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점근적 신뢰성이 있는 폐쇄직렬 생산시스템에 관한 연구

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On Asymptotically Reliable Closed Serial Production Systems

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Abstract

A problem of analysis and design of asymptotically reliable serial production systems, closed with respect to the number of carriers that transport parts between operations, is addressed. A case study of a paint shop operation at a modern automobile assembly plant is described. The results obtained indicate that optimization of the system with respect to the number of carriers available and the capacity of the feedback buffer leads to a substential improvement in the production rate.

1. INTRODUCTION

In [1] and [2], we considered the **open** asymptotically reliable serial production lines defined by the following assumptions:

(i) The system consists of M machines m_i, $i = 1, ..., M$, arranged in the consecutive order and $M-1$ buffers B_i seperating each two machines, m_i and m_{i+1}.

(ii) The machines have identical cycle time T. The time axis is slotted with the slot duration T. Each machine begins its operation at the beginning of the time slot.

(iii) Each buffer is characterized by its capacity, N_i, $i = 1, ..., M-1$, where N_i is a positive integer.

(iv) Machine m_i is starved during a time slot if buffer B_{i-1} is empty at the beginning of this time slot, machine m_i is blocked during a time slot if at the beginning of this time slot buffer B_i is full and machine m_{i+1} is either down or blocked. Machine m_1 is never starved, machine m_M is never blocked.

(v) Machine m_i, being not blocked and not starved, produces a part during any time slot with probability $q_i = 1-\epsilon k_i$ and fails to do so with probability ϵk_1, $i = 1, ..., M$, where $0 < \epsilon \ll 1$ and $k_i > 0$ is independent of ϵ. The k_i's are called the loss parameters.

In many practical situations, however, serial production lines are **closed**, i.e., have a feedback loop with respect to carriers on which the parts (jobs) are transported from one machine to another. This is, in particular, the case in assembly and painting operations in the automobile industry where car bodies and engine blocks are transferred between operations on carriers, and the number of carriers in the system is constant. To account for this situation, introduce the following assumptions:

(vi) The jobs are transported within the system on carriers. Each job is placed on a carrier at the input of machine m_1 and is removed from the carrier at the output of machine m_M. Empty carriers are returned to the empty carrier buffer, B_M, and are supplied to the input of m_1 instantaneously, given that B_M is not empty. The capacity of B_M is N_M. The total number of carriers in the system is S, where $M \leq S \leq \sum_{i=1}^{M} N_i$.

Manufacturing systems defined by (i)-(vi) are referred to as asymptotically reliable closed serial production lines. Performance of closed lines can be characterized by their production rate, PR_c, i.e., the average number of jobs

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produced in the steady state by the last machine, m_M. The problem of **analysis** of these lines is formulated as follows: Given, $k_1, N_1, ..., k_M, N_M$, and S, find $PR_c(k_1, N_1, ..., k_M, N_M, S)$. The probem of **design** is: Given $k_1, N_1, ..., k_{M-1}, N_{M-1}, k_M$, find the smallest S and N_M so that

$$PR_c(k_1, N_1, ..., k_M, N_M, S) = PR_o(k_1, N_1, ..., k_{M-1}, N_{M-1}, k_M) \quad (1)$$

Analyses of closed Markovian queuing systems have been reported in [3]-[14]. Most of them, however, address non-blocking systems with infinite buffers [3]-[10]. In [11], an approximate analysis of finite buffer systems has been carried out using a state space reduction technique. An application of mean value analysis has been reported in [12]. Papers [13] and [14] present another approximation technique based on a curve fitting approach.

Nevertheless, a relatively general treatment of performance characteristics of closed Markovian queuing systems is still missing. In this paper analytic methods are developed for performance evaluation of closed Markovian queuing systems.

2. ASYMPTOTIC ANALYSIS

Theorem 2.1: The performance of closed serial production lines defined by (i)-(vi) with $M = 2$ is as follows:

$$\begin{aligned} PR_c(k_1, N_1, k_2, N_2, S) &= PR_o(k_1, N_e, k_2) + O(\epsilon^2) \\ &= 1 - \epsilon[k_2 + k_1 Q(\tfrac{k_2}{k_1}, N_e)] + O(\epsilon^2) \\ &= 1 - \epsilon[k_1 + k_2 Q(\tfrac{k_1}{k_2}, N_e)] + O(\epsilon^2), \end{aligned} \quad (2)$$

where N_e, the effective buffer size, is given by

$$N_e = \begin{cases} S - 1, & \text{for } 2 \leq S \leq \min(N_1, N_2) \\ \min(N_1, N_2), & \text{for } \min(N_1, N_2) < S \leq \max(N_1, N_2) \\ N_1 + N_2 - S + 1, & \text{for } \max(N_1, N_2) < S \leq N_1 + N_2 \end{cases} \quad (3)$$

and $Q(\alpha, N) = \frac{1-\alpha}{1-\alpha^N}$, $\alpha \in R_+$, $N \in [1, \infty)$.

Remarks:

1. From (2), it follows that $PR_c(k_1, N_1, k_2, N_2, S) = PR_c(k_1, N_2, k_2, N_1, S)$, i.e., the permutation of buffers positions does not affect the production rate.

2. Alike open lines, closed lines under consideration are equivalent to a single, aggregated machine characterized, in isolation, by the parameter

$$\begin{aligned} q_{\text{aggregation}} &= 1 - \epsilon[k_2 + k_1 Q(\tfrac{k_2}{k_1}, N_e)] + O(\epsilon^2) \\ &= 1 - \epsilon[k_1 + k_2 Q(\tfrac{k_1}{k_2}, N_e)] + O(\epsilon^2). \end{aligned} \quad (4)$$

3. GENARAL CASE

Let q denote a vector with components $q_1, ..., q_M$, where q_i's are defined in assumption (v). Then, for fixed $N_1, ..., N_M$ and S, the prouction rate, $PR_c(k_1, N_1, ..., k_M, S)$, is a function of q_i; we denote this function as $F(q)$.

Let $I \triangleq \{i_1, ..., i_l\}$ and $\bar{I} \triangleq \{j_1, ..., j_{M-l}\}$ be ordered subsets of $\mu \triangleq \{1, ..., M\}$ such that $I \cup \bar{I} = \mu$ and $I \wedge \bar{I} = \emptyset$. Let q_I denote the vector $q_1, ..., q_M$ with $q_s = 1, \forall s \in I$ and $q_s < 1, \forall s \in \bar{I}$.

Assume, for instance, that the cardinality of I is 1, i.e., $I = \{i\}$. Then machine i introduces no losses and can be eliminated from consideration. Therefore, one might expect that

$$\begin{aligned} &PR_c(k_1, N_1, ..., k_M, N_M, S) \\ &= PR_c(k_1, N_1, ..., k_{i-1}, N_{i-1} + N_i, k_{i+1}, ..., k_M, N_M, S) + O(\epsilon^2) \end{aligned} \quad (5)$$

The following Lemma states that (5) is indeed true:

Lemma 3.1: Under assumptions (i)-(vi),

$$\begin{aligned} F(q_I) = \; & PR_c(k_{j_1}, \sum_{s=j_1}^{j_2-1} N_s, ..., k_{j_{M-l}}, \sum_{s=j_{M-l}}^{M} N_s \\ & + \sum_{s=1}^{j_1-1} N_s, S) + O(\epsilon^2), \quad \forall l \in \{1, ..., M-2\}. \end{aligned} \quad (6)$$

For all $I \subseteq \mu$, we can write:

$$F(q_I) = F(q_{I\cup\{j\}}) - \epsilon k_j \frac{\partial F(q_{I\cup\{j\}})}{\partial q_j} + O(\epsilon^2), \quad (7)$$

where $q_{I\cup\{j\}}$ is the vector $q_1, ..., q_M$ with $q_i = 1, \forall i \in I$ and $q_j \neq 1$, j is not in I. On the other hand,

$$F(q_I) = F(q_\mu) - \epsilon \sum_{s\in\bar{I}} k_s \frac{\partial F(q_\mu)}{\partial q_s} + O(\epsilon^2). \quad (8)$$

Finally,

$$\frac{\partial F(q_I)}{\partial q_s} = \frac{\partial F(q_\mu)}{\partial q_s} - \epsilon \sum_{n\in\bar{I}} k_n \frac{\partial^2 F(q_\mu)}{\partial q_n \partial q_s} + O(\epsilon^2). \quad (9)$$

Equations (7)-(9) result in

$$\begin{aligned} & F(q_{I\cup\{j\}}) - \epsilon k_j \frac{\partial F(q_{I\cup\{j\}})}{\partial q_j} \\ &= 1 - \epsilon \sum_{s\in\bar{I}} k_s \frac{\partial F(q_{I\cup\{j\}})}{\partial q_s} + O(\epsilon^2), \quad j \in \bar{I}. \end{aligned} \quad (10)$$

Equation (10) can be rewritten in the vector-matrix notations:

$$A_j x_j = b_j + O(\epsilon), \quad (11)$$

where

$$A_j = \begin{bmatrix} 0 & k_{j_2} & k_{j_3} & \cdots & k_{j_{M-l}} \\ k_{j_1} & 0 & k_{j_3} & \cdots & k_{j_{M-l}} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ k_{j_1} & k_{j_2} & k_{j_3} & \cdots & 0 \end{bmatrix}$$

$$x_j = \begin{bmatrix} \frac{\partial F(q_{I\cup\{j_1\}})}{\partial q_{j_1}} \\ \cdot \\ \cdot \\ \cdot \\ \frac{\partial F(q_{I\cup\{j_{M-l}\}})}{\partial q_{j_{M-l}}} \end{bmatrix}, \quad b_j = \begin{bmatrix} \frac{1-F(q_{I\cup\{j_1\}})}{\epsilon} \\ \cdot \\ \cdot \\ \cdot \\ \frac{1-F(q_{I\cup\{j_{M-l}\}})}{\epsilon} \end{bmatrix} \quad (12)$$

and $O(\epsilon)$ is an $(M-l)$-dimensional vector with components of the order ϵ. If b_j, i.e., $F(q_{I\cup\{j_r\}})$, $r=1,...,M-l$, is known, (10) is an equation for x_j. If x_j is known, $F(q_I)$ can be calculated from (7) or (8). This initiates the following inductive disaggregation procedure:

Procedure 3.1: Given a closed line $m_1, B_1, ..., m_M, B_M$,

1. Construct all two machines - two buffers systems by aggregating consecutive buffers, calculate their production rate estimates using (2) and neglecting $O(\epsilon^2)$ terms, i.e., find

$$PR_c^{est}(q_i, \sum_{j=i}^{i+k} N_j, q_{i+k+1}, \sum_{l=i+k+1}^{i+M-1} N_l, S)$$

for $i=1,...,M$ and $k=0,1,...,M-2$, where the following condition is used: $q_{i+M}=q_i$, and $N_{i+M}=N_i$.

2. Construct 3 machines - 3 buffers systems by aggregating consecutive buffers, calculate their production rate estimates using (11), (12), (7) and neglecting $O(\epsilon^2)$ terms, i.e., find

$$PR_c^{est}(q_i, \sum_{j=i}^{i+k} N_j, q_{i+k+1}, \sum_{l=i+k+1}^{i+g} N_l, q_{i+g+1}, \sum_{n=i+g+1}^{i+M-1} N_n, S)$$

for $i=1,...,M$, $k=0,1,...,M-2$, and $k<g<M-1$. Here again $q_{i+M}=q_i$, and $N_{i+M}=N_i$. Continue this disaggregation process until $(M-1)$ machines - $(M-1)$ buffers systems are obtained and their production rate estimates,

$$PR_c^{est}(q_i, N_i, ..., q_k, N_k+N_{k+1}, q_{k+2}, N_{k+2}, ..., q_{i+M-1}, N_{i+M-1}, S),$$
$$i=1,...,M, \quad i \le k \le i+M-1,$$

are found.

3. Calculate the production rate estimate, $PR_c^{est}(q_1, N_1, ..., q_M, N_M, S)$, of the original closed production line using (7) with $I=\emptyset$ and neglecting $O(\epsilon^2)$ terms.

Theorem 3.1: Procedure 3.1 results in $O(\epsilon^2)$-accurate estimate of the production rate of lines defined by (i)-(vi), i.e.,

$$PR_c^{est}(q_1, N_1, ..., q_M, N_M, S) = F(q) + O(\epsilon^2). \quad (13)$$

4. DESIGN

Consider the following problem: Given a closed serial production line defined by (i)-(vi) with $k_i, i=1,...,M$ and $N_i, i=1,...,M-1$ fixed and N_M and S free, find the smallest N_M and S such that $PR_c(k_1, N_1, ..., k_M, N_M, S)$ is maximized or, in other words, find N_M and S that do not impair the performance of the correponding open line. The solution is given in the following:

Theorem 4.1: The values of N_M and S that maximize $PR_c(k_1, N_1, ..., k_M, N_M, S)$ are given by

$$S = N_M = \sum_{i=1}^{M-1} N_i + 1.$$

5. CASE STUDY

In [1], a paint shop operation at an automobile assembly plant has been analyzed as an open serial production line, although in reality the system is closed with respect to the carriers (skids) (see Fig. 4 of [1]). In [2], an improvement measure for the paint shop has been developed, again based on the open loop approach. In both [1] and [2], the closed loop effects have been taken into account by heuristic considerations. Below, we apply the results of Sections 3 and 4 and develops an improvement measure based on the closed lines approach.

As it has been shown in [1], the paint shop description can be reduced to a closed serial production line shown in Fig. 6 of [1]. Thus, using Theorem 2.1,

$$\begin{aligned} PR_c &= 1-(k_1+k_2)\epsilon + O(\epsilon^2) \text{ (jobs/cycle)} \\ \text{or} \quad PR_c &= 63-(L_{P.O.}+L_{F.O.}) \text{ (jobs/hour)} \end{aligned} \quad (14)$$

where $L_{P.O.}$ and $L_{F.O.}$ are the average losses in P.O. and F.O., respectively, and ϵk_1 and ϵk_2 are defined as

$$\epsilon k_1 = \frac{L_{P.O.}}{63}, \quad \epsilon k_2 = \frac{L_{F.O.}}{63}.$$

The value of $L_{P.O.}$ and $L_{F.O.}$ for 5 consecutive monthly periods are given in Table 1. Using these values, we calculate the production rate estimate (14) and compare the results for the corresponding periods (Table 2). As it follows from this table, the accuracy of the prediction is sufficiently high, with the exception of period 2. It turned out that during this period a new car model has been introduced in the production, and we suppose that additional perturbations, not included in the model, played a crucial role.

Table 1: Average losses (jobs/hr)

Periods	Time Period 1	Time Period 2	Time Period 3	Time Period 4	Time Period 5
P.O.	3.77	3.94	4.46	3.25	2.95
F.O.	6.18	7.38	7.01	6.59	6.14

Table 2: Actual and Estimated Production Rates (jobs/hr)

Periods	Time Period 1	Time Period 2	Time Period 3	Time Period 4	Time Period 5
Actual PR	53.50	43.81	51.27	54.28	55.89
Estimated PR	53.05	51.68	51.53	53.16	53.91
Error(%)	0.8	18.0	0.5	2.1	3.5

Turning now to the improvement measures, from Theorem 4.1, we conclude that if S_e is chosen optimally, *i.e.*,

$$S_e = N_1 + 1 = 27,$$

the production rate becomes as shown in Table 3. Thus, if the total number of skids in the system is

$$S = S_e + 650 = 27 + 650 = 677,$$

the production rate is increased by 10.5%.

Combined with the improvement measures suggested in [2], where unbalanced times for operations in P.O. and F.O. have been assigned, the suggested above choice of S_e can bring the production rate estimate up to the planned value of 63 jobs/hour (17.2% improvement).

Table 3: Expected Production Rates (jobs/hr)

Periods	Time Period 1	Time Period 2	Time Period 3	Time Period 4	Time Period 5
Expected PR	59.23	59.06	58.54	59.76	60.05

6. CONCLUSIONS

Basically, the result of this paper shows that if there exists a buffer, $N_j, j \in \{1, ..., M\}$, such that

$$\sum_{i \neq j} N_i < S \leq N_j, \qquad (15)$$

the production rate, $PR_c(k_1, N_1, ..., k_M, N_M, S)$, of the closed line defined by (i)-(vi) can be calculated as the production rate, $PR_o(k_{j+1}, N_{j+1}, ..., k_M, N_M, k_1, N_1, ..., k_j)$ of the open line $m_{j+1}, B_{j+1}, ..., m_M, B_M, m_1, B_1, ..., m_j$ defined by (i)-(v). If (15) is satisfied for no j (which is typically the case in practice), the analysis of a closed line can be performed using the aggregation - disaggregation procedure of Section 3.

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