1991년도 봄
한국전산구조공학회 학술발표회 논문집

超有限 보간법에 의한 P-version 유한요소해석

Transfinite Interpolation Technique for P-version of F. E. M.

ㅇ 우 광 성 *
Woo Kwang Sung

ABSTRACT

An attempt has been made to generate a cured boundary by using a transfinite interpolation technique. In the following sections, it will be shown how to construct transfinite interpolants both in h-version and in p-version over polygonal and nonpolygonal regions. Numeririical test cases validate the applicability and superior capability with the help of several structural problems.

1. Introduction

The conventional finite element method involv-es the partitioning of a polygonal domain(Ω) into rectangular and/or triangular elements. Quite often, however, a structural engineer is faced with a boundary value problem over a nonpolygonal domain(Ω). The early approaches in finte element modeling required that the boundary, $\partial\Omega$, of Ω be approximated by a polygonal arc. Obviously, the accuracy of the F.E.M. is limited by the accuracy of the polygonal approximation to $\partial\Omega$. Thus, if a mesh with a regular polygon as its boundary serves to model a circular region, a refinement of the mesh causes the polygon to have more sides and to converge to a circle. In the h-version, all piecewise smooth boundaries can be approximated by a sufficient number of piecewise quadratic polynomials. In the p-version, however, the size of the element is usually large and hence the probability of distortions is more, especially if higher order parametric mapping is used, unless the boundary of an element is represented by a poynominal in the parametric form. In the case of nonpolygonal boundaries, like circles and ellipses, parametric mapping may not work at all. An attempt has been made to generate a curved boundary by using a transfinite interpolation technique. This technique has been discussed in detail by Gordon and Hall. So, this paper represent an approach to apply the transfinite interpolation technique based on p-version of finite element concepts to several structural problems.

2. Transfinite Interpolation Technique

Let f be a continuous function of two independent variables with domain L : [0,h] x [0,h] in the s-t plane as shown in Fig. 1. By a projector P, we mean a linear operator from the linear space T of all continuous bivariate function f, with domain L, onto a subspace of functions. For example, if the operator P_s is defined by the formula

$$P_s[f] = (1-s/h)f(0,t) + (s/h)f(h,t) \quad (1)$$

It can be expressed by the general form as follows:

$$P_s[f] = \sum_{i=0}^{m} f(s_i,t)\Phi_i(s) \quad (2)$$

where $0 = s_0 < s_1 < \ldots < s_m = h$ and

$$\Phi_i(s) = \prod_{j\neq i}(s-s_j)/\prod_{j\neq i}(s_i-s_j) \quad 0\le i\le m \quad (3)$$

are the fundamental functions for Lagrange polynomial interpolations.
For completeness and later reference, we displ-ay the analogous formula for P_t:

$$P_t[f] = \sum_{j=0}^{n} f(s,t_j)\psi_j(t) \quad (4)$$

where $0 = t_0 < t_1 \ldots < t_n = h$ and

$$\psi_j(t) = \prod_{i\neq j}(t-t_i)/\prod_{i\neq j}(t_j-t_i) \quad 0\le j\le n \quad (5)$$

There is a way to compound the projectors P_s and P_t by using Boolean sum.

$$P_s \oplus P_t \equiv P_s + P_t - P_sP_t \quad (6)$$

3. Transfinite Interpolants in P-version

The transfinite interpolants for curved bound-ary can be achieved by constructing blend

* 정회원 전남대학교 토목공학과 조교수

functions. First, each side of the element with arbitrary boundaries is defined by parametric equations in terms of standard coordinates shown in Fig. 2. and Fig. 3. The transfinite interpolants for each side of the element are expressed as:

$$\text{side 1} \quad \begin{cases} x = x_1(\xi) \\ y = y_1(\xi) \end{cases}$$
$$\text{side 2} \quad \begin{cases} x = x_2(\eta) \\ y = y_2(\eta) \end{cases}$$
$$\text{side 3} \quad \begin{cases} x = x_3(\xi) \\ y = y_3(\xi) \end{cases}$$
$$\text{side 4} \quad \begin{cases} x = x_4(\eta) \\ y = y_4(\eta) \end{cases}$$

The suitable transfinite interpolants of each side have been derived for the plane domain and cylindrical domain. The individual mapping functions are blended with the opposite sides of the element by means of projectors $(\eta-1)/2$, $(\xi+1)/2$, $(\eta+1)/2$, $(\xi-1)/2$.
In the process, the mapping functions take the following form by Boolean sum.

$$\begin{aligned} X = {} & x_1(\xi)(1-\eta)/2 + x_3(\xi)(1+\eta)/2 \\ & + x_2(\eta)(1+\xi)/2 + x_4(\eta)(1-\xi)/2 \\ & - x_1(1-\xi)(1-\eta)/4 - x_2(1+\xi)(1-\eta)/4 \\ & - x_3(1+\xi)(1+\eta)/4 - x_4(1-\xi)(1+\eta)/4 \end{aligned} \tag{7}$$

$$\begin{aligned} Y = {} & y_1(\xi)(1-\eta)/2 + y_3(\xi)(1+\eta)/2 \\ & + y_2(\eta)(1+\xi)/2 + y_4(\eta)(1-\xi)/2 \\ & - y_1(1-\xi)(1-\eta)/4 - y_2(1+\xi)(1-\eta)/4 \\ & - x_3(1+\xi)(1+\eta)/4 - x_4(1-\xi)(1+\eta)/4 \end{aligned}$$

4. Numerical Tests

4.1 Circular Plate

One quarter of a circular plate of radius, a, subjected to a central concentrated load shown in Fig. 4. , p=1.0 lb(or uniformly distributed load q_o=1.0 psi), is modeled with one p-version element which maps the circular boundary by transfinite interpolation technique as discussed earlier. The result by p-version of the finite element method are presented in Table 1.

4.2 Thick-Walled Cylinder

This is a thick-walled cylinder under plane strain conditions subjected to a unit internal pressure. The problem is shown in Fig.5. The geometry of the problem is the same as on example of SAP90 verification manual. For the pressure loading, the results obtained by theoretical and SAP90 analyses for the radial displacement and stresses at the inner surface are compared with the results from one element p-version model in Table 2 and Figs. 6, 7, 8 and 9.

5. Conclusions

It has been established a class of transfinite interpolation formula based on the use of blending functions. It may be noted that in the case of both triangular and quadrilateral elelments one or more of the boundaries may be curved, in which case mapping from standard elements becomes more important. Moreover, the concepts of exact mapping in the p-version can be stressed on since large elements are used. The p-version element based on transfinite interpolation technique is found to be successful in the case of plane and cylindrical shell problems.

6. References

(1) W.J. Gordon and C.A. Hall, "Transfinite Element Method:Blending-Function Interpola -tion over Arbitrary Curved Element Domains", Numer. Math. 21, 109-129, 1973
(2) W.J. Gordon and C.A. Hall, "Construction of Curvilinear Coordinate Systems and Applications to Mesh Generation", Int. J. Numer. Meth. Engng., Vol. 7, 461-477, 1973
(3) S.H. Lo, "Finite element mesh generation over curved surfaces", Comput. Struct., Vol. 29, pp.731-742, 1988
(4) K.K.Tamma and S.B. Railkar,"Transfininte element methodology towards a unified thermal structural analysis", Comput. Struct. Vol. 25, pp.649-660, 1987
(5) K.V.R. Vardhani and N.S. Prasad, "Mesh generation for spherical and conical surfaces using transfinite interpolation", Comput. Struct.,Vol. 32, No. 6, 18, 1359-1362, 1989

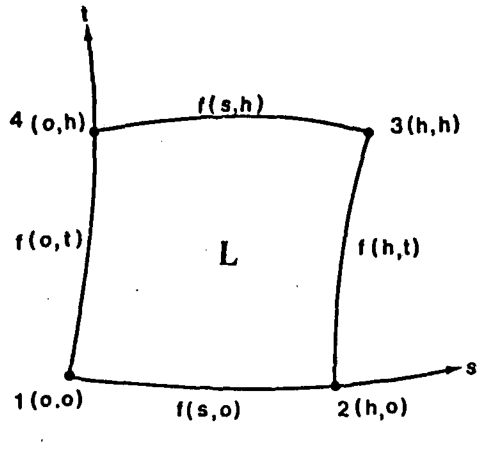

Fig. 1 Domain L in the s-t plane

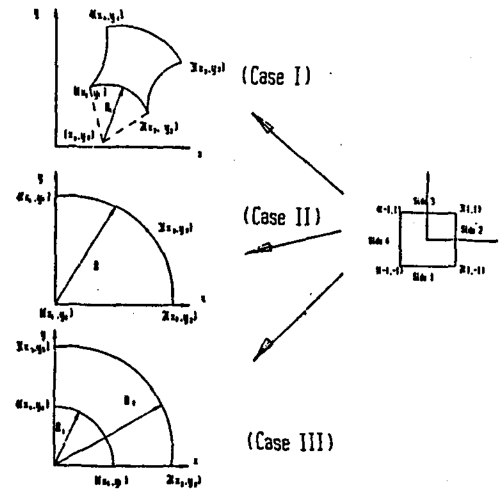

Fig. 2 Transfinite Interpolants From Standard Domain to Real Domain in the plane

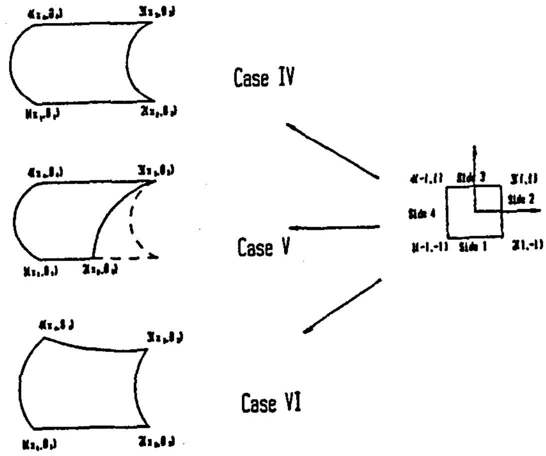

Fig. 3 Transfinite Interpolants Form Standard Domain to Real Domain in the Cylinder

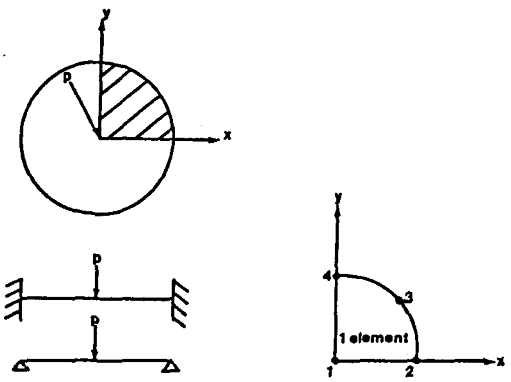

Fig. 4 Centrally Loaded Circular Plate with Clamped and Simply Supported Outer Edge

Table 1
Max. Deflection at the center for clamped and simply supported cicular plate

P-level	w_{max} (Uniform load)		w_{max} (Point load)	
	Clamped	S.S.	Clamped	S.S.
P=6	0.23680	1.01902	0.06850	0.19558
P=7	0.24511	1.02410	0.07594	0.19939
P=8	0.24978	1.02572	0.07769	0.20151
P=9	0.25011	1.02806	0.07856	0.20264
P=10	0.25026	1.03997	0.07906	0.20542
Timoshenko	0.25000	1.01923	0.07958	0.20200

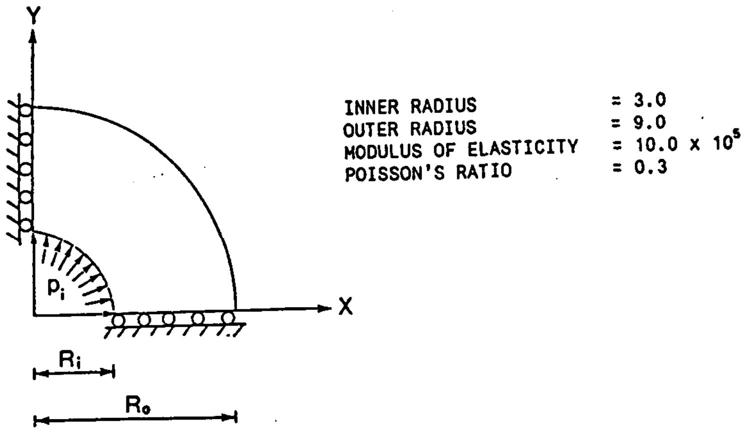

Fig. 5 Configuration and mesh refinement of thick-walled cylinder

Table 2
Convergence Characteristics of Displacement and Stresses

P-level	N D O F	Radial Displ.x10^{-5}	Radial Stress	Tangential Stress	Longitudinal Stress
1	4	0.352345	-0.267	1.48	0.502
2	8	0.438688	-0.298	1.43	0.354
3	16	0.455675	-0.704	1.35	0.199
4	24	0.457959	-0.889	1.30	0.122
5	34	0.458220	-0.961	1.27	0.092
6	46	0.458247	-0.987	1.26	0.081
7	60	0.458250	-0.996	1.25	0.077
8	76	0.458250	-0.999	1.25	0.077
9	94	0.458250	-1.000	1.25	0.075
S A P 90	396	0.4582	-0.99	1.26	0.08
Theoretical		0.4582	-1.0	1.25	0.075

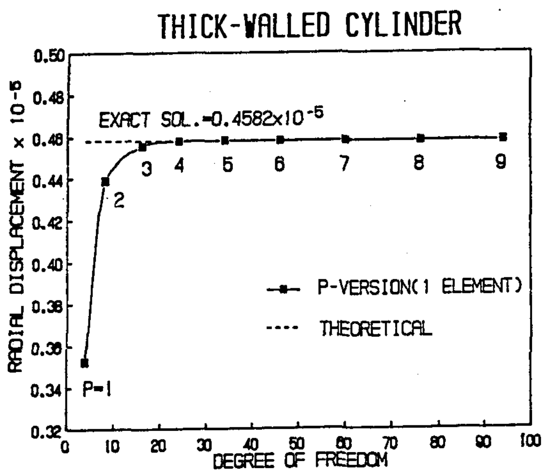

Fig. 6 Convergence of radial displacement

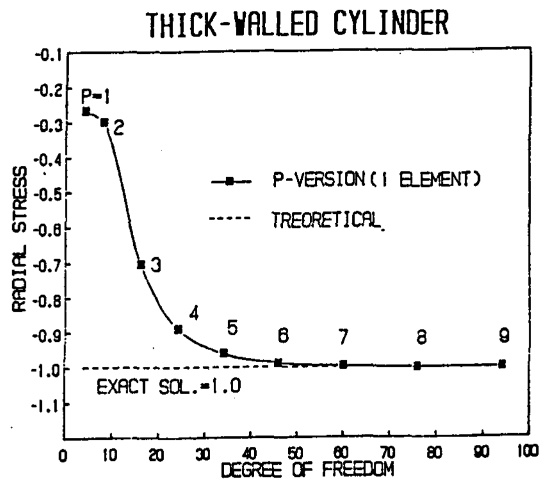

Fig. 7 Convergence of radial stress

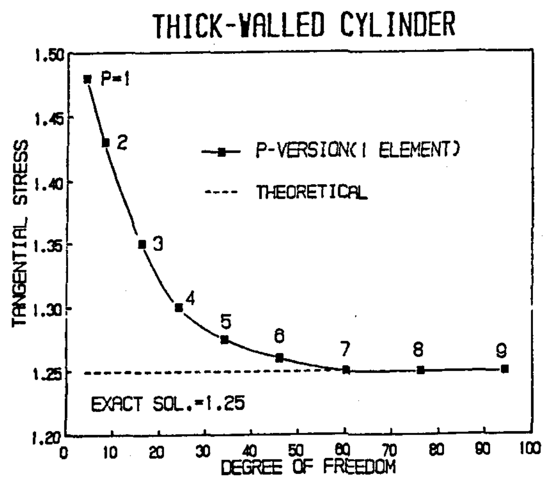

Fig. 8 Convergence of tangential stress

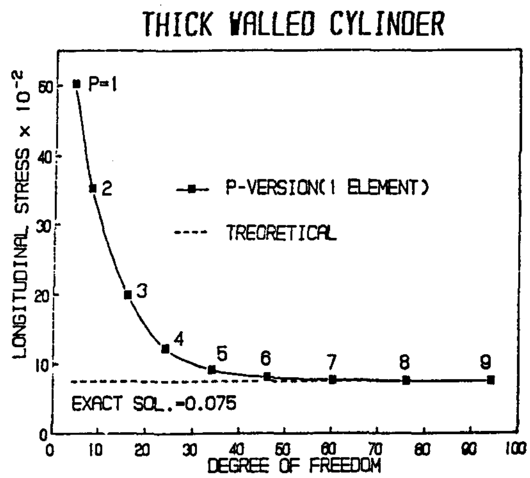

Fig. 9 Convergence of longitudinal stress