BOTTOM FRICTION OF WAVE-CURRENT FLOW ON A NATURAL BEACH

DONGHOON YOO*

INTRODUCTION

The bottom friction model of combined wave-current flow developed by Bijker, Yoo and O'Connor (hereafter it is called BYO model) is fully based on the Prandtl's mixing length theory. Although the Prandtl's theory is dependant on some rationalism, it is widely recognized that the theory is generally acceptable for the description of any turbulent flow. It is one further step from the Boussinesq's eddy viscosity concept for the quantification of turbulence intensity so that the constant, called von Karman constant, used in the Prandtl's theory is unversally constant while the eddy viscosity is still variable depending on the flow condition.

In principle BYO model employs the mixing length theory with some numerical manipulation for the computation of time integration of vectorial summation and some new rationalism for the estimation of current velocity reduction. The turbulence component of velocity is assumed proportional to the product of mixing length and velocity gradient, and the mixing length universally proportional to the distance from the sea bed. BYO model can be unreliable, when the flow condition of waves and current is not descriptable using the theory or violates the assumptions. Such violation may occur especially when large eddy motion develops over ripples. The mixing distance cannot be definately measured and the velocity gradient may not be dependant on the turbulence intensity at the same point particularly when ripples exist. Any theoretical effort may become in vane for the accurate estimation of bottom shear stresses in such violent condition. It is, however, expected that BYO model may still provide a reasonable answer for the case of some eddy motion over ripples, because even the large eddy motion developing over ripples is also reckoned to be heavily influenced by the flow motion very near the sea bottom.

In the previous works (Yoo and O'Connor 1987, O'Connor and Yoo 1988, Yoo 1989) it was suggested that the current velocity reduction is estimated to be the cube root of the ratio of current dissipation rate to combined flow dissipation rate without considering any arbitralily-angled interaction of urbulent motion. On the other hand in the Ph.D. work of Yoo (1986) it was proposed that the velocity reduction is estimated with considering the arbitrarily-angled interaction of turbulent motion, i.e.

$$\alpha = \begin{cases} \left(D_c / (D_c^* + D_w^*) \right)^{1/3} & \theta = \pi/2 \\ \left(D_c / (D_c^* + D_w^*/3) \right)^{1/3} & \theta = 0 \end{cases} \qquad (1)$$

where α is the reduction factor, D_c is the current dissipation rate, D_c^* is the enhanced current dissipation rate, D_w^* is the enhanced wave dissipation rate and θ is the angle between wave direction and direction normal to the current. The major reason for this implication is that the energy dissipation

* Department of Civil Engineering, Ajou University, Suwon 441-749 (Korea)

on the major direction is considered to be three times that on each minor direction, and in fact all computations were conducted on this assumption. It is further refined and corrected in the present work with considering the case of arbitrarily-angled interaction.

As ripples grow easily under the oscillatory wave motion, the description of ripple formation is a pre-requisite for the estimation of total bottom shear stresses of the combined wave-current flow on a natural sandy beach. The ripple formation is defined using the values of skin shear stresses and sediment sizes, and the computation of effective roughness height is conducted using the values of ripple height and steepness. Bijker's laboratory data measured on a mobile sandy beach is employed for the verification of the present model, and good comparison is obtained.

EDDY MOTION INTERACTION

Laboratory experiments suggest that the shear stresses k_s is only about 30 % of the normal stresses k_n. Since the cube of kinetic turbulence energy is equivalent to the square of turbulence energy dissipation rate, it is expected

$$D_s = 0.3^{3/2} D_n \qquad (2)$$

where D_n is the energy dissipation rate on a primary direction and D_s is the energy dissipation rate on each direction normal to the primary direction. Strictly speaking, D_n or D_s is the density of the energy dissipation rate on each direction, and the total energy dissipation rate can be evaluated by integrating the ellipsoid with the major axis D_n and the minor axis D_s. When the waves are interacted with a current with an arbitrary angle θ, the wave dissipation rate density along the current direction D_c may be approximately estimated by:

$$D_c = (0.16 + 0.84 \sin\theta) D_w \qquad (3)$$

where D_w is the density of wave energy dissipation rate along wave direction. Here we note that when waves are interacted with a current with the right angle D_c is only 16% of D_w.

It is assumed that the enhanced wave energy dissipation rate D_w^* would occur primarily along the wave direction while the enhanced current energy dissipation rate D_c^* primarily along the current direction. Thus the current velocity reduction factor is evaluated by:

$$\alpha = (D_c / (D_c^* + \xi D_w^*))^{1/3} \qquad (4)$$

or:

$$a\alpha^5 + b\mu^c\alpha^{5-c} + \xi r \eta \mu^{2+q}\alpha^{3-q} + 0.424\xi\eta\mu^2\alpha^3 - 1 = 0 \qquad (5)$$

instead of Eq. 1, where $\xi = 0.16 + 0.84 \sin\theta$, $\mu = \zeta\eta$, $\zeta = (\tilde{C}/\bar{C})^{1/2}$, $\eta = u_b/U$, C is the wave friction factor, C is the current friction factor, u_b is the maximum wave velocity at the sea bed, U is the depth-mean current velocity, and the constants a, b, r, q are the time integration factors obtained by regression (refer Yoo 1989).

BED FRICTION ON RIPPLED SANDY BEACH

Various empirical functions are proposed for the estimation of ripple height and ripple steepness, which may be the prime factors for the determination of the effective roughness height or $30z_o$, where z_o is the zero-velocity height. Within the authors' knowledge the ripple formation has rarely been investigated for the flow condition of combined waves and current, but the information for the waves alone or current alone seems to be abundant. As the ripples develop more easily in the oscillatory flow than in the one-directional flow, it may be better to choose the function of ripple formation for the wave condition than that for the current condition.

Based on a great number of laboratory and field data carried out by many investigators, Nielsen(1979) suggested that:

$$\Delta/\lambda = 0.182 - 0.24(\hat{\tau}/\gamma_s\Phi)^{1.5} \tag{6}$$

$$\Delta/A_b = 0.275 - 0.022\ u_b/(\gamma_s\Phi/\rho)^{0.5} \tag{7}$$

under regular laboratory waves, where Δ is the ripple height, λ is the ripple length, A_b is the maximum excursion length of wave motion at the bottom, γ_s is the immersed specific weight of sands, ρ is the water density, Φ is the grain diameter, and $\hat{\tau}$ is the maximum bed shear stress. He also concluded that the field measurements of ripples best confirm with the laboratory measurements when the field water motion is calculated from the significant wave height. In the present approach τ is replaced by the maximum skin shear stress of the combined wave-current flow, which is determined using a mean grain diameter for the effective roughness height R.

When the sea bed is flat, the roughness height of the boundary equals the grain diameter. When the sea bed is rough and has a wide distribution of various grain sizes, R is estimated by:

$$R = \alpha_r\Phi_m \tag{8}$$

where Φ_m is the mean grain diameter, and according to Engelund and Hansen(1972) α_r = 2.5, although values up to 4 have also been used. Equation 8 is also used for the estimation of skin shear stress on a rippled bed. When ripples exist on the bed, an empirical expression for R has been produced by Swart(1976), on the basis of a large number of field and laboratory tests, that is:

$$R = \beta_r\Delta^2/\lambda \tag{9}$$

where β_r is a constant, and it was suggested β_r = 8 - 25. For the estimation of total bed shear stress Eq.9 is finally used.

MODEL TEST

The modified BYO model has been tested for the fixed bed condition using all data employed in the previous tests (O'Connor and Yoo 1988; Yoo 1989), and some minor improvements were obtained for the case of right-angled interaction. In the present study the model is also tested against the laboratory data obtained by Bijker(1967) on a natural mobile beach. As previously mentioned in O'Connor and Yoo(1988), the laboratory results for the case of $\theta=\pi/6$ were noted to be less reliable than the case of fixed bed condition with $\theta=0$ because of the development of bed forms. In the present test, reasonable computation results are obtained when the roughness height is determined using Eq.9 with Eqs.6 and 7 for the estimation of ripple formation.

For the case of $\theta=\pi/6$ Bijker used natural mobile sands of Φ_m = 0.25 mm and Φ_{90} = 0.34 mm and tested various flow conditions with measuring the surface slope i for each condition. When the energy dissipation is purely due to bottom friction, the time-mean bed shear stress $\langle\tau\rangle$ is equal to ρgdi whether the uni-directional current flow is active alone or the combined wave-current flow is active together. But when the bed forms are not fixed and the bed materials are transported, ρgdi may be bigger than $\langle\tau\rangle$ because additional dissipation would occur due to the transportation of bed materials. As presented in Table 1, even on the same bed materials the ripple formation varies greatly with different flow condition. Using β_r=15, the effective roughness height is estimated from 3.3 mm to 40.1 mm. These values are far bigger than the one for the flat bed condition, but in the similar range detected by Bijker(1967) in his laboratory measurements. All data except one shows that the measured ρgdi are bigger than the computed $\langle\tau\rangle$, and the overall under-estimation is about 23 %. Considering the energy dissipation due to the sediment transport, the comparison is found to be satisfactory.

REFERENCES

Bijker, 1967, Some consideration about scales for coastal models with movable bed, Delft Hydraulics Lab., Publ. No. 50.

Engelund, F.A. and Hansen, E., 1972, A monograph on sediment transport, Teknisk Forlag, Copenhagen, 67 pp.

Nielsen, P., 1979, Some basic concepts of wave sediment transport, Inst. of Hydrod. and Hydraulic Eng., Tech. Univ. of Denmark, Series Paper No. 20.

O'Connor, B.A. and Yoo, D., 1988, Mean bed friction of combined wave-current flow, Coastal Engineering, 12: 1-21.

Swart, D.H., 1974, Offshore sediment transport and equilibrium beach profiles, Delft Hydraulics Lab., Publ. No. 131.

Yoo, D., 1986, Mathematical modelling of wave-current interacted flow in shallow waters, Ph.D. thesis, University of Manchester.

Yoo, D., 1989, Explicit modelling of bottom friction in combined wave-current flow, Coastal Engineering, 13: 325-340.

Yoo, D. and O'Connor, B.A., 1987, Bed friction model of wave-current interacted flow, Proc. Spec. Conf. Coastal Hydrodynamics, ASCE: 93-106.

ACKNOWLEDGEMENT

The present author is grateful for the financial support of Korea Industry University Cooperation Research Foundation for this study.

Table 1. Comparison of computed bed shear stress against Bijker laboratory data (β_r=15.0)

No	d m	U m/s	S degr	H m	T s	Φm mm	Δ mm	R mm	ρgdi N/m	⟨τ⟩ N/m
312	0.20	0.13	0.0	0.037	1.57	0.25	6.8	18.2	0.14	0.07
314	0.21	0.28	0.0	0.043	1.57	0.25	7.5	19.7	0.45	0.32
315	0.21	0.40	0.0	0.043	1.57	0.25	7.5	19.2	1.09	0.93
316	0.30	0.12	0.0	0.058	1.57	0.25	7.9	21.0	0.11	0.06
317	0.30	0.21	0.0	0.056	1.57	0.25	7.7	20.4	0.24	0.18
318	0.30	0.30	0.0	0.055	1.57	0.25	7.6	19.9	0.49	0.36
319	0.30	0.40	0.0	0.064	1.57	0.25	8.5	21.7	1.07	0.89
320	0.38	0.15	0.0	0.073	1.57	0.25	8.2	21.9	0.14	0.07
322	0.38	0.31	0.0	0.075	1.57	0.25	8.4	22.0	0.55	0.43
300	0.20	0.13	0.0	0.025	1.57	0.25	4.8	13.1	0.08	0.07
302	0.20	0.30	0.0	0.022	1.57	0.25	4.3	11.5	0.34	0.29
303	0.20	0.37	0.0	0.023	1.57	0.25	4.5	11.7	0.90	0.81
304	0.30	0.13	0.0	0.026	1.57	0.25	3.9	10.7	0.06	0.05
306	0.30	0.33	0.0	0.028	1.57	0.25	4.2	11.2	0.27	0.23
307	0.30	0.39	0.0	0.034	1.57	0.25	5.0	13.1	0.79	0.67
310	0.38	0.32	0.0	0.045	1.57	0.25	5.5	14.7	0.32	0.21
357	0.12	0.29	0.0	0.030	0.68	0.25	2.3	6.1	0.82	0.77
358	0.14	0.32	0.0	0.034	0.68	0.25	2.2	5.9	1.34	1.35
362	0.20	0.39	0.0	0.047	0.68	0.25	1.9	5.0	1.45	1.28
365	0.30	0.30	0.0	0.065	0.68	0.25	1.2	3.2	0.45	0.39
356	0.12	0.21	0.0	0.040	0.68	0.25	3.0	7.8	0.18	0.15
355	0.12	0.10	0.0	0.038	0.68	0.25	2.9	7.6	0.05	0.03
360	0.20	0.22	0.0	0.052	0.68	0.25	2.1	5.7	0.17	0.12
359	0.20	0.12	0.0	0.054	0.68	0.25	2.2	5.9	0.06	0.04
340	0.20	0.14	15.0	0.058	0.68	0.25	2.6	6.9	0.14	0.10
341	0.20	0.25	15.0	0.056	0.68	0.25	2.7	7.0	0.50	0.46
343	0.27	0.16	15.0	0.065	0.68	0.25	1.8	4.8	0.12	0.08
342	0.20	0.35	15.0	0.052	0.68	0.25	2.7	6.8	1.12	1.02
349	0.13	0.10	15.0	0.038	0.68	0.25	2.8	7.5	0.08	0.06
350	0.20	0.11	15.0	0.062	0.68	0.25	2.7	7.1	0.05	0.05
351	0.27	0.11	15.0	0.063	0.68	0.25	1.7	4.5	0.06	0.05
343	0.27	0.16	15.0	0.065	0.68	0.25	1.8	4.8	0.12	0.12
344	0.27	0.25	15.0	0.063	0.68	0.25	1.9	5.0	0.34	0.29
340	0.20	0.14	15.0	0.058	0.68	0.25	2.6	6.9	0.15	0.15
341	0.20	0.25	15.0	0.055	0.68	0.25	2.6	6.9	0.36	0.31
342	0.20	0.33	15.0	0.052	0.68	0.25	2.6	6.8	0.71	0.58
323	0.28	0.22	15.0	0.088	2.00	0.25	15.2	37.9	0.35	0.24
324	0.29	0.35	15.0	0.087	2.00	0.25	15.2	36.8	0.51	0.37
325	0.20	0.22	15.0	0.080	2.00	0.25	16.2	39.2	0.45	0.29
326	0.20	0.30	15.0	0.094	2.00	0.25	17.7	39.6	0.89	0.96
327	0.30	0.12	15.0	0.071	2.00	0.25	12.6	33.1	0.22	0.09
329	0.30	0.25	15.0	0.064	2.00	0.25	12.0	31.1	0.44	0.25
331	0.20	0.10	15.0	0.047	2.00	0.25	11.1	29.5	0.06	0.03
333	0.34	0.11	15.0	0.067	2.00	0.25	11.4	30.3	0.24	0.10
334	0.34	0.22	15.0	0.071	2.00	0.25	12.1	31.6	0.67	0.25
335	0.34	0.34	15.0	0.073	2.00	0.25	12.6	32.0	0.84	0.70
332	0.21	0.37	15.0	0.055	2.00	0.25	12.9	32.0	1.08	1.10
334	0.34	0.22	15.0	0.067	2.00	0.25	11.6	30.4	0.36	0.22

average deviation = 23.03 %