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Supervisor Reduction 과 관측함수 설계

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Supervisor Reduction and Observation Function Design

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Abstract

This paper investigates the relationship between the two problems, supervisor reduction and observation function (projection) design, which arise in supervisory control of DEDS. It is shown through an example that a reduced supervisor of minimal size does not necessarily result in a maximal projection when a projection design method which uses the transition structure of a supervisor is applied. Also, if an L-realizable projection P is available and if a supervisor has a special structural feature, a cover C for supervisor reduction can be easily obtained. By investigating the control-compatibility of states of the reduced supervisor based on C, we can also check maximality of P in a simple manner.

1. Introduction

In this paper we are concerned with two important problems that arise in the Ramadge and Wonham's supervisory control framework [1] for discrete event dynamic systems (DEDS). In the supervisory control framework, a DEDS is modeled by a finite automaton G, and controlled to perform a given orderly behavior L by a supervisor $\underline{S}$ which is realized by a finite automaton S and an output mapping ϕ of S. The output mapping ϕ assigns to each state of S a set of events that are to be allowed to occur in G. The desired behavior L is just a set of orderly strings of events that may occur in the controlled DEDS: thus L can be obtained by excluding from L(G), the set of strings of events that can occur in the uncontrolled DEDS G, all those strings which corresponds to disorderly evolutions of the system. L is frequently given by a finite automaton that generates L: this automaton is called a recognizer for L. A supervisor is often unable to observe the occurrences of certain events in G. This partial observation case can also be properly treated in this framework by placing an observation stage between the supervisor and G. The observation stage is modeled by a function, called an observation function or a projection, $P:\Sigma\rightarrow\Sigma_o\cup\{\varepsilon\}$, where Σ is a set of all discrete events in G, $\Sigma_o\subset\Sigma$, and ε symbolizes the null event, i.e., no occurrence of any event.

Fig.1. shows the mechanization of supervisory control with partial observations. We refer to [1] and [2] for detailed discussions of the framework. In the following, the reader is assumed to be familiar with the standard supervisory control framework.

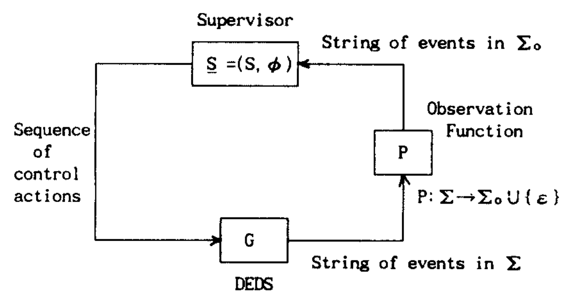


Fig.1. Supervisory Control of DEDS

Among many interesting problems posed in this framework, supervisor reduction and observation function design problem will be discussed in this paper. When a desired behavior L is realizable, there exist many supervisors that can do the job. Supervisor reduction problem ([3],[4]) addresses the question of how to obtain from a supervisor $\underline{S}$=(S, ϕ) a reduced supervisor $\underline{T}$=(T, ψ) where T has smaller number of states than S. In the observation function design problem ([5]), we consider the problem of obtaining an observation function with which a supervisor can be synthesized to realize L. These two seemingly independent problems at first look are in fact closely related. Indeed, in the supervisor reduction problem, we search for a set of states of S with "essentially the same" control actions $\phi(\cdot)$, and combine them into one state achieving reduction in number of states. However, the existence of such a set of states in S may imply that the supervisor need not to observe the occurrences of those events causing the transitions between the states in the set: and if this is the case, then we immediately obtain a proper projection P. Similarly, knowledge of a proper projection may lead to a supervisor reduction. In what follows, we elaborate on the above argument, and discuss when a maximal projection can be obtained and how maximality of a projection can be checked.

2. Normal Supervisors and Control-Compatible States

In this section, we introduce two important concepts concerning the structure of a supervisor. Throughout the paper we assume that L is closed, and $L \subset L(G)$. As usual, $\Sigma = \Sigma_u \cup \Sigma_c$ where Σ_u (Σ_c) denote the set of uncontrollable (controllable) events. Thus only the events in Σ_c can be disabled to occur in G by control actions from the supervisor. L is said to be $(\Sigma_u, L(G))$-invariant if $L\Sigma_u \cap L(G) \subset L$, and $(P, \Sigma_c, L(G))$-controllable if $\sigma \in \Sigma_c$, $s, t \in L$, $s\sigma \in L$, $t\sigma \in L(G)$ and $P(s)=P(t)$ together imply $t\sigma \in L$. $(\Sigma_u, L(G))$-invariance and $(P, \Sigma_c, L(G))$-controllability are necessary and sufficient conditions for L being realizable [2].

Let $P: \Sigma \rightarrow \Sigma_o \cup \{\varepsilon\}$ be a projection, and let $\underline{S}=(S, \phi)$ with $S=(X, \Sigma_o, \xi, x_o)$ be a supervisor. We write $\xi(w,x)!$ to mean that $\xi(w,x)$ is defined. In this paper, every automaton is assumed to be accessible (i.e., each state of the automaton can be reached from the initial state). Also, every supervisor is assumed to be complete unless otherwise stated (see [1] for details of completeness of a supervisor). Thus the closed loop system behavior, represented by a set of strings of events and denoted by $L(\underline{S}/G)$, can be defined recursively by ([5],[6])

i) $\varepsilon \in L(\underline{S}/G)$
ii) $w\sigma \in L(\underline{S}/G)$ iff $w \in L(\underline{S}/G)$, $\sigma \in \phi(\xi(P(w), x_0))$ and $w\sigma \in L(G)$.

In the above, we have used a tacit assumption that if $w \in L(\underline{S}/G)$, then $\xi(P(w), x_0)!$ In fact, this property follows immediately when the original definition of $L(\underline{S}/G)$ ([1],[2]) is used. We say that $\underline{S}$ *realizes* L *with* P if P is the observation function for $\underline{S}$ and $L(\underline{S}/G)=L$.

Definition 1.[3] A supervisor $\underline{S}$ is called *(P,L)-normal* if ϕ is given in the following form: $\phi=(\phi_0, \phi_1)$, where

$$\phi_0(x)=\{\sigma \in \Sigma_c: \exists\, s \in L \text{ such that } \xi(P(s), x_0)=x \text{ and } s\sigma \in L(G)-L\},$$

$$\phi_1(x)=\{\sigma \in \Sigma_c: \exists\, s \in L \text{ such that } \xi(P(s), x_0)=x \text{ and } s\sigma \in L\},$$

and $\phi_0(x) \cap \phi_1(x) = \emptyset$.

A (P,L)-normal supervisor will be sometimes called just *normal* when the associated P and L can be clearly identified from the context. Let $\phi_r(x)=\Sigma_c-(\phi_0(x) \cup \phi_1(x))$. Then it follows from the definition of ϕ_0 and ϕ_1 that

$$\phi_r(x)=\{\sigma \in \Sigma_c: \text{for all } s \in L \text{ such that } \xi(P(s), x_0)=x,\ s\sigma \notin L(G)\}.$$

When $\underline{S}$ is normal, we interprete the control mechanism of the closed loop system as follows: if $\sigma \in \phi_0(x)$, σ is disabled to occur in G. If $\sigma \in \phi_1(x) \cup \Sigma_u$, σ is allowed to occur. Finally if $\sigma \in \phi_r(x)$, the supervisor does not care because either the state x cannot be reached by a string $s \in P(L)$ or the event σ cannot occur after s in G. Therefore $\phi(x)$ in the definition of $L(\underline{S}/G)$ can be defined to be any set satisfying $\phi_1(x) \cup \Sigma_u \subset \phi(x)$ and $\phi_0(x) \cap \phi(x) = \emptyset$. In other words,

$$\phi(x)=\Sigma_u \cup \phi_1(x) \cup \phi_{rs}(x), \tag{1}$$

where $\phi_{rs}(x) \subset \phi_r(x)$.

Remark 1. The condition $\phi_0(x) \cap \phi_1(x) = \emptyset$ in Definition 1 is necessary for any supervisor being able to produce a proper control action. A slightly different definition of normality has been given in [3].

Let $A=(Y, \Sigma, \eta, y_0)$ be a finite automaton. *The image of* A *under* P is a finite automaton $B=Ac(Z, \Sigma_o, \zeta, z_0)$, where

$$Z = 2^Y, \quad z_0=\{\eta(s, y_0): P(s)=\varepsilon\},$$

$$\zeta(\sigma, z)=\begin{cases}\{\eta(s,y): y \in z,\ P(s)=\sigma\}, & \text{if this is nonempty} \\ \text{undefined}, & \text{otherwise.}\end{cases}$$

It is easy to verify [2] that $\zeta(w, z_0)=\{\eta(s, y_0): P(s)=w\}$ whenever the right hand side is nonempty, and $\zeta(w, z_0)$ is undefined otherwise. Frequently, the finite automaton S of a supervisor $\underline{S}=(S, \phi)$ which realizes L with P is the image under P of a recognizer for L. In this case, $\underline{S}$ can be made normal simply by redefining ϕ according to the rule in Definition 1. The following lemma shows that this is indeed possible.

Lemma 1. Let L be $(P, \Sigma_c, L(G))$-controllable, and let A be a recognizer for L. If S is the image of A under P, then the output mappings ϕ_0 and ϕ_1 defined as above at each state x of S have the property that $\phi_0(x) \cap \phi_1(x) = \emptyset$.

Proof Let $A=(Y, \Sigma, \eta, y_0)$. Suppose that there exists $\sigma \in \phi_0(x) \cap \phi_1(x)$. Then $\sigma \in \Sigma_c$, and there exist $s, t \in L$ such that $\xi(P(s), x_0)=\xi(P(t), x_0)=x$, $s\sigma \in L(G)-L$ and $t\sigma \in L$. Let $y=\eta(t, y_0)$. Since $t\sigma \in L$ and A is a recognizer for L, $\eta(\sigma, y)!$ Moreover, $y \in x=\xi(P(s), x_0)$. Hence there exists $w \in \Sigma^*$ such that $\eta(w, y_0)=y$ and $P(w)=P(s)$. Note that $\eta(w\sigma, y_0)!$, and therefore $w\sigma \in L$. Thus we have two strings $s, w \in L$ and $\sigma \in \Sigma_c$ which violate the condition of $(P, \Sigma_c, L(G))$-controllability of L. Hence we must have $\phi_0(x) \cap \phi_1(x)=\emptyset$. Q.E.D.

Proposition 1. Let L be closed and $(\Sigma_u, L(G))$-invariant. If $\underline{S}$ is complete and (P,L)-normal, $\underline{S}$ realizes L with P.

Proof ($L(\underline{S}/G) \subset L$) By induction. Suppose that $s\sigma \in L(\underline{S}/G)$. Then, by the definition of $L(\underline{S}/G)$, $s \in L(\underline{S}/G)$, $s\sigma \in L(G)$ and $\sigma \in \phi(x)$ where $x=\xi(P(s), x_0)$. Thus $s \in L$ by the induction hypothesis. Moreover, $\sigma \notin \phi_r(x)$, and therefore $\sigma \in \phi_1(x) \cup \Sigma_u$. Now if $\sigma \in \Sigma_u$, then $s\sigma \in L$ by the $(\Sigma_u, L(G))$-invariance of L. If $\sigma \in \Sigma_c$ and $s\sigma \in L(G)-L$, then $\sigma \in \phi_0(x)$ which violates the condition that $\phi_0(x) \cap \phi_1(x)=\emptyset$. Thus if $\sigma \in \Sigma_c$, we must have $s\sigma \in L$.

($L \subset L(\underline{S}/G)$) By induction. If $s\sigma \in L$, then $s \in L$ since L is closed. By the induction hypothesis, $s \in L(\underline{S}/G)$. Let $x = \xi(P(s), x_0)$. By the definition of ϕ_1, $\sigma \in \phi_1(x)$ so that $\sigma \in \phi(x)$. Again by the definition of $L(\underline{S}/G)$, $s\sigma \in L(\underline{S}/G)$. Q.E.D.

Definition 2 [3],[4] Two states x, $y \in X$ of a normal supervisor $\underline{S}=(S, \phi)$ are said to be *control-compatible*, written $x \sim y$, if

$$\phi_0(x) \cap \phi_1(y) = \emptyset = \phi_1(x) \cap \phi_0(y), \qquad (2)$$

and *progressively control-compatible*, written $x \sim_p y$, if (2) with x and y replaced by $\xi(w,x)$ and $\xi(w,y)$, respectively, holds for all $w \in \Sigma^*$ whenever $\xi(w,x)$ and $\xi(w,y)$ are both defined.

Remark 2. The relation $\sim$ and $\sim_p$ are not equivalence relations.

If $x \sim y$, then we can select some $\phi_{rs}(x)$ and $\phi_{rs}(y)$ in (1) so that $\phi(x) = \phi(y) \supset \Sigma_u \cup \phi_1(x) \cup \phi_1(y)$. In other words, if $x \sim y$, the control actions of the supervisor at x and y can be made exactly the same. Therefore if $x \sim_p y$, then we may combine the states $\xi(w,x)$ and $\xi(w,y)$ into a single state for each w without losing any control function over the DEDS G: this is in fact the essence of the supervisor reduction technique considered in [3] and [4]. More specifically, the supervisor reduction in [3] is based on the selection of a *cover* defined as below.

Definition 3. Let $\underline{S}=(S, \phi)$ be a normal supervisor. A *cover* of $\underline{S}$ is a family $C = \{X_i, i \in I\}$ of nonempty subsets of X satisfying

(i) $\cup X_i = X$,
(ii) each X_i consists of control-compatible states,
(iii) for all $x, y \in X_i$, $\xi(\sigma, x)$ and $\xi(\sigma, y)$ belong to the same X_j whenever they are both defined.

When a cover $C = \{X_i\}$ of a normal supervisor is given, a reduced supervisor $\underline{T}=(T, \psi)$ is constructed by letting $T = (\{X_i\}, \Sigma, \xi_r, X_{i0})$: X_{i0} can be any member of C that contains x_0. The transition function ξ_r and the output mapping ψ are defined in an obvious manner (see [3] or Section 4). Example 1 in Section 3 illustrates the use of the method.

3. Designing A Maximal Projection

From now on, a projection P will be called *L-realizable* if L is $(P, \Sigma_c, L(G))$-contollable. Also, an event $\sigma \in P^{-1}(\varepsilon)$ will be called *P-null*. Trivially, the identity mapping, denoted by I, is an L-realizable projection. Note that when L is $(\Sigma_u, L(G))$-invariant, the necessary and sufficient condition for L being realizable is that the projection P is L-realizable.

In the observation function design problem, we assume that L is $(\Sigma_u, L(G))$-invariant, and search for a L-realizable projection $P: \Sigma \rightarrow \Sigma_o \cup \{\varepsilon\}$ with as smallest Σ_o as possible. The set of L-realizable projections can be partially ordered by the relation $\leq$ defined as follows: $P_1 \leq P_2$ if $\Sigma_{o,1} \supset \Sigma_{o,2}$ where $P_i: \Sigma \rightarrow \Sigma_{o,i} \cup \{\varepsilon\}$, $i=1,2$ (we say that P_2 is coaser than P_1 if $P_1 \leq P_2$). The set does not in general have the coarsest element. However, it has a maximal element. Before proceeding further, we present a useful lemma, which can be easily verified.

Lemma 2. If $P_1 \geq P_2$, then for all $u, v \in \Sigma^*$,

(i) $P_1(u) = P_1(P_2(u))$
(ii) if $P_2(u) = P_2(v)$, then $P_1(u) = P_1(v)$.

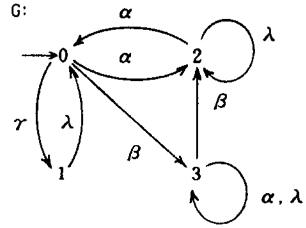
In [5], a simle method of obtaining an L-realizable projection is presented. The method makes use of the transition structure of S of a supervisor $\underline{S}$ which realizes L with the identity mapping I: thus a projection $P_{tr}: \Sigma \rightarrow \Sigma_{tr} \cup \{\varepsilon\}$ is defined based on S where

$$\Sigma_{tr} = \{\sigma: \xi(\sigma, x) = y \text{ for some } x, y \in X \text{ with } x \neq y\}, \quad (3)$$

and shown to be L-realizable. This method is simple and easy to implement. It, however, depends heavily on the transition structure of the supervisor: we expect a coarser projection P_{tr} when a reduced supervisor is used, and could expect a maximal projection if the reduced supervisor is of minimal size (i.e., has a minimal number of states). In the following, we focus on the question of precisely when the method results in a maximal L-realizable projection.

A little thought could lead to the following (false) statement: if the supervisor $\underline{S}=(S, \phi)$ has no progressively contrcl-compatible states, then the projection P_{tr} based on S is maximal. Of course, if there are progressively control-compatible states in $\underline{S}$, then the supervisor reduction technique in [3] (or [4]) can be applied to yield a reduced supervisor with possibly smaller number of states, which in turn may result in a coarser projection. However, the absence of progressively control-compatible states in $\underline{S}$ is not sufficient, as shown in the following example, for the resulting P_{tr} being maximal (in fact, it is not necessary either: a simple example can also be constructed to show this).

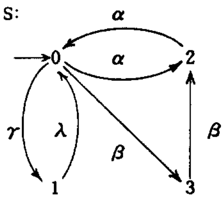
Example 1. Consider a DEDS G and a supervisor $\underline{S}=(S, \phi)$ given below. Here $\underline{S}$ realizes L with the identity mapping I, and S is a recognizer for L.



$\Sigma = \{\alpha, \beta, \gamma, \lambda\}$

$\Sigma_c = \{\alpha, \lambda\}$

S:

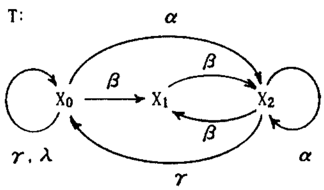


ϕ:

State	ϕ_0	ϕ_1
0	-	α
1	-	λ
2	λ	α
3	α, λ	-

Note that the supervisor $\underline{S}$ is (I,L)-normal. Also, $\Sigma_{tr}=\{\alpha,\beta,\gamma,\lambda\}$. Thus P_{tr} is just the identity mapping. Now it is easy to see that $0 \sim_p 1$ and $0 \sim_p 2$; there are no control-compatible states other than these pairs. Consider a cover $C=\{X_i,\ i=0,1,2\}$ where $X_0=\{0,1\}$, $X_1=\{3\}$ and $X_2=\{0,2\}$. The reduced supervisor $\underline{T}=(T,\psi)$ based on C is then

T:

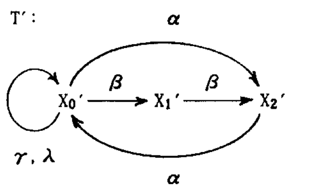


ψ:

State	ψ_0	ψ_1
X_0	-	α, λ
X_1	α, λ	-
X_2	λ	α

There are no control-compatible states in $\underline{T}$. In fact, $\underline{T}$ is a reduced supervisor with the smallest number of states. Now, $\Sigma_{tr}=\{\alpha,\beta,\gamma\}$. It turns out, however, that P_{tr} for this case is not maximal. Consider another cover $C'=\{X_i',\ i=0,1,2\}$ where $X_0'=\{0,1\}$, $X_1'=\{3\}$ and $X_2'=\{2\}$. The reduced supervisor $\underline{T}'=(T',\psi')$ based on C' is

T':



ψ':

State	ψ_0'	ψ_1'
X_0'	-	α, λ
X_1'	α, λ	-
X_2'	λ	α

Now $\Sigma_{tr}=\{\alpha,\beta\}$, giving the projection $P_{tr}':\Sigma\to\{\alpha,\beta\}\cup\{\varepsilon\}$. Note that P_{tr}' is strictly coarser than P_{tr} based on T. (end of example)

In order to obtain a maximal projection P_{tr}, we need to require some conditions to hold for the structure of the supervisor from which P_{tr} is obtained. Let $\underline{S}=(S,\phi)$ be (I,L)-normal. Consider the following condition (TT) on a transition triple (x,σ,y) of S. Here, a *transition triple* (x,σ,y) denotes the two states $x,y\in X$ with the transition $\xi(\sigma,x)=y$.

(TT) If $\sigma_c\in\phi_0(x)\cap\phi_1(y)$, then

(a) $\exists$ $s,w\in L$ such that $\xi(s,x_0)=x$, $\xi(w,x_0)=y$, $s\sigma_c\in L(G)-L$, $w\sigma_c\in L$ and $P_{tr}(w)=P_{tr}(s)\sigma$.

If $\sigma_c\in\phi_1(x)\cap\phi_0(y)$, then (a) holds with y and x replacing x and y, respectively.

<u>Proposition 2.</u> Let $\underline{S}=(S,\phi)$ be a (I,L)-normal supervisor. If for each $\sigma\in\Sigma_{tr}$, there exists a transition triple (x,σ,y) satisfying

(i) x is not control-compatible to y,

(ii) (x,σ,y) satisfies the condition (TT),

then the P_{tr} based on S is a maximal L-realizable projection.

<u>Proof</u> Let P_l be an L-realizable projection and $P_l\geq P_{tr}$. Suppose that $P_l\neq P_{tr}$. Then there exists a P_l-null event σ such that $\sigma\in\Sigma_{tr}$. By the definition of Σ_{tr} and the hypothesis of the proposition, there is a transition triple (x,σ,y) satisfying the conditions (i) and (ii). But (i) implies that there exists $\sigma_c\in\Sigma_c$ such that either $\sigma_c\in\phi_0(x)\cap\phi_1(y)$ or $\sigma_c\in\phi_1(x)\cap\phi_0(y)$. By (ii) and the condition (TT), $\sigma_c\in\phi_0(x)\cap\phi_1(y)$ implies that there exist $s,w\in L$ such that $s\sigma_c\in L(G)-L$, $w\sigma_c\in L$ and $P_{tr}(w)=P_{tr}(s)\sigma$. Using Lemma 2. we have $P_l(w)=P_l(P_{tr}(w))=P_l(P_{tr}(s))P_l(\sigma)=P_l(s)$. Thus P_l is not L-realizable, contradicting the assumption. Similarly, it is impossible to have that $\sigma_c\in\phi_1(x)\cap\phi_0(y)$. Hence we must have $P_l=P_{tr}$. Thus we have shown that P_{tr} is maximal. Q.E.D.

<u>Example 1 (continued).</u> We check if the supervisor $\underline{T}'$ satisfies the hypothesis of Proposition 2. Note that $\underline{T}'$ is (I,L)-normal. Recall that $\Sigma_{tr}=\{\alpha,\beta\}$. For α, consider a transition triple (X_0',α,X_2'). Recall that X_0' is not control-compatible to X_2'. We need to consider only $\lambda\in\psi_1'(X_0')\cap\psi'(X_2')$ in order to check if the condition (TT) holds. Now two strings γ and $\gamma\lambda\alpha$ lead to X_0' and X_2', respectively. Note that $\gamma\lambda\in L$ and $\gamma\lambda\alpha\lambda\in L(G)-L$. Moreover, $P_{tr}(\gamma\lambda\alpha)=\alpha=P_{tr}(\gamma)\alpha$. Thus (X_0',α,X_2') satisfies (TT). Now for β, take (X_1',β,X_2'). It can be verified in a similar way that (TT) holds for (X_1',β,X_2'). Hence $\underline{T}'$ satisfies the hypothesis of the proposition. Thus P_{tr}' is maximal. Finally, it is not difficult to see that the supervisors $\underline{S}$ and $\underline{T}$ do not satisfy the hypothesis of the proposition. (end of example)

In general, it is not easy to check if the condition (TT) holds for all transition triples of S under consideration. Thus it might be worth noting that if S is a subautomaton (see [7]) of G, then all transition triples of S satisfy (TT). This fact is, however, of little use since a supervisor constructed in such a way seldom satisfies the condition (i) of Proposition 2 (note that S in Example 1 is a subautomaton of G). In Section 4, we will present a simpler method to check if an L-realizable projection is maximal. Finally, we note that the projection P_{tr} based on an "efficient" supervisor introduced in [1] need not be maximal (a simple example can be constructed). We should also note that an efficient supervisor may have some progressively control-compatible states, and thus a reduced supervisor may result from it. The reason for an efficient supervisor not having a minimal number of states is that the finite automata of efficient supervisors are restricted to be recognizers for L.

4. Supervisor Reduction Using L-Realizable Projections

Assume that L is $(\Sigma_u, L(G))$-invariant. Let $P: \Sigma \rightarrow \Sigma_o \cup \{\varepsilon\}$ be an L-realizable projection. Then we can construct a supervisor which realizes L with P (a standard construction procedure can be found in [2]). Now suppose that we already have an (I,L)-normal supervisor $\underline{S}=(S, \phi)$, which realizes L with the identity mapping. When the projection P is known to be L-realizable, a natural question to ask would be the following: can we devise a simpler supervisor reduction method which uses the (additional) information that P is L-realizable? In this section, we concern ourselves with this question and give an answer.

We start with the following lemma which can be easily proved.

Lemma 3. Let $\underline{S}=(S, \phi)$, $S=(X, \Sigma, \xi, x_0)$, be an (I,L)-normal supervisor. Assume that S has the following properties:

(a) S is a recognizer for L,
(b) if $\xi(s,x_0)=\xi(t,x_0)$, then for all $\sigma \in \Sigma_c$, $s\sigma \in L(G)-L$ iff $t\sigma \in L(G)-L$.

If $\xi(u,x_0)=x$, then $u \in L$ and

(i) if $\sigma \in \phi_0(x)$, then $u\sigma \in L(G)-L$,
(ii) if $\sigma \in \phi_1(x)$, then $u\sigma \in L$.

Remark 3. There are at least two methods of constructing S which has the properties in Lemma 3. First, if S is a *subautomaton* of G, then (a) and (b) in Lemma 3 hold (see [7] for details of subautomata and for a construction procedure). The second method can be found in [3].

Let $S=(X, \Sigma, \xi, x_0)$, and let P be a projection. For $w \in P(L)$, let $X_P(w)=\{\xi(s,x_0): P(s)=w\}$. Thus $X_P(w)$ is a state of the image of S under P. Note that we may have $X_P(w)=X_P(w')$ while $w \neq w'$.

Lemma 4. Let $\underline{S}=(S, \phi)$ be as in Lemma 3, and let P be an L-realizable projection. Then for all $w \in P(L)$ and for all $x,y \in X_P(w)$, $x \sim_P y$.

Proof Let $x,y \in X_P(w)$, $w \in P(L)$. First, we show that $x \sim y$. Note from the definition of $X_P(\cdot)$ that there exist $u,v \in \Sigma^*$ such that $\xi(u,x_0)=x$, $\xi(v,x_0)=y$ and $P(u)=P(v)=w$. Suppose that $\sigma \in \phi_0(x) \cap \phi_1(y)$. By Lemma 3, $u,v \in L$, $u\sigma \in L(G)-L$ and $v\sigma \in L$. Since $P(u)=P(v)$, P is not L-realizable, which is a contradiction. Hence $\phi_0(x) \cap \phi_1(y)=\emptyset$. Similarly, $\phi_1(x) \cap \phi_0(y)=\emptyset$. Thus we have proved that $x \sim y$.

Note that if $x,y \in X_P(w)$, then $\xi(w',x)$ and $\xi(w',y)$ belong to the same set $X_P(w'')$, $w''=wP(w')$, whenever they are defined. Thus $\xi(w',x) \sim \xi(w',y)$ for all w', which shows that $x \sim_P y$. Q.E.D.

Proposition 3. Let $\underline{S}$ be as in Lemma 3, and let P be L-realizable. Then $C=\{X_P(w): w \in P(L)\}$ is a cover of $\underline{S}$.

Proof Clearly, each member of C is nonempty. We check if the conditions in Definition 3 hold for C.

If $x \in X$, then there is $s \in \Sigma^*$ such that $\xi(s,x_0)=x$ (S is accessible). But $s \in L$ since S is a recognizer for L. Thus $P(s) \in P(L)$ and therefore x belongs to a member of C. Also, it is clear that if $x \in X_P(w)$ for some w, then $x \in X$. Hence C satisfies (i). Now that (ii) holds by Lemma 3, it remains to check (iii). Let $x,y \in X_P(w)$, $w \in P(L)$. If $\xi(\sigma,x)$ and $\xi(\sigma,y)$ are defined, then it is easy to see from the definition of X_P that they belong to $X_P(w')$, $w'=wP(\sigma)$. Hence (iii) holds. Q.E.D.

Let $\underline{S}$ be as in Lemma 3, and let P be L-realizable. Proposition 3 then shows that a cover of $\underline{S}$ for a supervisor reduction can be immediately obtained: the cover is the state space of the image of S under P. It is thus clear that the construction of such a cover is much simpler than the method used in [3]. However, the computational complexity is still exponential in time. We should also note that the simplicity in obtaining a cover in this case has been achieved by the additional knowledge of an L-realizable projection. Following [3], we describe below a reduced supervisor $\underline{T}=(T, \psi)$ based on the cover $C=\{X_P(w), w \in P(L)\}$. A useful feature of $\underline{T}$ will then be presented.

Let $S'=(Z, P(\Sigma), \zeta, z_0)$ be the image of S under P. Thus $Z=C$, and $z_0=X_P(\varepsilon)$. Also, $\zeta(w,z_0)=X_P(w)$ if $w \in P(L)$. Now let $T=(Z, \Sigma, \eta, z_0)$, where η is defined by

$$\eta(\sigma,z)=\begin{cases}\zeta(P(\sigma),z), & \text{if } \exists\ x \in z \text{ such that } \xi(\sigma,x)! \\ \text{undefined}, & \text{otherwise.}\end{cases}$$

It is easy to see that for all $z \in Z$, (i) if $\eta(s,z)!$, then $\eta(s,z)=\zeta(P(s),z)$, and (ii) if $\xi(s,x)!$ and $x \in z$, then $\eta(s,z)!$ Also, we have

Lemma 5. Let P_{tr} be the projection based on T. Then $P \leq P_{tr}$: i.e., if σ is P-null, then σ is also P_{tr}-null.

Proof Let σ be P-null. For each $z \in Z$, if $\eta(\sigma,z)!$ then $\eta(\sigma,z)=\zeta(P(\sigma),z)=\zeta(\varepsilon,z)=z$. Thus there is no transition triple (z_1,σ,z_2) of T such that $z_1 \neq z_2$. Hence $\sigma \notin \Sigma_{tr}$, so that σ is P_{tr}-null. Q.E.D.

Define $\psi=(\psi_0, \psi_1)$ by

$$\psi_0(z) = \cup_{x \in z} \ \phi_0(x), \quad \psi_1(z) = \cup_{x \in z} \ \phi_0(x).$$

Then $\underline{T}=(T, \psi)$ is a reduced supervisor based on C. Indeed,

Theorem 1 [3] $L(\underline{T}/G)=L(\underline{S}/G)$ and $\underline{T}$ is complete.

Moreover, the supervisor $\underline{T}$ so constructed has the following useful properties.

Proposition 4. (i) $\underline{T}$ is (I,L)-normal.
(ii) Each transition triple (z_1, σ, z_2), $z_1 \neq z_2$, of T satisfies (TT).

Proof (i) Let $z \in Z$. By Lemma 4, $\phi_0(x) \cap \phi_1(y) = \emptyset$ if $x, y \in z$. It follows that $\psi_0(z) \cap \psi_1(z) = \emptyset$. Now let

$$\Sigma_{z0} = \{\sigma \in \Sigma_c : \exists\, u \in L \text{ such that } \eta(u, z_0) = z \text{ and } u\sigma \in L(G) - L\}.$$

We are to show that $\psi_0(z) = \Sigma_{z0}$. Suppose that $\sigma \in \psi_0(z)$. Then there is $x \in z$ such that $\sigma \in \phi_0(x)$. Let $z = X_P(w)$. Then there also exists $t \in \Sigma^*$ such that $\xi(t, x_0) = x$ and $P(t) = w$. By Lemma 3, $t\sigma \in L(G) - L$. Note that $\eta(t, z_0) = \zeta(w, z_0) = X_P(w) = z$. Hence $\sigma \in \Sigma_{z0}$. Suppose now $\sigma \in \Sigma_{z0}$. Then there exists $u \in L$ such that $\eta(u, z_0) = z$ and $u\sigma \in L(G) - L$. Note that if we let $x = \xi(u, x_0)$, then $\sigma \in \phi_0(x)$ and $x \in \zeta(P(u), z_0) = \eta(u, z_0) = z$. By the definition of ψ_0, $\sigma \in \psi_0(z)$. Hence we have shown that $\psi_0(z) = \Sigma_{z0}$. Similarly, $\psi_1(z)$ is the set as defined in Definition 1.

(ii) Consider a transition triple (z_1, σ, z_2), $z_1 \neq z_2$, of T. Clearly, $\sigma \in \Sigma_{tr}$ where Σ_{tr} is the set defined for T as in (3). By Lemma 5, σ is not P-null. Suppose now that $\sigma_c \in \psi_0(z_1) \cap \psi_1(z_2)$. By the definition of ψ, there exist $x \in z_1$, $y \in z_2$ such that $\sigma_c \in \phi_0(x) \cap \phi_1(y)$. Let $z_1 = X_P(w)$ $(= \zeta(w, z_0))$. Then $z_2 = \eta(\sigma, z_1) = \zeta(P(\sigma), z_1) = \zeta(\sigma, z_1) = \zeta(w\sigma, z_0) = X_P(w\sigma)$. Thus there exist $s, t \in \Sigma^*$ such that $\xi(s, x_0) = x$, $\xi(t, x_0) = y$, $P(s) = w$ and $P(t) = w\sigma$. By Lemma 3, we have that $s, t \in L$, $s\sigma_c \in L(G) - L$ and $t\sigma_c \in L$. Note that, using Lemma 5 and Lemma 2, we have $P_{tr}(t) = P_{tr}(P(t)) = P_{tr}(w\sigma) = P_{tr}(w)\sigma = P_{tr}(P(s))\sigma = P_{tr}(s)\sigma$. Also, $\eta(s, z_0) = \zeta(P(s), z_0) = X_P(w) = z_1$. Similarly, $\eta(t, z_0) = z_2$. Hence we have shown that if $\sigma_c \in \psi_0(z_1) \cap \psi_1(z_2)$, then there exist $s, t \in L$ such that $\eta(s, z_0) = z_1$, $\eta(t, z_0) = z_2$, $s\sigma_c \in L(G) - L$, $t\sigma_c \in L$ and $P_{tr}(t) = P_{tr}(s)\sigma$.

Similary, if $\sigma_c \in \psi_1(z_1) \cap \psi_0(z_2)$, the above conclusion holds with z_2 and z_1 replacing z_1 and z_2, repectively. Therefore (z_1, σ, z_2) satisfies (TT). Q.E.D.

By Lemma 6 and Proposition 2, the following result is immediate.

Corollary 1. Let Σ_{tr} be the set defined for T as in (3). If for each $\sigma \in \Sigma_{tr}$ there exists a transition triple (z_1, σ, z_2) of T such that z_1 is not control-compatible to z_2, then P is maximal.

Therefore we have a fairly simple means of checking if an L-realizable projection P is maximal.

Example 1 (continued) Recall that $P: \Sigma \to \{\alpha, \beta\} \cup \{\varepsilon\}$ is an L-realizable projection. We check if P is maximal. Note that S is a subautomaton of G so that $\underline{S}$ has the properties in Lemma 3. Now it is easy to see that $\{X_P(w), w \in P(L)\} = \{\{0,1\}, \{2\}, \{3\}\}$. Thus the reduced supervisor based on the cover $\{X_P(w), w \in P(L)\}$ is nothing but $\underline{T}' = (T', \psi')$. Note that there are no control-compatible states in $\underline{T}'$. By Corollary 1, we conclude that P is maximal. (end of example)

5. Conclusion

The sufficiency condition (Section 3) for maximality of an L-realizable projection P_{tr} based on a supervisor $\underline{S} = (S, \phi)$ become checkable when S has special structural properties. Since such an S can be easily constructed (with polynomial time complexity [7]), the result is expected to be useful in the observation function design problem. The result (Proposition 3) that the states of the image of S under an L-realizable projection P form a cover for a supervisor reduction may find its use only in restricted cases, since such P is seldom available when supervisor reduction problem is considered. Nevertheless, it gives an insight into the relationship between supervisor reduction and observation function design problems.

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