1991년도 봄
한국전산구조공학회 학술발표회 논문집

SFRC 보에 대한 System Identification

System Identification on SFRC Beam

o 이 차 돈 *
Lee, Cha-Don

ABSTRACT

Considering the relatively large amount of stable flexural test results available for steel fiber reinforced concrete (SFRC) and their dependency on the constitutive behavior of the material, a technique called "System Identification" is used for interpretating the flexural test data in order to obtain basic information on the tensile constitutive behavior of steel fiber reinforced concrete. "System Identification" was successful in obtaining optimum sets of parameters which provide satisfactory matches between the measured and predicted flexural load-deflection relationships.

1. INTRODUCTION

Flexural load-deflection relationships for SFRC are dependent on the tensile and compressive constitutive behavior of the material. The relative ease of conducting flexural tests compared with direct tension tests and large amount of available flexural test results are reflected as an inverse problem in this study on deriving basic SFRC tensile constitutive behavior of SFRC under flexure. Inverse problem is solved in this investigation by using the method of "System Identification." The derived values through "System Identification" are then compared with analytically, and experimentally obtained values, and some discussions are made regarding the strain gradient effects on constitutive behavior of steel fiber reinforced concrete.

2. SYSTEM IDENTIFICATION

In "System Identification" the response of the system to a given input is known from experiments and a mathematical model is to be found which will describe the material behavior. The mathematical models which can simulate both the physical flexural behavior of SFRC and constitutive behavior of the material must be well established. The characteristic material values in constitutive models are then adjusted until the best possible correlation is achieved between the predicted and measured responses of SFRC under flexure.

A mathematical form for error function is needed to measure the correlation between test results and predictions of the mathematical model for a given set of characteristic values. The error function should be able to quantify the differences in important flexural characteristics of SFRC. "System Identification" deals with finding the location on the surface with minimum error, the coordinates of which will be the desired optimizing parameters.

In order to simulate the steel fiber reinforced concrete behavior under flexural loading, compressive and tensile constitutive models of SFRC (Soroushian and Lee 1989; 1990) were incorporated into the flexural analysis procedure developed by the authors (Soroushian and Lee 1990). Three important charactersitic parameters of the tensile constitutive behavior of SFRC were then selected out of the ten material-related and ten constitutive behavior-related factors of SFRC, which will be discussed later, and these three parameters were then optimized, while other factors were kept constant as "statndard" values. The "standard" values of the factors have been chosen either on the basis of test results or considering practical ranges applicable to SFRC.

The error function (E) is defined to measure the correlation in overall flexural behavior between the experimentally measured and theoretically predicted load-deflection relationships. The characteristic values expressing the flexural behavior of SFRC are peak flexural load (P), flexural ductility (D), and flexural toughness (A). The differences in these characteristic values set the bases for computing the error between predicted and experimental flexural load-deflection relationships:

$$E = \sum_{i=1}^{3} \omega_i \cdot e_i^2 \qquad (1)$$

where ω_i = weighing coefficients for each factor = 1.0 in this investigation; $e_1 = (P_e - P_t)/P_e$; $e_2 = (D_e - D_t)/D_e$; $e_3 = (A_e - A_t)/A_e$; P = ultimate load (Fig. 1); D = ductility = P/P_r (see Fig. 1); A = toughness = area under load-deflection curve as defined in Fig. 1; and subscripts "e" and "t" represent "experiment" and "theory," respectively.

The error function derived above is an objective measure of how well the model fits the experimental data. The error function should be minimized in the N-parametric space. Nonlinear programming techinques can be used for this purpose. The nature of the present study suggests that the minimum point lies in the interior of the feasible region of the parameter space rather than on its bondary, and thus

* 정회원 중앙대학교 건설대 건축공학과 조교수

unconstrained nonlinear programmings suit this problem.

An iterative minimization algorithm was used in the related unconstrained nonlinear programming approach. The algorithm must be able to converge to a stationary point in the global sense and should also converge rapidly when it is in the neighborhood of a local minimum (Luenberger 1973). The iterative minimization approach adopted in this investigation is described below. Starting from the point in the parameter space selected after k steps (x_k), choose the next point as follows:

$$x_{k+1} = x_k + \mu \cdot d \qquad (2)$$

where d = direction search vector; and μ = step length. Individual methods vary in their choice of μ and d and this choice in general determines the efficiency of the method. Calculation of the gradient numerically rather than analytically may be desirable or even necessary. As the calculation of partial derivatives of a given function is, in general, at least as complicated as calculation of function itself, a method which avoids the calculation of derivatives has the possibility of being more efficient as well as having the advantage of being more convenient to use. One such method has been given by Powell 1964. The basic Powell's algorithm chosen for use in this study is presented below and it is modified in this study to properly choose the direction vectors in order to avoid possible break down due to linear dependency of the direction vectors (refer to Walsh 1975 for details). The k^{th} iteration of this method starts with a current point x_k and n directions, $d_{k,j}$, j = 1,2,··· n. At the beginning, x_1 and $d_{1,j}$ are assumed to be given.

1. Let $y_{k,0} = x_k$
2. Find λ_j^* which minimizes function $f(y_{k,j-1} + \lambda_j \cdot d_{k,j})$ and let $y_{k,j} = y_{k,j-1} + \lambda_j^* \cdot d_{k,j}$ for j = 1,2, ··· ,n.
3. Let $\delta_k = y_{k,n} - x_k$
4. Find λ_n^* which minimizes $f(y_{k,n} + \lambda_n \cdot \delta_k)$ and let $x_{k+1} = y_{k,n} + \lambda_n^* \cdot \delta_k$.
5. Let $d_{k+1,j} = d_{k,j+1}$, j = 1,2, ··· ,n-1 and $d_{k+1,n} = \delta_k$. The direction $d_{k,1}$ is discarded in favor of a new direction δ_k.
6. Go to step 1 and restart for $(k+1)^{th}$ step.

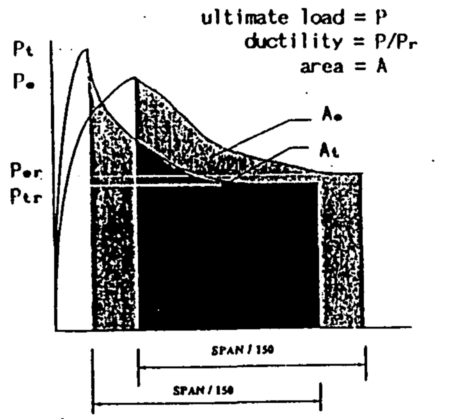


Fig. 1. Definitions of three different criteria

The k^{th} cycle which contains (n+1) subcycles for finding minimum along the given direction is schematically shown in Fig. 2 for n = 2. In this figure, superscripts and subscripts represent the subcycle number and iteration number in a certain subcycle, respectively. In Powell's method, (n+1) line searches are needed to generate one conjugate direction. Therefore, to find the global minimum point (assuming that the given function is quadratic and positive definite) a total of n(n+1) line searches are required. Since in the Powell's method, the error function is being approximated by a quadratic function, it seems to be appropriate to use quadratic line search. In the present study, the method of quadratic line search described by Powell (Powell 1964) has been used.

2. SELECTION OF PARAMETERS

The flexural model containes ten material-related and ten constitutive behavior-related factors (Fig. 3). . The variations in some of these factors have significant effects on the behavior of SFRC under flexure, while variations in other factors result in negligible effects on the flexural behavior of SFRC. Since it is not practical to optimize all these factors in the process of "System Identification," factors whose variations result in significant effects on the flexural behavior of SFRC need to be selected as the "System Identification" parameters.

Soroushian and Lee (Soroushian and Lee 1990) have examined the influence of each factor on the flexural peak load (P), flexural ductility

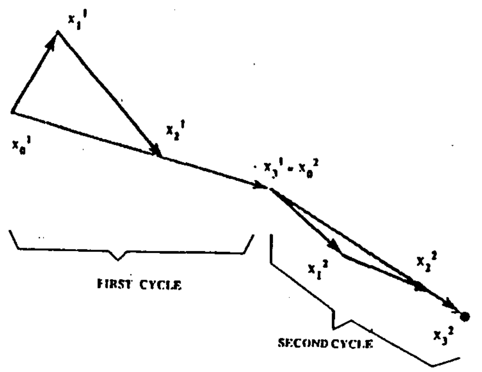


Fig. 2. Main theorems in Powell's Algorithm for n = 2

(D), flexural toughness (A) and overall flexural behavior of SFRC which was described by combination of P,D and A defined above. It was observed that in the case of material-related factors, the fiber peak pull-out strength (τ_u), fiber diameter (d_f), fiber length (l_f), fiber volume fraction (V_f), matrix tensile strength (σ_m'), and fiber slip at residual pull-out strength (S_r) are the most influential factors deciding the flexural behavior of SFRC. As far as the constitutive behavior-related factors are concerned, it was shown that their effects are negligible when compared with those of the material-related factors (Soroushian and Lee 1990).

Among the six influential material-related factors, those representing fiber dimensions (i.e., d_f and l_f) as well as the volume fraction of fibers (V_f) should be known inputs while analyzing some flexural test data obtained for SFRC. This further reduces the number of "System Identification" parameters and leaves only three material-related factors to be entered as parameters in "System Identification:" fiber peak pull-out strength (τ_u), fiber slip at residual pull-out strength (S_r) and matrix tensile strength (σ_m'). It is worth mentioning that the tensile stregnth of SFRC can be determined once the values of these three factors are obtained through analysis of flexural results using "System Identification."

3. RESULTS OF "SYSTEM IDENTIFICATION"

Table 1 summarizes conditions of the SFRC flexural tests considered for "System Identification," and also presents the optimized values of the three main parameters obtained from "System Identification." Fig. 4 illustrates some typical comparisons between the experimentally obtained and theoretically optimized flexural load-deflection curves. Satisfactory correlations are observed in these figures. From Table 1, the optimized values of three parameters are found to be larger than the values obtained from direct tension and material tests (see the comparison presented in Tabel 2). The experimental data presented in Table 2 are the averages obtained from several direct tension and fiber pull-out test performed on materials comparable to those used in flexual tests. The matrix tensile strength (σ_m') and performance of fibers obtained from the analysis of flexural test results may be improved in comparison with those obtained from direct tension and pull-out tests due to the strain gradient effects under flexural loading condition, which generally lead to improved tensile performance of the material (Swamy et al. 1974). The improvements in pull-out performance in fleuxural test specimens over those obtained from single fiber pull-out tests may also be attributed to the positive effects of fiber reinforcement at the surrounding matrix (noting that single fiber pull-out tests are generally conducted using non-fibrous surrounding matrices) in flexural test specimens. Swamy et al. 1974, using an analysis of experimental data, has also reported increase in pull-out strength under flexure when compared with pull-out strength under tension.

Large variations in the values of parameters (τ_u, σ_m', and S_r) obtained from "System Identification" in Table 1 suggest that the highly variable (and unreliable) measurements of flexural deflections in the pre-peak region have some influence on the analysis of flexural test data using the "System Identification" approach. These variations may also partly result from the fact that some flexural test results reported in the literature were not accompanied by reliable information on basic material properties and thus some assumptions had to be made on these properties through the course of "System Identification."

4. CONCLUSION

An analysis of the results indicated that:

(1) The improvements in pull-out performance in flexural tests over those obtained from single fiber pull-out tests (where fibers are generally pulled out of non-fibrous matrices) may also be attributed to the positive effects

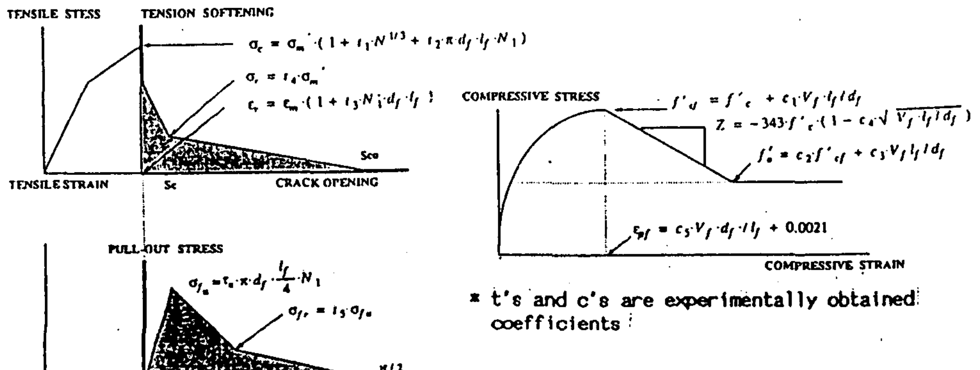


Fig. 3. Factors considered in flexural analysis

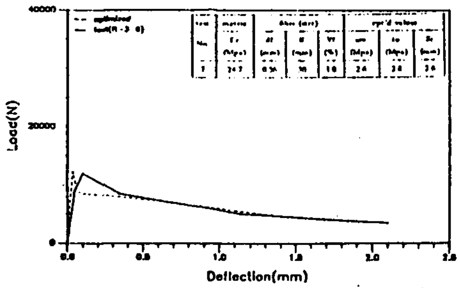
of fiber reinforcement of the surrounding matrix in flexural test specimens.

(2) The matrix tensile stregnth (σ_m') and pull-out performance of fibers obtained from the analysis of flexural test results were superior to those obtained from direct tension and pull-out tests. This may be attributed to the positive effect of strain gradient under flexural loads.

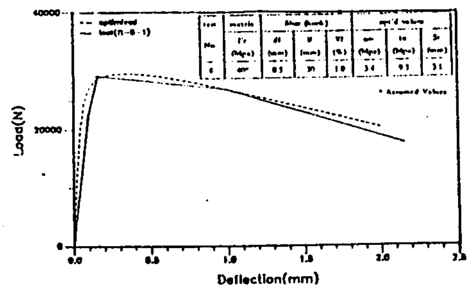
(3) Large variations were observed in the values of parameters (τ_u, σ_m' and S_r) obtained from "System Identification." This could result from both unreliable measurements of flexural deflections in the pre-peak region in some test results reported in the literature, and also from the lack of information on some basic material properites for flexural tests conducted by other investigators.

REFERENCES

1. Soroushian, P. and Lee, C.D. (1989). Constitutive Modeling of Steel Fiber Reinforced Concrete under Direct Tension and Compression. Report MSU-ENGR-89-002, pp. 1-44.
2. Soroushian, P. and Lee, C.D. (1990). Flexural Analysis of Steel Fiber Reinforced Concrete. Report, Michigan State University.
3. Luenberger, D.G. (1973). Introduction to Linear and Nonlinear Programming, Mass. Addison Wesley Publishing Company.
4. Powell, M.J.D. (1964). An Efficient Method for Finding the Minimum of a Function of Several Variables without Calculating Derivatives. Computer Journal. Vol. 7.
5. Walsh, G.R. (1975). Method of Optimization, John Wiley and Sons Ltd.



(a) test results from Soroushian and Ateff. 1989



(b) test results from Sakai et al. 1986

Fig. 4. Comparisons between experimentally obtained and theoretically optimized flexural load-deflection curves

Table 1. Test conditions and optimized values from "System Identification"

Ref.	Test No.	Specimen width	Specimen depth	Specimen length	Fiber type	Fiber df	Fiber lf	Fiber Vf	f'c	Opt'd Values σm'	Opt'd Values τu	Opt'd Values Sr	Error	It. No
Sakai and Nakamura, 1986	1	100	100	300	str	0.56	30	0.01	(40)	5.032	6.174	2.72	0.000456	4
	2	100	100	300	str	0.56	30	0.015	(40)	5.895	5.036	3.441	0.000024	3
	3	100	100	300	str	0.56	30	0.02	(40)	7.132	4.413	3.447	0.000011	6
Cho and Kobayashi, 1982	4	100	100	300	str	0.56	30	0.01	34.6	3.332	3.831	2.198	0.010869	6
	5	100	100	300	str	0.56	30	0.015	34.6	4.649	3.933	3.121	0.000273	3
	6	100	100	300	str	0.56	30	0.01	48	3.032	5.0	3.0	0.026306	2
	7	100	100	300	str	0.56	30	0.01	24.7	2.564	2.752	2.56	0.000494	7
Soroshian and Ateff, 1989	8	100	100	300	hook	0.5	30	0.01	(40)	3.444	9.291	3.085	0.000231	3
	9	100	100	300	hook	0.5	30	0.01	(40)	3.831	7.73	6.247	0.000967	4
	10	100	100	300	hook	0.5	30	0.01	(40)	3.695	5.371	2.957	0.004180	2
	11	100	100	300	hook	0.5	30	0.01	(40)	2.57	6.25	2.887	0.003061	3

Values in parenthesis are assumed ones.

Table 2. Comparison of the tension test results with the optimum values of parameters in analysis of flexural test results using "System Identification"

Ref.	Test No.	Fiber type	Fiber df	Fiber lf	Fiber Vf	σ_m' $(0.332\sqrt{f'c})$	Ratios σ_m^o/σ_m'	Ratios τ_u^o/τ_u	Ratios Sr^o/Sr
Sakai and Nakamura, 1986	1	str	0.56	30	0.01	2.1	2.4	2.35	0.97
	2	str	0.56	30	0.015	2.1	2.8	1.92	1.23
	3	str	0.56	30	0.02	2.1	3.4	1.68	1.23
Cho and Kobayashi, 1982	4	str	0.56	30	0.01	1.95	1.7	1.45	0.97
	5	str	0.56	30	0.015	1.95	2.4	1.50	0.11
	6	str	0.56	30	0.01	2.30	1.32	1.90	1.07
	7	str	0.56	30	0.01	1.65	1.55	1.05	0.91
Soroshian and Ateff, 1989	8	hook	0.5	30	0.01	2.1	1.63	2.07	1.10
	9	hook	0.5	30	0.01	2.1	1.82	1.72	2.23
	10	hook	0.5	30	0.01	2.1	1.76	1.20	1.05
	11	hook	0.5	30	0.01	2.1	1.22	1.35	1.03

6. Swamy, R.N. and Mangat, P.S. (1974). A Theory for the Flexural Strength of Steel Fiber Reinforced Concrete. Cement and Concrete Research. Vol. 4. pp. 313-325.
7. Soroushian, P. and Ateff, K. (1989). Mechnical Performance of Latex Modified Steel Fiber Reinforced Concrete. Report, Michigan State University.
8. Sakai, M. and Nakamura, N. (1986). Analysis of Flexural Behavior of Steel Fiber Reinforced Concrete, RILEM
9. Cho, R. and Kobayashi, K. (1982). Flexural Characteristics of Steel Fiber and Polyethylene Fibre Hybrid-Reinforced Concrete. Composites. pp.164-168.