단면 반사율이 Bragg Reflector 구조의 전체 반사율 스펙트럼에 미치는 효과

The Effect of Front Facet Reflections on the Reflectivity Spectrum of Bragg Reflector structures

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We present an analytic equation for the reflectivity spectrum of a Bragg reflector in terms of the front mirror reflectivity, due to the refractive index difference between the refractive index of outside medium and the average refractive index of Bragg reflector structures, and the reflectivity of a Bragg reflector calculated by the coupled wave method. We show that even Fresnel reflection causes the reflectivity spectrum of a Bragg reflector to be very different from that of Bragg reflectors calculated by the coupled wave method. The reflectivity spectrum of a Bragg reflector is dramatically changed because the interference effect between the reflected wave from the front facet and that from the Bragg reflector is changed due to the difference of a phase change from a Bragg reflector when the sequence of layers in a Bragg reflector is changed.

1 Introduction

Many researchers studying the characteristics of optical devices employing distributed Bragg reflector (DBR) structures have ignored the effect of end facet reflections on the reflectivity spectrum of the Bragg reflector. However, this effect can be very important when using semiconductors, due to the high Fresnel reflection at the boundaries of devices. Winful considered a lossless nonlinear DBR with the same reflectivity at both ends of the device and indicated that end reflections had a detrimental effect on the switching capability [1]. Milsom et al. showed that both the linear and nonlinear response of a lossless DBR structure in the case of the same reflectivity at both ends of the device can be altered by changing the grating position relative to the end mirror reflectors [2]. Streifer et al. showed that the end reflections occuring in distributed feedback (DFB) lasers affect a lot the characteristics of longitudinal modes of DFB lasers [3].

Since the coupled-wave method assumes that the beam is incident into the DFB structure from a medium with an average refractive index of the DFB structure [4], the reflectivity spectrum calculated by the coupled wave method is not valid when the beam is incident into the Bragg reflector structure from a medium which does not have an average refractive index of the Bragg reflector structure. Thus, we have to consider the reflection due to the refractive index difference between the refractive index of outside medium and the average refractive index of Bragg reflector structures to calculate the reflectivity spectrum of Bragg reflector structures. We will call this reflection front facet reflection.

Here we investigate the effect of front facet reflections and phase changes from a Bragg reflector on the reflectivity spectrum of a mirrored Bragg reflector (MBR) which has one of its end mirror reflectivities as nonzero and the other as zero. Since a MBR which has a back mirror with the wavelength dependent reflectivity and phase change, a spacer layer of zero thickness, and a front mirror with the front facet reflection operates on multipass interference, similar to a Fabry-Perot, we explained the results in terms of a Fabry-Perot structure. We present an analytic equation for the reflectivity spectrum of a MBR in terms of front mirror reflectivity and the reflectivity of a Bragg reflector calculated by the coupled wave method. We show that even Fresnel reflection causes the reflectivity spectrum of a MBR to be very different from that of Bragg reflectors embedded within a semiconductor medium, (such as calculated by the coupled wave method). If the sequence of layers in a Bragg reflector is changed, the phase of the reflecton coefficient changes by π radian. Thus, the reflectivity spectrum of a MBR is dramatically changed because the interference effect between the reflected wave from the front facet and that from the Bragg reflector is changed.

2 Derivation of the Reflection Coefficient

Consider the Bragg reflectors shown in Fig. 1. Using the coupled-wave method, we obtain an analytic expression for the reflection coefficient of a Bragg reflector in the simultaneous presence of index modulation and absorption modulation [5]

$$r = \frac{E_b(0)}{E_f(0)} = i(K - iF)\frac{sinh(sL)}{s\, cosh(sL) + i(\delta - ia\frac{\alpha}{2})sinh(sL)} \quad (1)$$

where

$$s = \sqrt{\{(K^2 - F^2) - \delta^2 + \frac{a^2\alpha^2}{4}\} + i\{\delta a\alpha - 2KF\}}. \quad (2)$$

Here $K = sin(a\pi)(n_h^2 - n_L^2)/(n_{av}\lambda_B)$, $F = sin(a\pi)\alpha/(2\pi)$, $\delta = \beta - (\pi/\Lambda)$, $n_{av} = 2n_h n_L/(n_h + n_L)$, and a is a measure of the difference of adjacent layer thickness of a Bragg reflector defined by $a = n_L/(n_h + n_L)$.

This reflection coefficient of Eq. (1) is obtained when a half of a quarter wavelength layer with low refractive index is on the top of a multilayer Bragg reflector which begins with a high index layer. This is because we chose the origin of coordinate system $z = 0$ in the middle of the layer with low refractive index in order to calculate the Fourier components of the complex dielectric constant step as real values. We could think of this low refractive index layer whose thickness is one-eighth of an optical wavelength as a phase shifter. Then we could obtain the reflection coefficient of a Bragg reflector when the outermost layer has the high refractive index, r_h, as

$$r_h = \frac{E_b(0)e^{i\frac{\pi}{4}}}{E_f(0)e^{-i\frac{\pi}{4}}} = re^{i\frac{\pi}{2}} = ir. \quad (3)$$

If the layer sequence is changed (outermost layer has the low refractive index), the reflection coefficient, r_L, is given by

$$r_L = \frac{E_b(0)e^{-i\frac{\pi}{4}}}{E_f(0)e^{i\frac{\pi}{4}}} = re^{-i\frac{\pi}{2}} = -ir = -r_h. \quad (4)$$

When the layer sequence is changed, the phase of the reflection coefficient changes by π radian; however the magnitude of the reflection coefficient is the same.

3 The Effect of Front Facet Reflections and Phase change

We investigate the effect of front facet reflections and phase change from a Bragg reflector on the reflectivity spectrum of a MBR. Since the reflection from the interface between a Bragg reflector and a substrate is negligible when the absorbing substrate left intact, we will assume that one of end mirror reflectivities is finite and the other is zero. We will approach the MBR as a Fabry-Perot, with a back mirror as a Bragg reflector having a wavelength dependent reflection coefficient and phase change, a spacer layer of zero thickness, and a front mirror with the front facet reflection. We present an analytic equation for the reflectivity spectrum of a MBR in terms of the front facet reflectivity and the reflectivity of a Bragg reflector calculated by the coupled wave method. We show that even ordinary Fresnel reflection causes the reflectivity spectrum of a MBR to be very different from that of a Bragg reflector embedded within a semiconductor medium (such as calculated by the coupled wave method). If the sequence of layers in a Bragg reflector is changed, the reflectivity spectrum of a MBR is dramatically changed due to the difference of phase change from a Bragg reflector.

The expression for the reflectivity (R) of a Fabry-Perot in which the spacer is characterized by an absorption coefficient α and refractive index n, and having front and back mirror reflectivities of R_f and R_b, and the phase change from front and back mirror of ϕ_f and ϕ_b are given by:

$$R = \frac{E + F sin^2(\phi + \phi_f)}{1 + F sin^2(\phi)}, \qquad (5)$$

where $F = 4R_e/(1-R_e)^2$, $E = (R_f - R_e)^2/R_f(1-R_e)^2$, and

$R_e = \sqrt{R_f R_b} e^{-\alpha D}$. Here ϕ_f is the phase change of wave which is incident from the spacer to the outside medium. The half round trip phase change ϕ is given by

$$\phi = \frac{2\pi}{\lambda} nD - \frac{\phi_f}{2} - \frac{\phi_b}{2}, \qquad (6)$$

where D is the spacer thickness.

In the case of a MBR, R_b is the reflectivity of a Bragg reflector and ϕ_b is the phase change from a Bragg reflector calculated by the coupled wave method, R_f is the reflectivity of the front facet and ϕ_f is the phase change from the front facet due to the refractive index difference between the refractive index of outside medium and the average refractive index of Bragg reflector structures, and the spacer thickness D is zero. If we wish to ignore the effect of end facet reflections, we set the front mirror reflectivity to zero. Then, Eq. (5) becomes the reflectivity of a Bragg reflector, R_b, (because R_e is zero). Thus, we could expect that the reflectivity spectrum of a MBR is different from that of a Bragg reflector calculated by the coupled wave method due to the existence of a nonzero value of the front facet reflectivity and the wavelength dependent phase change of waves from a Bragg reflector.

The phase change from the front facet, ϕ_f, will be either 0 or π. We showed that, when only the sequence of layers in a Bragg reflector is changed, the phase change as a function of detuning is the same except that the phase of the reflection coefficient changes by π radian in Section 2. In the case of a MBR, the half round trip phase change ϕ is given by $-(\phi_f + \phi_b)/2$. Thus, ϕ in a MBR when a Bragg reflector has a low refractive index outermost layer when $\phi_f = 0$ is the same as that when a Bragg reflector has a high refractive index outermost layer when $\phi_f = \pi$ and the opposite case is the same. Thus, for our calculations we may assume, without the loss of generality, that $\phi_f = 0$. If $\phi_f = \pi$, the results in the MBR obtained by $\phi_f = 0$ should be reversed as explained above.

We obtained analytic equations for the reflectivity spectrum as a function of detuning of a Bragg reflector in the case of simultaneous refractive index and absorption modulation using the coupled wave method in Section 2. When a Bragg reflector has the outermost layer with high refractive index, the reflection coefficient is given by Eq. (3) and when a Bragg reflector has the outermost layer with low refractive index, the reflection coefficient is given by Eq. (4). From these equations, we obtain the analytic equation for the reflectivity of a back mirror, R_b, and the phase change from a back mirror, ϕ_b, as a function of detuning. Then, we obtain the reflectivity spectrum of a MBR as a function of detuning using Eqs. (5) and (6).

At zero detuning, we obtain simple analytic expressions for R_b and ϕ_b when the refractive index modulation is larger than the absorption modulation ($K > F$) as

$$R_l = \frac{cosh(2KL) - cos(2FL)}{cosh(2KL) + cos(2FL)},$$

$$\phi_{l,L} = -tan^{-1}\{\frac{sin(2FL)}{sinh(2KL)}\} \text{ and } \phi_{l,h} = \pi + \phi_{l,L}, \quad (7)$$

where h and L represent the outermost layer of a Bragg reflector as the high refractive index and the low refractive index, respectively. Then, we obtain the reflectivity of a MBR at zero detuning using Eqs. (5), (6), and (7).

The phase change from a lossless Bragg reflector is antisymmetric with respect to detuning [6]. Also, we showed that the reflectivity spectrum of a lossless Bragg reflector is symmetric with respect to detuning in Section 2. Furthermore, the reflectivity spectrum of a Fabry-Perot is symmetric as a function of ϕ with respect to resonance ($\phi = 0$), and antiresonance ($\phi = \pm\pi/2$). Then, the reflectivity spectrum of a lossless MBR is symmetric with respect to detuning. Thus, we plot the reflectivity spectrum only in the positive detuning region in a lossless case. Of course, if a MBR has a loss in the Bragg reflector, the reflectivity spectrum is not symmetric with respect to detuning.

At zero detuning in a lossless case, we obtain R_l and ϕ_l from Eq. (7) as

$$R_l = tanh^2(KL),$$

$$\phi_{l,L} = 0 \text{ and } \phi_{l,h} = \pi. \quad (8)$$

When a Bragg reflector has a low refractive index outermost layer, the half round trip phase change of a MBR is zero from Eq. (6). Then, the reflectivity of a lossless MBR at zero detuning when $\phi = 0$ is given by from Eqs. (5) and (8)

$$R_L = \frac{\{\sqrt{R_f} - tanh(KL)\}^2}{\{1 - tanh(KL)\sqrt{R_f}\}^2}. \quad (9)$$

Fig. 2 shows the reflectivity of a MBR at zero detuning when $\phi = 0$ for various KL as a function of R_f using Eq. (9). From Fig. 2, we can see that the zero detuning reflectivity of a MBR decreases to zero as the front facet reflectivity increases until it equals the back mirror reflectivity. Zero overall reflectivity is due to the destructive interference of waves from the front facet and the Bragg reflector. As the front facet reflectivity increases further, the reflectivity of a MBR increases.

When a Bragg reflector has a high refractive index outermost layer, the half round trip phase change of a MBR is $-\pi/2$ from Eq. (6). Then, the reflectivity of a lossless MBR at zero detuning when $\phi = -\pi/2$ is given by from Eqs. (5) and (8)

$$R_h = \frac{\{\sqrt{R_f} + tanh(KL)\}^2}{\{1 + tanh(KL)\sqrt{R_f}\}^2}. \quad (10)$$

Fig. 3 shows the reflectivity of a MBR at zero detuning when $\phi = -\pi/2$ for various KL as a function of R_f using Eq. (10). The zero detuning reflectivity of this MBR increases as the front facet reflectivity increases due to the constructive interference of waves from the front facet and the Bragg reflector.

The reflectivity difference (ΔR) between R_h and R_L is given by from Eqs. (9) and (10)

$$\Delta R = \frac{4\, tanh(KL)\sqrt{R_f}(1 - R_F)\{1 - tanh^2(KL)\}}{\{1 - R_f tanh^2(KL)\}^2}. \quad (11)$$

Fig. 4 shows the reflectivity difference of a MBR at zero detuning, when the sequence of layers in a Bragg reflector is changed, for various KL as a function of R_f using Eq. (11). The maximum of the reflectivity difference occurs at the front mirror reflectivity which equals the back mirror reflectivity as we expected. In the region which the front mirror reflectivity is small, the reflectivity difference increases when the back mirror reflectivity decreases because the interference effect between the front facet reflection and the reflection from a Bragg reflector to the reflectivity of a MBR increases as the back mirror reflectivity decreases.

We calculated the reflectivity spectrum of a lossless MBR as a function of detuning for various front facet reflectivities when the Bragg reflector has a low refractive index outermost layer using the matrix method. The result is shown in Fig. 5. The case of $R_f = 0$ is the same as that obtained by the coupled wave method. Since the phase change from a Bragg reflector is zero at zero detuning, the half round trip phase change ϕ is zero at zero detuning (Bragg wavelength) from Eq. (6). Thus, the reflectivity of a MBR at zero detuning is given by Eq. (9). The reflectivity of a MBR decreases to zero as the front facet reflectivity increases until it equals the back mirror reflectivity. As the front mirror reflectivity increases further, the reflectivity of a MBR increases. From Fig. 5, we can see that the reflectivity of a MBR increases even though R_b decreases as detuning increases around zero detuning. The reason is as follows: We know that the reflectivity of a MBR is determined by R_e and ϕ. Since R_b and ϕ_b depend on detuning, R_e and ϕ determining the reflectivity of a MBR depend on detuning. Thus, there is a competition between R_e and ϕ to determine the reflectivity of a MBR as a function of detuning. Since $\phi_b = 0$ at zero detuning and ϕ_b increases in the stop band, ϕ of Eq. (6) decreases from zero as detuning increases. The reflectivity of a MBR increases as detuning increases around zero detuning because a MBR goes from resonance to antiresonance even though R_e decreases due to the decrease of R_b as detuning increases.

For comparison, we repeated the calculation of Fig. 5 except that the sequence of layers in a Bragg reflector is changed. This means that the Bragg reflector used in this case has a high refractive index outermost layer. The phase change from such a Bragg reflector is π radian at zero detuning. Thus, the MBR is now on antiresonance at zero detuning from Eq. (6) ($\phi = -\pi/2$).

The result is shown in Fig. 6. We note that, when $R_f = 0$, Fig. 6 is the same as in Fig. 5, as expected. Since the reflectivity of a conventional Fabry-Perot has the maximum value on antiresonance ($\phi = \pm\pi/2$), the MBR has a maximum reflectivity at zero detuning. At zero detuning the reflectivity of the MBR increases as the front facet reflectivity increases because R_e increases. The reflectivity of the MBR decreases as detuning increases because the MBR goes from antiresonance to resonance and R_e decreases with increasing detuning due to the wavelength dependent R_b and ϕ_b. Since the minimum reflectivity of a MBR occurs on resonance, a phase condition which is independent of the front facet reflectivity, the detuning at which the minimum reflectivity occurs is the same independent of the front facet reflectivity.

We conclude that the differences in the reflectivity spectrum of a MBR between Figs. 5 and 6 come from the different interference effect due to the dependence of phase changes from a Bragg reflector dependent on the sequence of layers in a Bragg reflector.

4 Conclusion

We investigated the effect of front facet reflections and phase changes from a Bragg reflector on the reflectivity spectrum of a Bragg reflector. We presented an analytic equation for the reflectivity spectrum of a Bragg reflector in terms of the front mirror reflectivity and the reflectivity of a Bragg reflector calculated by the coupled wave method. We showed that even Fresnel reflection causes the reflectivity spectrum of a Bragg reflector to be very different from that of Bragg reflectors calculated by the coupled wave method. The reflectivity spectrum of a Bragg reflector is dramatically changed because the interference effect between the reflected wave from the front facet and that from the Bragg reflector is changed due to the difference of a phase change from a Bragg reflector when the sequence of layers in a Bragg reflector is changed.

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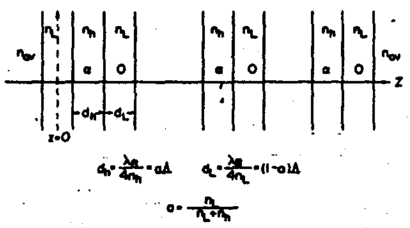

Fig. 1: The geometry of a Bragg reflector.

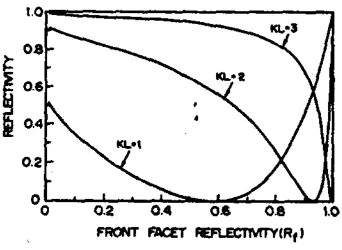

Fig. 2: The reflectivity of a MBR at zero detuning when $\phi = 0$ as a function of R_f for various KL.

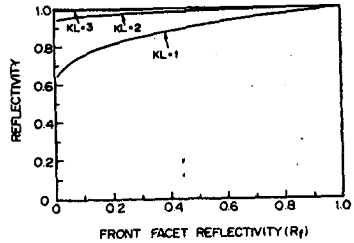

Fig. 3: The reflectivity of a MBR at zero detuning when $\phi = -\pi/2$ as a function of R_f for various KL.

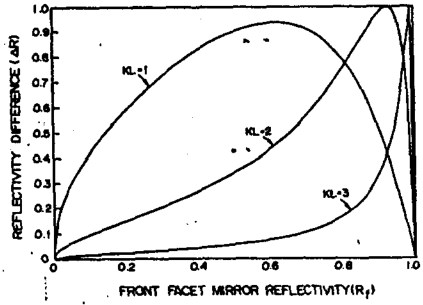

Fig. 4: The reflectivity difference of a MBR at zero detuning as a function of R_f for various KL.

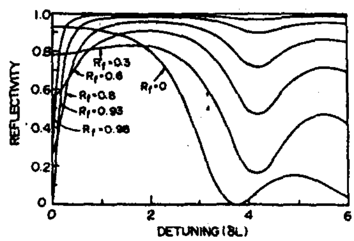

Fig. 5: The reflectivity spectrum of a MBR which has a low refractive index top layer with $KL = 2$ as a function of detuning for various R_f.

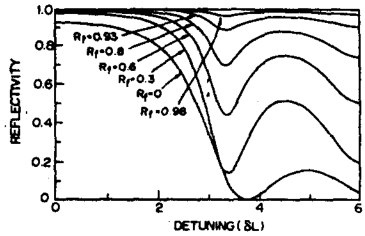

Fig. 6: The reflectivity spectrum of a MBR which has a high refractive index top layer with $KL = 2$ as a function of detuning for various R_f.