1991년도 봄
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有限要素法에 의한 溫度 荷重의 解析

(A Finite Element Analysis Of Thermal Load On The Concrete Highway Pavement)

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Abstract

In the recent years,a rigid pavement composed of a flat concrete slab has been constructed due to the desirable structural strength of concrete, durability and economy. However, despite of precise design and construction of concrete highway pavement, some sections of the 88 Olympic express highway, Jung-bu express highway, and Kyung-bu express highway, which have shown premature cracking, faulting, and pumping before the end of their intended service life, have already been viewed with great concerns by highway officials and engineers.
Since environmental variations and traffic loads might be considered as major factors to cause pavement failure problems, the thermal load due to temperature variations between top and bottom surface of the concrete slab was highlighted to verify analytical behavior of concrete slab using the finite element method.

1. Skewed Element.

The finite element formulation of the parallelogram plate bending element is presented based on the MZC rectangular plate bending element. The four node element as shown in Figure 1 has three independent displacements at each node, which are a vertical deflection, w, and two rotations about the X and Y axes, for a total of twelve degrees of freedom per element.

The array of nodal displacements at the ith node is :

$$q_i = \begin{bmatrix} q_{i1} \\ q_{i2} \\ q_{i3} \end{bmatrix} = \begin{bmatrix} w_i \\ w_i \\ y_i \end{bmatrix} = \begin{bmatrix} w_i \\ -dw_i/dy \\ dw_i/dx \end{bmatrix}$$

The array of corresponding nodal forces is :

$$p_i = \begin{bmatrix} p_{i1} \\ p_{i2} \\ p_{i3} \end{bmatrix} = \begin{bmatrix} Fz_i \\ Mx_i \\ My_i \end{bmatrix}$$

(i = 1,2,3,4 node number)

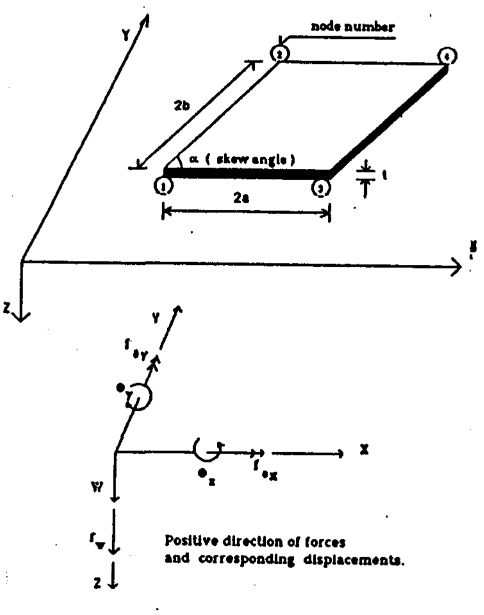

Fig. 1 Dimensions and Positive direction of a Parallelogram Plate Bending Element.

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2. Initial strain due to thermal load

Let the generic displacement function, vertical displacement w, with an element be :

$$W = C_1 + C_2X + C_3Y + C_4X^2 + C_5XY + C_6Y^2 + C_7X^3 + C_8X^2Y + C_9XY^2 + C_{10}Y^3 + C_{11}X^3Y + C_{12}XY^3$$

$$(-dw/dy)_i = Q_{xi} = -C_3 + C_5X + \ldots.$$
$$(dw/dy)_i = Q_{yi} = C_2 + 2C_4X + \ldots.$$

Displacement shape functions {f} relate generic displacements (U) to nodal displacements { q }.

$$U = \{ f \} \{ q \} \qquad (2.1)$$

Strain can be obtained by differentiation of the generic displacements (U),

$$\{ \epsilon \} = d\, U, \qquad (2.2)$$

where, $\epsilon_x = dU / d_x$, $\epsilon_y = dV / d_y$, $\epsilon_{xy} = dU / d_y + dV / d_x$

Substituting (2.1) to (2.2),

$$\{\epsilon\} = d\, U = d\{f\}\{q\} = B\{q\} \qquad (2.3)$$

where B Strain - Displacement operator (B = d { f }) relates nodal displacements at each point to strain matrix and can be obtained by the differentiation of shape functions.

Simple continuum mechanics shows

$$\sigma = [\, E\,] \quad (\epsilon - \epsilon_0) \qquad (2.4)$$

Substituting (2.3) to (2.4),

$$\sigma = [\, E\,] \quad \{ (Bq) - \epsilon_0 \} \qquad (2.5)$$

in which the matrix product E B, which is called stress matrix, gives stresses at a generic point.

By the definition, the virtual work of external actions is equal to the virtual strain energy of internal stresses.

$$dU_i = dw_e \qquad (2.6)$$

Assume a set of nodal virtual displacements, $\{ q^v \}$.

The resulting displacements and strains within the element are

$$[\, U^v\,] = [\, f\,] \quad \{ q^v \}$$
$$[\, \epsilon^v\,] = [\, B\,] \quad \{ q^v \}$$

Equating the external work with the total internal work and integrating over the volume of the element,

$$\{q^v\}^T\{F\} = \int_v [\epsilon^v]^T\{\sigma\}\, dv = \{q^v\}^T \left(\int [B]^T\{\sigma\}\, dv - \int [f]^T w\, dv \right)$$

Substituting Eqs. (2.1) through (2.5),

$$\{F\} = \int_v [B]^T [E] [B]\, dv \,)\quad \{ q \} - \int_v [B]^T [E] \{ \epsilon_0 \}\, dv - \int_v [f]^T w\, dv \qquad (2.7)$$

The stiffness matrix [K] becomes,

$$[K] = \int_v [B]^T [E] [B]\, dv \,) \qquad (2.8)$$

Nodal forces due to distributed loads are

$$\{F_w\} = - \int_v [f]^T \; w\, dv \,) \qquad (2.9)$$

Nodal forces due to initial strains are

$$\{F\epsilon_0 = - \int_v [B]^T [E] \; \{\epsilon_0\} \; dv \qquad (2.10)$$

3. Equivalent nodal load due to thermal gradients.

Assuming that the temperature varies linearly from the top to the bottom of the plate with a temperature differential of dT, the curvatures of the unrestrained plate element will be :

$$\epsilon_0 = \begin{bmatrix} -d_2w/dx^2 \\ -d_2w/dy^2 \\ 2d^2w/dxdy \end{bmatrix} = \begin{bmatrix} -\alpha\, dT/t \\ -\alpha\, dT/t \\ 0 \end{bmatrix}$$

α = Coeff. of thermal expansion.
dT = Temperature differential.
t = Thickness of plate element.

The virtual work theory yields to the equivalent nodal loads due to the uniform temperature variations as follows :

$$F^T = \int_A B^T \; E \; \epsilon_0 \; dA$$
$$= ab\, Sin(\alpha) \iint B^T \; E \; \epsilon_0\, dR\, dS$$
$$= ab\, Sin(\alpha) \iint B^T \begin{bmatrix} Dx & D1 & 0 \\ D1 & Dy & 0 \\ 0 & 0 & Dxy \end{bmatrix} (-\alpha\, dT/t) \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} dRdS$$

$$= \frac{E\, t^2\, (\alpha\, dT)}{12\, (1-PR)} \begin{bmatrix} -2\, Cot(\alpha) \\ a/Sin(\alpha) \\ -b/Sin(\alpha) \\ 2\, Cot(\alpha) \\ -a/Sin(\alpha) \\ -b/Sin(\alpha) \\ -2\, Cot(\alpha) \\ a/Sin(\alpha) \\ -b/Sin(\alpha) \\ 2\, Cot(\alpha) \\ -a/Sin(\alpha) \\ -b/Sin(\alpha) \end{bmatrix}$$

4. Temperature variations and response of pavement

Displacements of pavement slabs are mainly associated with changes in concrete temperature. The extent and direction of slab curling and joint movement change according to daily variation of temperature. Therefore, accurate and realistic evaluation of structural response of concrete pavements requires a thorough understanding of pavement temperatures.

Florida Department of Transportation (FDOT) constructed the test road to measure air temperature and pavement temperature in Gainesville florida using thermocouples and Linear Variable Differential Transducers(LVDT'S). The report shows that daily pavement temperatures are generally 15°F to 25°F (8°C to 14°C) higher than air temperatures and minimum air temperatures occur early in the morning between 6:00 A.M. and 8:00 A.M.,and maximum temperature occur during mid-afternoons between 1:00 P.M. and 3:00 P.M., regardless of the season. Minimum and maximum pavement temperatures occur approximately one hour after air temperature has reached, respectively, its minimum and maximum. The range between daily maximum and minimum temperature of the pavement is between 10°F and 20°F (5.5°C and 11°C)

The temperature differential between surface and bottom of concrete slab is responsible for the magnitude and direction of slab curling. The maximum negative temperature differential occurs most freguently between 5:00 A.M. and 7:00 A.M. with the slab curling upward at the edges while the center is in contact with the subgrade. On the other hand, the maximum positive temperature differential occures between 12:00(noon)and 3:00 P.M. with the slab curling at the center of the slab and downward curling along the edges.

5. Effects of thermal load on Concrete Pavement

Using the finite element analysis program, the effects of thermal load on concrete pavement were analyzed with the temperature differential of 10°F (5.6°C) and 20°F(11.1°C) Figure 2 shows that there is no initial stress in the slab with zero temperature differential case. As temperature curling grows, -10°F (5.6°C) temperature differential creates about -150 psi (10.5 kg/cm^2) flexural stress in x - direction (σ_x) and + 20 F (+ 11.1 C) temperature differential produces about 300psi (21 kg/cm^2) flexural stress in x-direction(σ_x), concave up and down. Figure 3 shows how thermal load affect traffic load,which located on the joint edge.

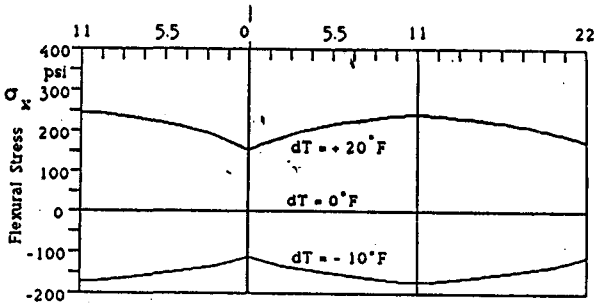
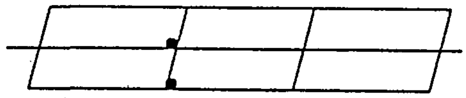


FIGURE 2 INITIAL STRESS DUE TO TEMPERATURE DIFFERNTIAL



1/4 Point	Joint	1/4 Point	Stress
-183 psi	-157.8 psi	-180.6 psi	σ (dT+D.L)
-54	205	-44.6	σ (L.L)
-237	47.2	-225.2	σ (dT+D.L) + σ (L.L)
-238	47.4	-226.2	σ (dT+D.L +L.L)

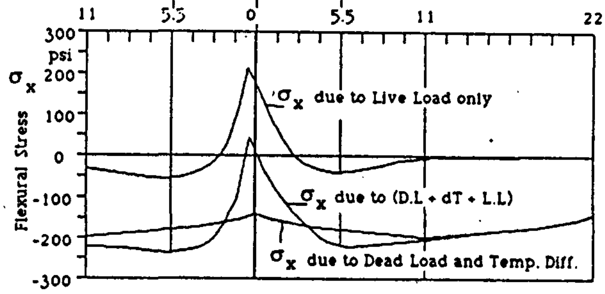


FIGURE 3 COMPARISON OF σ(dT+D.L)PLUS σ(L.L) WITH σ(dT+D.L+L.L)

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