r_2 = \cdots = r_m$. Under the equal-relative degree condition, we can globally find all r_i's for the robot system. Let us first extend Lie derivative notation for a vector $h(q) = (h_1(q) \cdots h_m(q))^T$ to

$$L_\eta h(q) = \begin{pmatrix} L_\eta h_1(q) \\ \vdots \\ L_\eta h_m(q) \end{pmatrix}.$$

It can be seen that each $L_{g_j}h(q) = 0$, because in each column g_j of $G(x)$, the top n components are all zero, while in the gradient of each $h_i(q)$, the last n components are zero. Furthermore, since $L_f h(q) = (J \;\; O)f = J\dot{q}$,

$$(L_{g_1}L_f h(q) \cdots L_{g_m}L_f h(q)) = \left(\tfrac{\partial J\dot{q}}{\partial q} \;\; \tfrac{\partial J\dot{q}}{\partial \dot{q}}\right)G(x) = JT_{br}\hat{W}_{rr}^{-1} = \hat{J}_r\hat{W}_{rr}^{-1}.$$

The resultant m by m matrix is actually the decoupling matrix $D(x)$ for the robot system. Clearly, if $\det(Q) \ne 0$, then $D(x) = \hat{J}_r\hat{W}_{rr}^{-1}$ is nonsingular so that the relative degree $r_1 = \cdots = r_m = 2$. Therefore, each output channel of the space robot system has the relative degree of two. This also explains why it is always required to know up to the desired output acceleration in trajectory-tracking control design.

In a space robot system, the extended robot model possesses n joint variables, and the state vector is $2n$-dimensional, while the input $u = \tau$ and output $y = h(q)$ are both $m = (n-6)$-dimensional vectors. Because $r_i = 2$ in each output channel, based on nonlinear control system theory [14,15], there can be a $2m$-dimensional subsystem which is *input-output exactly linearizable*, while the remaining $2n - 2m = 12$-dimensional subsystem is unlinearizable by the input-output procedure. Such an unlinearizable subsystem is referred to as the *internal dynamics*. Unless the base is also driven by attitude thrust jets or gyroscopic devices, the internal dynamics for the space robot system has 12 dimensions. For the $2m$-dimensional linearizable portion, we may define a new state vector $\zeta = \binom{y}{\dot{y}}$, and the new dynamic equation has *Brunovski canonical form* shown as follows:

$$\dot{\zeta} = \begin{pmatrix} \dot{y} \\ \ddot{y} \end{pmatrix} = \begin{pmatrix} O & I \\ O & O \end{pmatrix}\begin{pmatrix} y \\ \dot{y} \end{pmatrix} + \begin{pmatrix} O \\ I \end{pmatrix}v, \tag{26}$$

where I and O are the m by m identity and zero matrices, respectively, and $v \in \Re^m$ is the new input vector. Clearly, this linear equation is equivalent to $\ddot{y} = v$. Moreover, if knowledge on all system parameters is available, the real robot input u can be resolved in terms of the new input v,

$$u = D^{-1}(x)(v - b(x)) = \alpha(x) + \beta(x)v, \tag{27}$$

where $b(x)$ is an m-dimensional vector defined by $b(x) = L_f^2 h(x)$, and $\alpha(x) = -D^{-1}(x)b(x)$ and $\beta(x) = D^{-1}(x)$. Equation (27) is known as *static state-feedback control* [14].

To determine the state-feedback coefficients $\alpha(x)$ and $\beta(x)$, it is needed to obtain up to the second order of Lie derivatives in $D(x)$ and $b(x)$. We may use an alternative way to derive the static feedback (27) through the dynamic equation (8) to get rid of the direct computation of all Lie derivatives. Namely, according to (8) and (25), we have

$$J\ddot{q} + JW^{-1}C\dot{q} = JW^{-1}\begin{pmatrix} 0 \\ u \end{pmatrix} = \hat{J}_r\hat{W}_{rr}^{-1}u = D(x)u. \tag{28}$$

Using $v = \ddot{y} = J\ddot{q} + \dot{J}\dot{q}$ and substituting $J\ddot{q} = v - \dot{J}\dot{q}$ into (28) result in

$$\begin{aligned} \alpha(x) &= \hat{W}_{rr}\hat{J}_r^{-1}(JW^{-1}C\dot{q} - \dot{J}\dot{q}) \\ \beta(x) &= \hat{W}_{rr}\hat{J}_r^{-1}. \end{aligned} \tag{29}$$

Their counterparts, in a full-actuated fixed-base robot system that is exactly linearizable, can be determined by

$$\begin{aligned} \alpha_s(x) &= C\dot{q} - WJ^{-1}\dot{J}\dot{q} \\ \beta_s(x) &= WJ^{-1}. \end{aligned} \tag{30}$$

Comparing (29) to (30), and noting the definitions of $\hat{W}_{rr}$ and $\hat{J}_r$ in (15) and (19), we find that for a space robot system, due to the existence of internal dynamics, the property of linear parameterization in $\alpha(x)$ and $\beta(x)$ is now no longer valid. The fact of the above nonlinearized parameterization has been revealed by authors in [5] and here we demonstrate this fact in a different angle. The nonlinear parameterization poses a tough problem to future adaptive control design, because most of the existing algorithms in nonlinear adaptive control area are based on the linearity of parameterization. To overcome the parameter-nonlinearity problem, we will develop a normal form-augmentation approach in the next section.

5 Normal Form-Augmentation Approach and Augmented State-Feedback Control Model

Since Cartesian displacement of a robot end-effector can be specified by tasks in most space applications, it is ideal to be chosen as a system output function $y = h(q) \in \Re^m$. Furthermore, $2n - 2m$ variables are unobservable, and constitute the states of the internal dynamics. The six joint positions in q_b of the floating base along with their time-derivatives may be the best choice of states to represent the internal dynamics. Therefore, we now define an augmented output vector $y_a = \binom{y}{q_b} \in \Re^n$, and its time-derivative becomes

$$\dot{y}_a = \begin{pmatrix} \dot{y} \\ \dot{q}_b \end{pmatrix} = \begin{pmatrix} J_b & J_r \\ I & O \end{pmatrix}\begin{pmatrix} \dot{q}_b \\ \dot{q}_r \end{pmatrix} = J_{eq}\dot{q}, \tag{31}$$

where I is the 6 by 6 identity matrix and O is the 6 by m zero matrix. The n by n square Jacobian matrix J_{eq} defined in (31) can be inverted by

$$J_{eq}^{-1} = \begin{pmatrix} O & I \\ J_r^{-1} & -J_r^{-1}J_b \end{pmatrix} \tag{32}$$

if J_r in $J = (J_b \;\; J_r)$ is nonsingular. Using $\ddot{y}_a = J_{eq}\ddot{q} + \dot{J}_{eq}\dot{q}$ and substituting $\ddot{q} = J_{eq}^{-1}(\ddot{y}_a - \dot{J}_{eq}\dot{q})$ into (21), we obtain

$$WJ_{eq}^{-1}\ddot{y}_a - WJ_{eq}^{-1}\dot{J}_{eq}\dot{q} + C\dot{q} = \begin{pmatrix} 0 \\ u \end{pmatrix}. \tag{33}$$

Premultiplying (33) by $J_{eq}W^{-1}$ yields

$$\ddot{y}_a - \dot{J}_{eq}\dot{q} + J_{eq}W^{-1}C\dot{q} = J_{eq}W^{-1}\begin{pmatrix} 0 \\ u \end{pmatrix}. \tag{34}$$

The above equation can be decomposed into two parts

$$\ddot{y} - \dot{J}\dot{q} + JW^{-1}C\dot{q} = JW^{-1}\begin{pmatrix} 0 \\ u \end{pmatrix} = \hat{J}_r\hat{W}_{rr}^{-1}u, \quad \text{and} \tag{35}$$

$$\ddot{q}_b + (I \;\; O)W^{-1}C\dot{q} = (I \;\; O)W^{-1}\begin{pmatrix} 0 \\ u \end{pmatrix}. \tag{36}$$

Clearly, (35) represents the linearizable subsystem of the space robot system, while (36) describes the internal dynamics. If the static state-feedback control law (27) with (29) is applied to the subsystem (35), it can be immediately obtained that $\ddot{y} = v$, provided that link parameters are known. Therefore, if we define $e(t) = y_d(t) - y(t)$, the output error between the desired output $y_d(t)$ and the actual trajectory $y(t)$, and then define

$$v = \ddot{y}_d + k_v\dot{e} + k_p e, \tag{37}$$

the dynamics of the linearizable subsystem of the space robot is equivalent to

$$\ddot{e} + k_v \dot{e} + k_p e = 0, \qquad (38)$$

where k_v and k_p are constant control gains. Obviously, k_v and k_p should be chosen such that the linear error equation (38) is Hurwitz, i.e., all roots of the corresponding characteristic equation have negative real parts.

The internal dynamic equation (36) derived by the above augmentation way reveals the dynamic behavior of the base interacted by the robot arm due to the principle of momentum conservation. This portion is unlinearizable and q_b is unobservable through the output channels defined at Cartesian level. Since $q = \begin{pmatrix} q_b \\ q_r \end{pmatrix}$, using $\ddot{q}_b = (I\ O)\ddot{q}$, we can see that (36) is actually the first six components of the entire dynamic equation. Namely,

$$(I\ O)(\ddot{q} + W^{-1}C\dot{q}) = (I\ O)W^{-1}\begin{pmatrix} 0 \\ u \end{pmatrix}. \qquad (39)$$

Since the complete set of equations including the linearized portion and the internal dynamics portion is called the *normal form* [14,15], we refer to the definitions of y_a and J_{eq} and the derivation of equation (33) as a **normal form-augmentation approach.**

By the concept of the augmented square Jacobian matrix J_{eq}, we can further define a **Cartesian inertial matrix** as follows,

$$M = J_{eq}^{-T} W J_{eq}^{-1}, \qquad (40)$$

where $(\cdot)^{-T} = [(\cdot)^{-1}]^T = [(\cdot)^T]^{-1}$ for any nonsingular square matrix $(\cdot)$. Thus, if J_{eq} is nonsingular, the dynamic equation (33) of the space robot system can be rewritten as

$$M\ddot{y}_a - M\dot{J}_{eq}\dot{q} + J_{eq}^{-T}C\dot{q} = J_{eq}^{-T}\begin{pmatrix} 0 \\ u \end{pmatrix}. \qquad (41)$$

Substituting the inverse kinematics $\dot{q} = J_{eq}^{-1}\dot{y}_a$ into (41), we obtain a dynamic equation in terms of the augmented velocity $\dot{y}_a$ and acceleration $\ddot{y}_a$,

$$M\ddot{y}_a - M\dot{J}_{eq}J_{eq}^{-1}\dot{y}_a + J_{eq}^{-T}CJ_{eq}^{-1}\dot{y}_a = M\ddot{y}_a + G\dot{y}_a = J_{eq}^{-T}\begin{pmatrix} 0 \\ u \end{pmatrix}, \qquad (42)$$

where

$$G = J_{eq}^{-T}CJ_{eq}^{-1} - M\dot{J}_{eq}J_{eq}^{-1}. \qquad (43)$$

Based on the dynamic equation (42) represented in Cartesian space for the space robot system, we can see that if one defines a new augmented input

$$v_a = \ddot{y}_a = \begin{pmatrix} \ddot{y} \\ \ddot{q}_b \end{pmatrix}, \qquad (44)$$

then the static state-feedback control can be considered as

$$\begin{pmatrix} 0 \\ u \end{pmatrix} = J_{eq}^T[M\ddot{y}_a + (-M\dot{J}_{eq}J_{eq}^{-1} + J_{eq}^{-T}CJ_{eq}^{-1})\dot{y}_a] = J_{eq}^T(Mv_a + G\dot{y}_a). \qquad (45)$$

Obviously, to realize the above **augmented state-feedback control model** associated with the linearized augmented system given by (44) for the entire space robot, in addition to the knowledge of y_d and the invertibility of J_{eq}, the following two conditions must be met:

1. Either desired q_b, $\dot{q}_b$ and $\ddot{q}_b$ can be known, or their actual values are measurable;

2. All the top six components of the resultant vector on the right-hand side of (45) are equal to zero.

If these two conditions are satisfied, then we can *remodel* the entire space robot to be a *full-actuated robot system*. All the six joints in the pseudo robot model of the floating base can be conceived to have motor-drivers, but generate zero torques at every sampling time. With such a new model, we can utilize many conventional control algorithms that are applicable to regular (full-actuated) robot systems to deal with the space robot control problem, such as adaptive control design.

In summary, when we *extend* the input vector from $u \in \Re^m$ to $\begin{pmatrix} 0 \\ u \end{pmatrix} \in \Re^n$, and the output vector from $y \in \Re^m$ to $y_a = \begin{pmatrix} y \\ q_b \end{pmatrix} \in \Re^n$, the entire space robot system becomes a full-actuated robot system as if the internal dynamics disappears, as long as the above mentioned requirement can be satisfied.

6 Adaptive Control of the Space Robot System Using Cartesian Space Representation

As discussed in the previous sections, the state-feedback coefficients $\alpha(x)$ and $\beta(x)$ can be determined by equation (29) for a space robot system. Due to the nature of the under-actuated system, both $\alpha(x)$ and $\beta(x)$ are nonlinear functions of physical parameters of robot links. This results in difficulty for adaptive control realization against parameter uncertainty. In this section, we will demonstrate that the normal form-augmentation approach expressed in (31) and (33) and the augmented state-feedback control (45) can overcome the nonlinear parameterization problem. Let us start with the assumption that q_b, $\dot{q}_b$ and $\ddot{q}_b$ are measurable. Under this assumption, the augmented output error function between the desired $(y_a)_d = \begin{pmatrix} y_d \\ q_b \end{pmatrix}$ and the actual $y_a = \begin{pmatrix} y \\ q_b \end{pmatrix}$ can be written as $e_a = (y_a)_d - y_a = \begin{pmatrix} e \\ 0 \end{pmatrix}$. Let an *extended augmented error vector* be defined as

$$s = \dot{e}_a + k_v e_a = \begin{pmatrix} \dot{e} + k_v e \\ 0 \end{pmatrix} \in \Re^n, \qquad (46)$$

where $e = y_d - y \in \Re^m$ is the output error function, and $k_v > 0$ is the constant gain. Then, we define a reference output velocity η and a reference output acceleration $\dot{\eta}$ as follows,

$$\eta = \begin{pmatrix} \dot{y}_d + k_v e \\ \dot{q}_b \end{pmatrix} \quad \text{and} \quad \dot{\eta} = \begin{pmatrix} \ddot{y}_d + k_v \dot{e} \\ \ddot{q}_b \end{pmatrix}. \qquad (47)$$

Thus, comparing (47) with (46), we have

$$s = \eta - \dot{y}_a, \quad \text{and} \quad \dot{s} = \begin{pmatrix} \ddot{e} + k_v \dot{e} \\ 0 \end{pmatrix} = \dot{\eta} - \ddot{y}_a. \qquad (48)$$

Let us now define $E = \frac{1}{2}s^T M s$ to represent an extended error energy, and then,

$$\dot{E} = s^T M \dot{s} + \frac{1}{2}s^T \dot{M} s = s^T M \dot{\eta} - s^T M \ddot{y}_a + \frac{1}{2}s^T \dot{M} s. \qquad (49)$$

The second term on the right-hand side of (49) can be determined by (42),

$$-s^T M \ddot{y}_a = s^T G \dot{y}_a - s^T J_{eq}^{-T}\begin{pmatrix} 0 \\ u \end{pmatrix}. \qquad (50)$$

In order to derive the third term, we first take time-derivative for the Cartesian inertial matrix (40), and then substitute $\dot{M}$ into this term. Then, recalling (10), we obtain

$$\begin{aligned} \frac{1}{2}s^T \dot{M} s &= -s^T M \dot{J}_{eq} J_{eq}^{-1} s + \frac{1}{2}s^T J_{eq}^{-T} \dot{W} J_{eq}^{-1} s \\ &= -s^T M \dot{J}_{eq} J_{eq}^{-1} s + s^T J_{eq}^{-T} C J_{eq}^{-1} s \\ &= s^T G s = s^T G \eta - s^T G \dot{y}_a. \end{aligned} \qquad (51)$$

Finally, (49) turns out to be

$$\dot{E} = s^T M \dot{\eta} + s^T G \eta - s^T J_{eq}^{-T}\begin{pmatrix} 0 \\ u \end{pmatrix}. \qquad (52)$$

We now define a following control law

$$\begin{aligned} \begin{pmatrix} 0 \\ u \end{pmatrix} &= W_m J_{eq}^{-1}\dot{\eta} + (C_m J_{eq}^{-1} - W_m J_{eq}^{-1}\dot{J}_{eq}J_{eq}^{-1})\eta + J_{eq}^T\begin{pmatrix} H(\dot{e} + k_v e) \\ \delta \end{pmatrix} \\ &= J_{eq}^T[M_m \dot{\eta} + G_m \eta + \begin{pmatrix} H(\dot{e} + k_v e) \\ \delta \end{pmatrix}], \end{aligned} \qquad (53)$$

where W_m and C_m, respectively, represent the inertial matrix W and the matrix $C = \frac{1}{2}\dot{W} + \frac{1}{2}(W_d^T - W_d)$ in the *model plant*, and accordingly,

$$M_m = J_{eq}^{-T} W_m J_{eq}^{-1}$$

and

$$G_m = J_{eq}^{-T} C_m J_{eq}^{-1} - M_m \dot{J}_{eq} J_{eq}^{-1}.$$

In (53), H is an m by m positive-definite, symmetric constant weighting matrix. The vector $\delta \in \Re^6$ in the control law (53) plays a key important role in realizing the second condition for the augmented state-feedback control proposed in the last section. In fact, since

$$J_{eq}^T = \begin{pmatrix} J_b^T & I \\ J_r^T & O \end{pmatrix},$$

the control law (53) can be splitted into two portions,

$$0 = (W_{bb}\ W_{br})_m J_{eq}^{-1}\dot{\eta} + (J_b^T\ I)G_m\eta + J_b^T H(\dot{e} + k_v e) + \delta \quad \text{and} \qquad (54)$$

$$u = (W_{br}^T\ W_{rr})_m J_{eq}^{-1}\dot{\eta} + (J_r^T\ O)G_m\eta + J_r^T H(\dot{e} + k_v e). \qquad (55)$$

It is clear that since δ is *only* contained in (54), δ can simply be evaluated to ensure that (54) vanishs. Namely, the second condition of the augmented state-feedback control can always be satisfied without affecting the control command u determined by (55). Therefore, the control law (53) is feasible.

Let ξ be the parameter column vector that lists all real physical objective parameters to be identified. Let ξ_m be the corresponding parameter vector for the model plant of the space robot system. Now, substituting the control law (53) into (52), we further obtain

$$\begin{aligned} \dot{E} &= s^T[(M - M_m)\dot{\eta} + (G - G_m)\eta - \begin{pmatrix} H(\dot{e} + k_v e) \\ \delta \end{pmatrix}] \\ &= s^T Y\phi - s^T\begin{pmatrix} H(\dot{e} + k_v e) \\ \delta \end{pmatrix}, \end{aligned} \qquad (56)$$

where

$$Y\phi = (M - M_m)\dot{\eta} + (G - G_m)\eta,$$

and Y is a matrix function of q, $\dot{q}$, $\ddot{q}_b$, and y_d, $\dot{y}_d$ and $\ddot{y}_d$, and is independent of the objective physical parameters, while $\phi = \xi - \xi_m$ is the parameter deviation vector between the real plant and the model plant.

We now define an adaptation law for the space robot system,

$$\dot{\phi} = -\Gamma Y^T s, \qquad (57)$$

where Γ, referred to as the *adaptation gain matrix*, is a constant positive-definite, symmetric matrix with the same dimension as ξ or ϕ. Then, a following Lyapunov function can be adopted to justify the stability of the space robot system with the control law (53) and the adaptation law (57),

$$V_L = E + \frac{1}{2}\phi^T\Gamma^{-1}\phi = \frac{1}{2}s^T M s + \frac{1}{2}\phi^T\Gamma^{-1}\phi. \qquad (58)$$

Clearly, $V_L > 0$, and $V_L = 0$ only at the equilibrium point of this adaptive system, i.e., $\begin{pmatrix} e \\ \dot{e} \end{pmatrix} = 0$ and $\phi = 0$. Taking time-derivative for V_L, we have

$$\begin{aligned} \dot{V}_L &= \dot{E} + \dot{\phi}^T\Gamma^{-1}\phi \\ &= s^T Y\phi - s^T\begin{pmatrix} H(\dot{e} + k_v e) \\ \delta \end{pmatrix} - s^T Y\phi \\ &= -(\dot{e} + k_v e)^T H(\dot{e} + k_v e) \end{aligned} \qquad (59)$$

which is negative-definite and is zero only at the equilibrium point $\begin{pmatrix} e \\ \dot{e} \end{pmatrix} = 0$.

Therefore, the control law (53) and the adaptation law (57) *asymptotically stabilize* the entire space robot system to track a desired trajectory (or to approach a desired point, as a special case of the trajectory-tracking) described in terms of y_d, $\dot{y}_d$ and $\ddot{y}_d$. Since J_{eq}^{-1} is heavily involved in the control law and adaptation law, the stability also requires that J_r be nonsingular due to the definition of J_{eq} in (31) and its inversion (32). Whereas the feasibility of the above adaptive control law depends on whether the desired joint trajectory of the pseudo robot representing the floating base is available, or q_b, $\dot{q}_b$ and $\ddot{q}_b$ are measurable. In general, it is not so easy to determine the desired q_b and its time-derivatives *kinematically* based on a given desired Cartesian trajectory, because all joint variables are also constrained by the dynamic equation (22). Thus, with absence of the desired joint trajectory of the pseudo robot, the floating base must be equipped with sensors capable of monitoring not only the base centroid position, orientation and their velocities, but also measuring their accelerations. This may be interpreted as the *cost* we have to pay for achieving linear parameterization in order to control the space robot system adaptively against parameter uncertainty.

7 Simulation Study

We now discuss a simulation study to verify the augmented state-feedback control model and adaptive control scheme developed in previous sections. The space robot system to be simulated is a robot arm with two revolute joints mounted on a free-motion base moving on 2D plane. Since a rigid body on 2D plane possesses 3 d.o.f.: two for its centroid position and one for the orientation about the axis normal to the plane, the base can be viewed as the "end-effector" of a three-joint "pseudo robot". We consider that the first two joints of the pseudo robot are prismatic along two sliding axes that are perpendicular to each other, while the third joint is revolute about the axis normal to the 2D plane. Based on the pseudo robot model, the total mass and inertia moments of the floating base are only concentrated on the third link, and the first two links have zero masses. Combining with the two-joint planar robot arm, the entire extended robot model has totally five joints, i.e., $n = 5$ and $m = 2$.

The system input $u \in \Re^2$ contains two torques of joint 4 and joint 5. Whereas the output $y = h(q) \in \Re^2$ represents translational motion of the robot arm end-effector with respect to the orbit fixed base. A schematic diagram of the space robot system for the case study is shown in Figure 1. The Denavit-Hartenberg (D-H) table including joint variables and link length parameters of the extended robot model is also given in Figure 1.

We can derive the inertial matrix W and dynamic equation for this planar space robot system via (11), (4) and (21). As we have seen, the adaptive control scheme developed in the preceding section assumes that only the dynamic parameters appearing in the inertial matrix W are to be identified, and those geometric parameters, such as the robot link lengths a_4 and a_5 and the base centroid x-coordinate a_3 involved in the Jacobian matrix J are not included for adaptation. Moreover, in order to more efficiently illustrate the design procedure, we assume that the centroid of each robot link has only one non-zero coordinate which is along x-axis of its individual coordinate frame, denoted by $-\bar{x}_i$ for $i = 4, 5$. We also define the remaining length of each link as $l_i = a_i - \bar{x}_i$. Under such conditions, each inertia tensor Φ_i of the base and two robot links is reduced to be diagonal and only the inertia moment about z-axis will be used, i.e., $m_i k_{zi}^2$, where k_{zi} is the gyration radius about z-axis and m_i is the mass of link i.

Therefore, totally eight parameters involved in W are to be identified. The eight-dimensional parameter vector ξ can thus be defined as

$$\xi = (m_3 \; m_4 \; m_5 \; m_3k_{z3}^2 \; m_4k_{z4}^2 \; m_5k_{z5}^2 \; m_4l_4 \; m_5l_5)^T. \tag{60}$$

According to the definition of ξ, the inertial matrix W, and W_d and $\dot{W}$ can be decomposed to

$$W = \sum_{i=1}^{8} \xi_i W^i, \quad W_d = \sum_{i=1}^{8} \xi_i W_d^i \quad \text{and} \quad \dot{W} = \sum_{i=1}^{8} \xi_i \dot{W}^i, \tag{61}$$

where each W^i, W_d^i and each $\dot{W}^i$ are independent of ξ and are the coefficient matrices of each ξ_i. Once the symbolic form of each W^i is derived, the corresponding W_d^i and $\dot{W}^i$ can be computed by (4) and (5) so that the matrix C can further be found via (9).

For the space robot kinematics, the output vector is defined by using the robot arm tip position with respect to the fixed base,

$$y = h(q) = \begin{pmatrix} d_2 - a_3 s_3 - a_4 s_{34} - a_5 s_{345} \\ d_1 + a_3 c_3 + a_4 c_{34} + a_5 c_{345} \end{pmatrix}. \tag{62}$$

Taking partial derivative of $y = h(q)$ with respect to q, we obtain the Jacobian matrix of the robot,

$$J = \begin{pmatrix} 0 & 1 & -a_3c_3 - a_4c_{34} - a_5c_{345} & -a_4c_{34} - a_5c_{345} & -a_5c_{345} \\ 1 & 0 & -a_3s_3 - a_4s_{34} - a_5s_{345} & -a_4s_{34} - a_5s_{345} & -a_5s_{345} \end{pmatrix} = (J_b \;\; J_r), \tag{63}$$

where s_i and c_i are $\sin\theta_i$ and $\cos\theta_i$, s_{ij} and c_{ij} are $\sin(\theta_i + \theta_j)$ and $\cos(\theta_i + \theta_j)$, and s_{ijk} and c_{ijk} are $\sin(\theta_i + \theta_j + \theta_k)$ and $\cos(\theta_i + \theta_j + \theta_k)$, respectively, for $i, j, k = 3, 4, 5$. Once J is computed, we can form J_{sq} and J_{sq}^{-1} by (31) and (32), and further determine the 5 by 8 matrix Y required by the adaptation law (57) through (40), (43) and (56).

In the simulation study, the desired trajectory of the space robot tip point is defined to be a circle with radius $R = 1.2$ and a constant speed $\omega = 1.5$ rad./sec. in clockwise direction. We set a large initial tracking error for both y and $\dot{y}$ to simulate how the space robot control system can catch up to the desired trajectory after interruption by some disturbance. The real plant parameters in vector (60) are fixed to be

$$\xi = (10 \;\; 2 \;\; 1 \;\; 37.5 \;\; 6 \;\; 3 \;\; 3 \;\; 1.5)^T.$$

All the model plant parameters in ξ_m used for adaptive control simulation are simply defined to be 1. Moreover, the adaptation gain is defined by a following diagonal matrix:

$$\Gamma = \text{diag}(2,\ 0.1,\ 0.1,\ 4,\ 0.1,\ 0.1,\ 0.2,\ 0.2).$$

The control gains are set to be $k_v = 16$ and $H = 10$ for the simulation study.

As the simulation results, Figure 2(a) shows the desired and actual trajectories of the robot tip point A, and the traces of the base centroid C and the fourth joint center (the top point of the base) B in the case without parameter deviation. Figure 2(b) plots the same trajectories in the case with parameter deviation but without adaptive control. In Figure 3, part (a) shows the resultant trajectories with parameter adaptation, and part (b) gives the input (two joint torques) plot versus time in the same condition. The tracking errors versus time in the cases with and without adaptive control are shown in Figure 4. Figure 5 demonstrates two parameter adaptation processes in the space robot system with the adaptive control scheme. All these resultant plots verify that the proposed model and adaptive control for the space robot system are feasible and effective.

8 Conclusion

The adaptive control problem of a space robot system as the base is freely floating in space has been discussed. By applying an extended robot model on the space robot system and through a nonlinear control system analysis, we first concluded that the fundamental difference between the space robot and the fixed-base robot lies on the existence of the non-trivial internal dynamics. Under the extended robot model, the space robot system can be categorized to a class of under-actuated robot systems. Then, we applied the input-output exact linearization procedure to the space robot system, and found that a subsystem with double dimension of the input or the output is exactly linearizable, while the remaining subsystem is unlinearizable. The unlinearizable part is referred to as the internal dynamics.

It has also been shown that in the space robot system, the linear parameterization in the state-feedback control law is no longer valid due to the under-actuated nature. The proposed normal form-augmentation approach is feasible for overcoming the nonlinearity of parameterization as any adaptive control scheme is implemented for the space robot system. Based on the normal form-augmentation approach, we further developed an augmented state-feedback control model. Using this control model and under two conditions being all satisfied, the entire extended robot model of the space robot system can be remodeled as a full-actuated robot system as if the internal dynamics disappears. In this way, the parameterization nonlinearity problem can be eliminated, and the adaptive control scheme developed in the paper becomes realizable.

As indicated above, one of the two conditions is that the augmented state-feedback control model requires all joint positions, velocities and accelerations of the floating base pseudo-robot model to be measurable. This may be explained by the fact that the motion variables of the floating base constitute the states of the internal dynamics, and they are unobservable through all the output channels defined in Cartesian space.

Under the assumption of measurable base trajectories, we have shown that the developed adaptive control scheme represented in Cartesian space can asymptotically stabilize the entire space robot system to track a given trajectory or to reach a desired point. Through the simulation study, it is evidently seen that even if the mass ratio between the base and the robot arm is small, the tracking convergence can still be achieved. The control program, however, will blow up when the Jacobian block J_r approaches its singularity along the actual trajectory. Moreover, the simulation results have also shown the feasibility and effectiveness of the adaptive control scheme developed. Based on the work presented in this paper, we are currently focusing on the following research topics:

- How to reduce the computational complexity of the proposed control algorithms, specifically for real-time implementation;
- What happens if the robot and/or the base is of light-weight and presents certain degrees of flexibility for energy efficiency and orbit life concerns [16].

References

[1] R.W. Longman, "The Kinetics and Workspace of a Satellite-Mounted Robot", *Journal of Astronautical Science*, Vol.38(4), 1990.

[2] Z. Vafa and S. Dubowsky, "On the Dynamics of Space Manipulators Using the Virtual Manipulator, with Applications to Path Planning", *Journal of Astronautical Science*, Vol.38(4), 1990.

[3] E. Papadopoulos and S. Dubowsky, "On the Dynamic Singularities in the Control of Free-Floating Space Manipulators", in *Proc. of ASME Winter Ann. M.*, ASME DSC-Vol. 15, San Francisco, CA., Dec. 1989.

[4] Y.S. Xu and H.Y. Shum, "Dynamic control of a space robot system with no thrust jets controlled base", *The Robotics Institute, Carnegie-Mellon University*, No. CMU-RI-TR-91-33, 1991.

[5] Y.S. Xu, H.Y. Shum, J.J. Lee and T. Kanade, "Adaptive control of a space robot system with an attitude controlled base", *The Robotics Institute, Carnegie-Mellon University*, No. CMU-RI-TR-91-14, 1991.

[6] B.A. Dubrovin, A.T. Fomenko and S.P. Novikov, *Modern Geometry-Methods and Applications*, Springer-Verlag, New York, 1984.

[7] E. Cartan, *Geometry of Riemannian Space*, Math Sci Press, Brookline, MA., 1983.

[8] Y.L. Gu, "Modeling and Simplification for Dynamic Systems with Testing Procedures and Metric Decomposition", *Proceedings of 1991 IEEE International Conference on Systems, Man, and Cybernetics*, Charlottesville, VA., October 13-16, 1991, pp. 487-492.

[9] Y.L. Gu and N.K. Loh, "Control System Modeling for Robot Manipulators by Use of a Canonical Transformation", *Proceedings of 1987 IEEE International Conference on Robotics and Automation*, Raleigh, N. Carolina, March, 1987, pp. 484-489.

[10] Y.L. Gu and N.K. Loh, "Dynamic Modeling and Control by Utilizing an Imaginary Robot Model", *IEEE Journal of Robotics and Automation*, Vol.RA-4, No.5, October, 1988, pp. 532-540.

[11] R.A. Horn and C.A. Johnson, *Matrix Analysis*, Cambridge University Press, New York, 1985.

[12] P. Lancaster and M. Tismenetsky, *The Theory of Matrices with Application*, 2nd Edition, Academic Press, Inc., New York, 1985.

[13] S.P. Banks, *Mathematical Theories of Nonlinear Systems*, Prentice Hall, New York, 1988.

[14] A. Isidori, *Nonlinear Control Systems*, 2nd Edition, Springer-Verlag, New York, 1989.

[15] J. Slotine and W. Li, *Applied Nonlinear Control*, Prentice Hall, New Jersey, 1991.

[16] Y.S. Xu, H.Y. Shum, J.J. Lee and T. Kanade, "Control System of Self-Mobile Space Manipulator", will appear in *Proceedings of 1992 IEEE International Conference on Robotics and Automation*, Nice, France, May 10-15, 1992.

[17] H.K. Khalil, *Nonlinear Systems*, Macmillan Publishing Co., New York, 1992.