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Possibility of Using the Classical Mechanics for the Preliminary Design of Laminated Composite Structures for Civil Construction

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Abstract

At the preliminary design stage, the orientations of laminae in a laminate are not known. This fact discourages the most of engineers from the beginning. If the quasi-isotropic constants are used, it halps the design engineer greatly to start his work. If conventional mechanics and elasticity theories can be used, the effort for design and analysis is greatly reduced. This paper reports the possibility of using the classical mechanics at the preliminary design stage for the laminated composite primary structure for civil construction. The result is quite promissing

1. The Importance of the Subject Problem

The highest specific strength and stiffness of composites can be obtained by arranging long fiber reinforcements in straight fashion, and forming a laminate made of several laminae. Design and analysis of a laminate is so much complicated that considerable number of structural engineers are simply allergic to composite design. In analysis, even boundary conditions are not so simple as with the classical mechanics or elasticity cases. Both simple and clamped boundaries have eight possible types.

For simple support ;

$$\begin{aligned}
&\text{Type 1 : } w=0,\ M_n=0,\ u_n=\bar{u}_n,\ u_t=\bar{u}_t \\
&\text{Type 2 : } w=0,\ M_n=0,\ N_n=\bar{N}_n,\ u_t=\bar{u}_t \\
&\text{Type 3 : } w=0,\ M_n=0,\ u_n=\bar{u}_n,\ N_{nt}=\bar{N}_{nt} \\
&\text{Type 4 : } w=0,\ M_n=0,\ N_n=\bar{N}_n,\ N_{nt}=\bar{N}_{nt}
\end{aligned} \quad (1)$$

For clamped edge ;

$$\begin{aligned}
&\text{Type 1 : } w=0,\ \frac{\partial w}{\partial n}=0,\ u_n=\bar{u}_n,\ u_t=\bar{u}_t \\
&\text{Type 2 : } w=0,\ \frac{\partial w}{\partial n}=0,\ N_n=\bar{N}_n,\ u_t=\bar{u}_t \\
&\text{Type 3 : } w=0,\ \frac{\partial w}{\partial n}=0,\ u_n=\bar{u}_n,\ N_{nt}=\bar{N}_{nt} \\
&\text{Type 3 : } w=0,\ \frac{\partial w}{\partial n}=0,\ N_n=\bar{N}_n,\ N_{nt}=\bar{N}_{nt}
\end{aligned} \quad (2)$$

where the upper bar indicates the given value. Even when the transverse shear deformation is neglected, the related equations are three simultaneous fourth order partial differential equations, given as Eqns(7-72), (7-73), and (7-74) in Ref(1).

$$A_{11}\frac{\partial^2 u}{\partial x^2}+2A_{16}\frac{\partial^2 u}{\partial x\partial y}+A_{66}\frac{\partial^2 u}{\partial y^2}+A_{16}\frac{\partial^2 v}{\partial x^2}$$

$$(A_{12}+A_{66})\frac{\partial^2 v}{\partial x\partial y}+A_{26}\frac{\partial^2 v}{\partial y^2}-B_{11}\frac{\partial^3 w}{\partial x^3}-3B_{16}\frac{\partial^3 w}{\partial x^2\partial y}$$

$$-(B_{12}+2B_{66})\frac{\partial^3 w}{\partial x\partial y^2}+B_{26}\frac{\partial^3 w}{\partial y^3} = 0 \quad (3)$$

$$A_{16}\frac{\partial^2 u}{\partial x^2}+(A_{12}+A_{66})\frac{\partial^2 u}{\partial x\partial y}+A_{26}\frac{\partial^2 u}{\partial y^2}+A_{66}\frac{\partial^2 v}{\partial x^2}$$

$$+2A_{26}\frac{\partial^2 v}{\partial x\partial y}+A_{22}\frac{\partial^2 v}{\partial y^2}-B_{16}\frac{\partial^3 w}{\partial x^3}$$

$$-(B_{12}+2B_{66})\frac{\partial^3 w}{\partial x^2\partial y}-3B_{26}\frac{\partial^3 w}{\partial x\partial y^2}-B_{22}\frac{\partial^3 w}{\partial y^3}=0 \quad (4)$$

$$D_{11}\frac{\partial^4 w}{\partial x^4}+4D_{16}\frac{\partial^4 w}{\partial x^3\partial y}+2(D_{12}+2D_{66})\frac{\partial^4 w}{\partial x^2\partial y^2}$$

$$+4D_{26}\frac{\partial^4 w}{\partial x\partial y^3}+D_{22}\frac{\partial^4 w}{\partial y^4}-B_{11}\frac{\partial^3 u}{\partial x^3}-3B_{16}\frac{\partial^3 u}{\partial x^2\partial y}$$

$$-(B_{12}+2B_{66})\frac{\partial^3 u}{\partial x\partial y^2}-B_{26}\frac{\partial^3 u}{\partial y^3}-B_{16}\frac{\partial^3 v}{\partial x^3}$$

$$-(B_{12}+2B_{66})\frac{\partial^3 v}{\partial x^2 \partial y}-3B_{26}\frac{\partial^3 v}{\partial x \partial y^2}-B_{22}\frac{\partial^3 v}{\partial y^3}=q(x,y) \quad (5)$$

Considerable simplification can be made in preliminary analysis, if

A. classical mechanics and elasticity theories can be used.

B. the bending-extension coupling matrix, B_{ij}, vanishs so that related equation becomes one fourth order partial differdrtial equation.

2.Possibility of Simplified Approaches

The classical theories and formulas can be used if the normalized extensional stiffness equals the normalized bending stiffness, that is

$$A^* = D^* \quad (6)$$

where

$$A^* = A/h \quad \text{in GPa}$$
$$B^* = 2B/h^2 \quad \text{in GPa} \quad (7)$$
$$D^* = 12D/h^3 \quad \text{in GPa}$$

in which

$$A_{ij}=\sum_{k=1}^{n} (\bar{Q}_{ij})_k (h_k-h_{k-1}),$$

$$B_{ij}=\frac{1}{2}\sum_{k=1}^{n} (\bar{Q}_{ij})_k (h^2_k-h^2_{k-1}), \quad (8)$$

$$D_{ij}=\frac{1}{3}\sum_{K=1}^{n} (\bar{Q}_{ij})_k (h^3_k-h^3_{k-1}),$$

h=the thickness of the laminate

where the $\bar{Q}_{ij}$ is the reduced stiffness matrix for the plane stress cases given as

$$\bar{Q}_{11} = Q_{11}m^4+2(Q_{12}+2Q_{66})m^2n^2+Q_{22}n^4$$

$$\bar{Q}_{12} = (Q_{11}+Q_{22}-4Q_{66})m^2n^2+Q_{12}(m^4+n^4)$$

$$\bar{Q}_{13} = Q_{13}m^2+Q_{23}n^2$$

$$\bar{Q}_{16} = -Q_{22}mn^3+Q_{11}m^3n-(Q_{12}+2Q_{66})mn(m^2-n^2)$$

$$\bar{Q}_{22} = Q_{11}n^4+2(Q_{12}+2Q_{66})m^2n^2+Q_{22}m^4$$

$$\bar{Q}_{23} = Q_{13}n^2+Q_{23}m^2$$

$$\bar{Q}_{26} = -Q_{22}m^3n+Q_{11}mn^3+(Q_{12}+2Q_{66})mn(m^2-n^2)$$

$$\bar{Q}_{33} = Q_{33}$$

$$Q_{36} = (Q_{13}-Q_{23})mn \quad (9)$$

$$\bar{Q}_{44} = Q_{44}m^2+Q_{55}n^2$$

$$\bar{Q}_{45} = (Q_{55}-Q_{44})mn$$

$$\bar{Q}_{55} = Q_{55}m^2+Q_{44}n^2$$

$$\bar{Q}_{66} = (Q_{11}+Q_{22}-2Q_{12})m^2n^2+Q_{66}(m^2-n^2)^2$$

in which Q_{ij} is given as

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}}$$

$$Q_{12} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} = \frac{\nu_{21}E_1}{1-\nu_{12}\nu_{21}} \quad (10)$$

$$Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}}$$

$$Q_{66} = G_{12}$$

and, $m=\cos\alpha$ and $n=\sin\alpha$, where α is the angle of the transformation.

It is generally known that the bending-extension coupling matrix, [B], vanishes only if the cross-section of a laminate is symmetrical, in both material, and geometry and orientation, with respect to its midsurface. However, a sufficient condition to eliminate the bending-extention coupling is that the sum of the normalized weighting factors of each group of orientation is equal to zero (Ref 10-210, 10-211, and 10-212 of ref 1). In addition to such condition, increase of the number of layers for certain orientations for such as the thick laminates of the primary structures for the civil construction may result in negligibly small quantity of B-matrix.

3. Quasi-isotropic Concept

In his recent book(1), D.H. Kim proposes to use the quasi-istropic constants by Tsai for the preliminary design of the composite primary structures for the civil construction. This concept is indirectly supported by the recent paper of Verchery et al(2).

Every anisotropic material has quasi-isotropic constants derived from the invariants of coordinate transformation, These constants represent the lower bound of each composite performance, and are given by Tsai(1) as

$$[Q]^{iso} = \begin{vmatrix} U_1 & U_4 & 0 \\ U_4 & U_1 & 0 \\ 0 & 0 & U_5 \end{vmatrix}$$

where

$$U_1 = \frac{1}{8}(3Q_{xx}+3Q_{yy}+2Q_{xy}+4Q_{ss})$$

$$U_4 = \frac{1}{8}(Q_{xx}+Q_{yy}+6Q_{xy}-4Q_{ss}) = U_1 - 2U_5$$

$$U_5 = \frac{1}{8}(Q_{xx}+Q_{yy}-2Q_{xy}+4Q_{ss})$$

When quasi-isotropic constants sre used we always have $A^* = D^*$, $B^* = 0$.

4. Numerical Studies

In order to

A. study the validity of use of the quasi-isotropic constants,

B. find the laminates with $A^* = D^*$, and $B_{ij} \cong 0$, several laminate configurations with different orientations and numbers of layers are studied. The result is rather promising and shown in next article.

The material property used is as follows

$E_m = 3.8$ GPa

$E_f = 70$ GPa

$\nu_m = 0.35$

$\nu_f = 0.22$

$V_m = 0.4$

$V_f = 0.6$

From these values, we obtain

$E_1 = 67.36$ GPa

$E_2 = 8.12$ GPa

$\nu_{12} = 0.272$

$\nu_{21} = 0.0328$

$G_{12} = 3.02$ GPa

5. Study Result

A. Quasi-homogeneous laminates($A^*=D^*$)

1)When quasi-isotropic constants are used.

$A^* = D^*$

2)Angle ply laminates ,$[\pm\theta]_r$.

$A^*=D^*$

Table 1. $[\pm\theta]_r$ $\theta=\pm15°$

r	3	5	8	11	14	17	20
$\frac{A_{11}^*}{D_{11}^*}$	1	1	1	1	1	1	1
$\frac{B_{16}^*}{D_{11}^*}$	0.039	0.023	0.014	0.01	0.008	0.007	0.0056
$\frac{B_{26}^*}{D_{11}^*}$	0.0026	0.0016	0.001	0.0007	0.0006	0.0005	0.0004
$\frac{D_{11}}{D_{11\,iso}}$	1.96	1.96	1.96	1.96	1.96	1.96	1.96

3) Quasi-isotropic orientation, $[90, +45, -45, 0]_r$

$A^* \cong D^*$ A_{11}^* is constant.

The differences are 4.4% when r=2 and 0.2% when r=9.

Table 2. $[90,+45,-45,0]_r$

r	1	2	3	4	5	6	7	9
$\frac{A_{11}^*}{D_{11}^*}$	0.845	0.956	0.98	0.99	0.993	0.995	0.996	0.997
$\frac{B_{11}^*}{D_{11}^*}$	0.31	0.18	0.12	0.09	0.07	0.06	0.053	0.04
$\frac{D_{11}}{D_{11\,iso}}$		1.05	1.02	1.01	1.007	1.005	1.0039	1.002

4) Special Orthotropic laminates, [0,90,0] orientation

The differences are 27% when N=3 and 3% when N=51.

Table 3. [0,90,0] Orientation

Ply number (N)	3	7	11	15	19	27	51
$\frac{A_{11}^*}{D_{11}^*}$	0.731	0.835	0.883	0.909	0.926	0.907	0.97

(5) Special Orthotropic laminates, [90,0,90] orientation.

The differences are 45% when N=5 and 3.6% when N=45.

Table 4. Special Orthotropic Laminates [90,0,90] Orientation

Ply number (N)	5	9	13	17	21	45
$\frac{A_{11}^*}{D_{11}^*}$	1.56	1.233	1.146	1.107	1.083	1.036

(6) $[ABBCAAB]_r$ orientation with A=45°, B=-45°, C=0°

Table 5. $[ABBCAAB]_r$ A=45°· B=-45°, C=0°

r(N)	1(7)	2(14)	3(21)	4(28)	5(35)
$\frac{A_{11}^*}{D_{11}^*}$	1.268	1.056	1.024	1.013	1.008
B_{ij}	≒0	≒0	≒0	≒0	≒0

This orientation has fairly good quasi-homogeneous characteristics when r ≥ 2.

(7) $[ABCCABBCA]_r$ orientation with A=45°, B=-45°, C=0°.

Table 6. $[ABCCABBCA]_r$ A=45°, B=-45°, C=0°.

r(N)	1(9)	2(18)	3(27)	4(36)	5(48)
$\frac{A_{11}^*}{D_{11}^*}$	1.13	1.03	1.013	1.007	1.003
B_{ij}	≒0	≒0	≒0	≒0	≒0

This arrangement has fairly good quasi-homogeneous characteristics when r ≥ 2

(8) Antisymmetric Angle-Ply $[ABBAAB]_r$ A=+15° B=-15°

r number	3	7	11	15	19	27	51
$\frac{B_{16}^*}{D_{11}^*}$	0.0105	0.0052	0.0035	0.0026	0.0021	0.0015	0.0011
$\frac{B_{26}^*}{D_{11}^*}$	0.0015	0.00075	0.0005	0.00037	0.00030	0.00021	0.000166

(9) Symmetric Angle-Ply $[[45,-90,30,0]_r]_s$

r number	2	4	6	8	10	12
$\frac{D_{16}}{D_{11}^*}$	0.3454	0.2016	0.1960	0.1935	0.1922	0.1913
$\frac{D_{26}^*}{D_{11}^*}$	0.3058	0.1402	0.1338	0.1309	0.1293	0.1284

B. Elimination of the bending - extension coupling stiffness, B_{ij}.

(1) when quasi-isotropic constants are used.

(2) When the cross-section is symmetric with respect to the midsurface of the laminate.

(3) Angle ply laminate with N=36 number of plies.

$B = 0$.

(4) [ABBCAAB]. A=45°, B=-45°, C=0°. (See Table 5)

$B_{ij} = 0$.

(5) [ABCCABBCA]. A=45°, B=-45°, C=0°. (See Table 6)

$B_{ij} = 0$.

(6) $[\pm\theta]_r$, $\theta=15°$. B_{16} is 2-30% of D_{11} when r=5 and 0.56% when r=20.

(7) Antisymmetric Angle-Ply $[ABBAAB]_r$, A=15°, B=-15°

$B_{16} \fallingdotseq 0$, $B_{26} \fallingdotseq 0$ as r increases.

See Figure (1) and (2)

(8) Symmetric Angle-Ply $[[+45,-90,30,0,]_r]_s$

$D_{16} \neq 0$, $D_{26} \neq 0$.

As r increase,

$D_{16}^*/D_{11}^* \to 0.2$

$D_{26}^*/D_{11}^* \to 0.13$

See Figures(3) and (4)

6. Conclusion

The classical mechanics and elasticity theories can be used for the preliminary design of the laminated composite structures if

(1) quasi-isotropic constants are used,

(2) laminates with certain orientations are used, or

(3) certain "thick" laminates are used especially for civil construction.

This will greatly reduce the calculation effort at the early stage of the design. Materials, orientations, and sizes for the preliminary design can be decided by the formulas obtained by the use of classical theories. With the chosen sections, the stresses and strains, with stability and dynamic behavior taken into account, can be found by rigorous theory. The strength/failure theory will be applied then. If necessary, the sections can be easily modified. This is possible because of versatility and flexibility of composite design.

Figure1.Antisymmetric angle-ply
[+15,-15,-15,15,15,-15]r

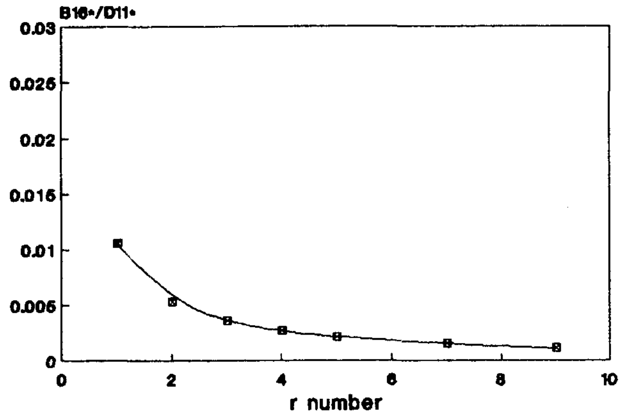

Figure2.Antisymmetric Angle-Ply
[15,-15,-15,15,15,-15]r

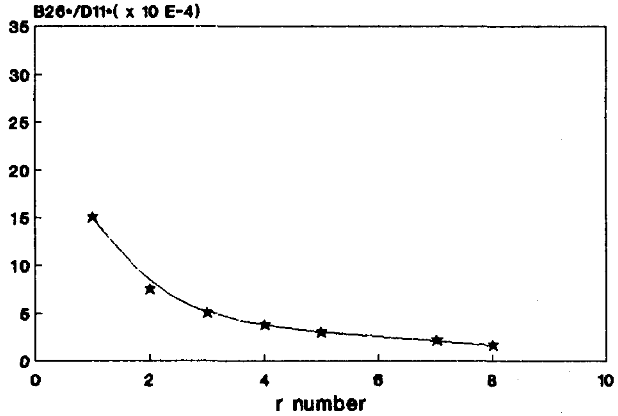

Figure2.Symmetric Angle-Ply
[[45,-90,30,0]r]s

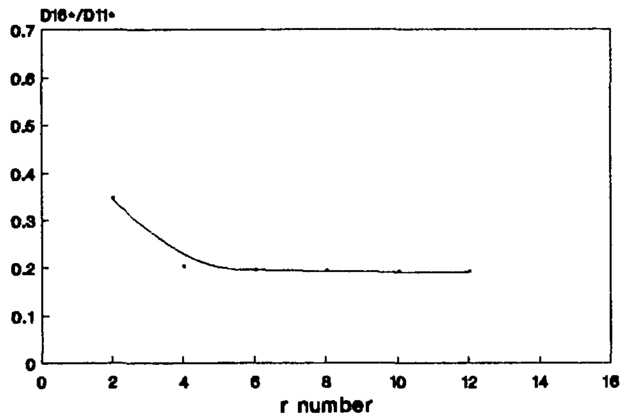

Figure3.Symmetric Angle-Ply
[[45,-90,30,0]r]s