'91 KACC 1991. 10. 22~24

Sliding Mode Control of Induction Motors Based on Reduced Order Model

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Abstract

It is difficult to design the controller of an induction motor because of its non-linearity and high order dynamics. But it is possible to get reduced order system using the theory of singular pertubation because the dynamics of induction motor consists of fast stable mode and slow one. On the other hand, the sliding mode control is well-known for its performance of robustness. This paper deals with the sliding mode controller of induction motors based on the reduced order system.

1 Introduction

The dynamics of an induction motor is governed by nonlinear system of differential equations [1] as follows

$$\begin{aligned} \dot{\psi}_1 &= -R_1 i_1 - \dot{\theta} \mathrm{J} \psi_1 + u \\ \dot{\psi}_2 &= -R_2 i_2 + (\dot{\theta} - \omega_2) \mathrm{J} \psi_2 \end{aligned} \quad (1)$$

$$I\dot{\omega}_2 = T - T_L \quad (2)$$

$$T = {i_1}^T \mathrm{J} \psi_1 = -{i_2}^T \mathrm{J} \psi_2 = M {i_1}^T i_2 \quad (3)$$

$$\mathrm{J} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

where $u, i_1, \psi_1, i_2, \psi_2 \in R^2$ are voltage, stator and rotor current and flux vectors, T and T_L are the motor and load torques, ω_2 is a rotation speed, $\dot{\theta}$ is a rotation speed of the coordinate system, I is inertia. The flux vectors are expressed as

$$\left\{ \begin{aligned} \psi_1 &= L_1 i_1 + M i_2 \\ \psi_2 &= M i_1 + L_2 i_2 \end{aligned} \right. \quad or \quad \left\{ \begin{aligned} \psi_1 &= L_{\sigma 1} i_1 + M i_M \\ \psi_2 &= L_{\sigma 2} i_2 + M i_M \end{aligned} \right. \quad (4)$$

$R_1, R_2, L_1, L_2, M, L_{\sigma 1}, L_{\sigma 2}$ are paremeters of the stator and rotor windings, I_M is a magnetizing current. Futher it is assumed that $R_1 = R_2 = R; L_{\sigma 1} = L_{\sigma 2} = L$.

Efficient speed (positiion) control algorithms imply decoupling the overall motion into two components depending on the orientation of the motor flux and then correspondingly design of control components providing desired values of the motor flux and torque [2]. The field oriented control design methods need, on one hand, information on the current values of flux components, obtained with the help of sensors or observers, and, on the other hand, nonlinear state dependent coordinate transformations. These reasons may hinder implementation of induction motor control systems in particular for low power electric drives when application of complex control algorithms may prove to be unjustified.

One of the possible ways of simplification of control algorithms consists in using reduced order models. The dynamic processes in inductioin motors may consist of partial motions of different rates. The rate of varying of a magnetizing current may be much faster than that of mechanical rotation; the time constant associated with stator and rotor currents is much less than a magnetizing current one. It follows from the theory of singularly perturbed systems[3] that the existence of rate separated motions enables order reduction of the system and as a result simplification of the desigh procedure.

The paper deals with two versions of induction motor control systems based on reduced order models — of the first and of the third orders. In the first case the electromagnetic dynamics is neglected, in the second — the processes associated with dissipation fluxes. The motor slip and phase are handled as control actrions and designed as discontinuous functions of control error which is steered to zero due to enforcing sliding modes.

2 First–Order Model

The design method being developed in this section is oriented to induction motors with a high inertia moment reduced to the rotor shaft. The motion equations (1) (2) (3) with respect to new time and the coordinate system fixed with of the voltage vector ($\dot{\theta} = \omega_1$)

$$t' = t/I \quad (5)$$

may be written as

$$\begin{aligned} \mu\dot{\psi}_1 &= u - R i_1 - \omega_1 \mathrm{J} \psi_1 \\ \mu\dot{\psi}_2 &= -R i_2 - (\omega_1 - \omega_2)\psi_2 \\ I\dot{\omega}_2 &= T - T_L \\ T &= {i_1}^T \mathrm{J} \psi_1 \end{aligned} \quad (6)$$

where $\mu = 1/I$. Substitution the stator and rotor currents as functions of fluxes $i_1 = \Delta^{-1}(L_2\psi_1 - M\psi_2)$, $i_2 = \Delta^{-1}(-M\psi_1 - L_1\psi_2)$, $\Delta = L_1 L_2 - M^2$ into (6) yields

$$\begin{aligned} \mu\dot{\psi}_1 + \frac{RL_2}{\Delta}\psi_1 + \omega_1 \mathrm{J}\psi_1 - \frac{RM}{\Delta}\psi_2 - u &= 0 \\ \mu\dot{\psi}_2 - \frac{RM}{\Delta}\psi_1 + \frac{RL_1}{\Delta}\psi_2 + (\omega_1 - \omega_2)\mathrm{J}\psi_2 &= 0 \end{aligned} \quad (7)$$

or in a matrix form

$$\dot{\psi} = -\frac{1}{\mu} A\psi + B, \quad (8)$$

where

$$\psi = \begin{bmatrix} \psi_{1\alpha} \\ \psi_{1\beta} \\ \psi_{2\alpha} \\ \psi_{2\beta} \end{bmatrix}, \quad B = \begin{bmatrix} u_\alpha \\ u_\beta \\ 0 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} \frac{L_2R}{\Delta} & -\omega_1 & -\frac{MR}{\Delta} & 0 \\ \omega_1 & \frac{L_2R}{\Delta} & 0 & -\frac{MR}{\Delta} \\ -\frac{MR}{\Delta} & 0 & \frac{L_1R}{\Delta} & -(\omega_1-\omega_2) \\ 0 & -\frac{MR}{\Delta} & \omega_1-\omega_2 & \frac{L_1R}{\Delta} \end{bmatrix} \quad (9)$$

For high enough value of I or small μ the system motion may be decoupled into fast and slow components — with respect to vector ψ and ω_2 — rotor speed correspondingly.

Approximate solution to (9) within μ accuracy may be obtained from (9) with $\dot{\psi} = 0$, if the fast motion is asymptotically stable under the assumption that the slow component is constant or matrices A and B are time invariant. It means that asymptotic stability of the equation in deviations

$$\delta\dot{\psi} = -\frac{1}{\mu}A\delta\psi, \quad (10)$$

where $\psi = \psi_0 + \delta\psi, \quad \psi_0 = \mu A^{-1}B$ should be studied.

Time derivative of Lyapunov function $v = \frac{1}{2}\delta\psi^T\delta\psi > 0$ on the solutions of (10)

$$\dot{v} = -\frac{1}{\mu}\delta\psi^T\frac{A+A^T}{2}\delta\psi$$

is negative definite which testifies to asymptotic stability, since the matrix

$$\frac{A+A^T}{2} = \frac{1}{\Delta}\begin{bmatrix} L_2R & 0 & -MR & 0 \\ 0 & L_2R & 0 & -MR \\ -MR & 0 & L_1R & 0 \\ 0 & -MR & 0 & L_1R \end{bmatrix} \quad (11)$$

has positive dioganal determinants.Hence to derive a reduced model μ should be made equal to zero

$$\begin{aligned} 0 &= u - Ri_1 - \omega_1 J\psi_1 \\ 0 &= -Ri_2 - (\omega_1-\omega_2)J\psi_2 \\ \psi_1 &= L_1i_1 + Mi_2 \\ \psi_2 &= Mi_1 + L_2i_2 \\ \dot{\omega}_2 &= \tfrac{1}{I}[i_1J\psi_1 - T_L] \end{aligned} \quad (12)$$

the vectors i_1 and ψ_1 should be found as functions of the voltage vector u. Neglecting the electromagnetic dynamics means that the rotation speeds of the flux and voltage coincide and

$$\omega_2 + s = \omega_1$$

where s is a motor slip. The above procedure results in

$$I\dot{\omega}_2 = T(s) - T_L \quad (13)$$

where $T(s)$ is the well-known induction motor "torque–slip "characteristics (Fig.1).

$$T(s) = \frac{|u|^2M^2}{R^3} \cdot \frac{s}{[1+s^2(\frac{L_2}{R})^2] + 2s\omega_1\frac{L_1L_2}{R^2}(1-\eta) + [1+\eta^2(\frac{L_2}{R})^2s\omega_1^2(\frac{L_1}{R})^2]} \quad (14)$$

$$|u|^2 = u_\alpha^2 + u_\beta^2 \quad (15)$$

$$\eta = 1 - \frac{M^2}{L_1L_2}$$

The maximum value of the motor torque T_{cr} corresponds to the critical value of slip

$$s_{cr} = \frac{1+\omega_1^2(\frac{L_1}{R})^2}{1+\eta^2\omega_1^2(\frac{L_1}{R})^2}(\frac{R}{L_2})^2 \quad (16)$$

For $|s| \leq s_{cr}$

$$T(s) \approx \frac{T_{cr}}{s_{cr}}s. \quad (17)$$

In the framework of the model (13) and (15), the sliding mode control is a discontinuous function of the control error

$$s = s_{cr}\text{sgn}[w_0(t) - w_2], \quad (18)$$

$w_0(t)$ – refernece input.

For $T_{cr} >| T_L + I\dot{\omega}_0 |$ the values $\sigma = \omega_0 - \omega_2$ and $\dot{\sigma}$ have different signs therefore after a finite time interval sliding mode occurs [4] and the motor rotation is equal to the reference input identically.

3 Third–Order Model

The second approach to the design of sliding mode control algorithm is based on the assumption that the time constant related to the motor flux is considerably greater than that of the dissipation flux.

The possibility of motion separation in this case becomes transparent if the motor motion equations (1) (2) (3) are written with respect to the stator and magnetixing currents:

$$\begin{aligned} (L_2+M)\dot{i}_M &= -L_\sigma\omega_2Ji_1 - [RI_2 + (M+L_2)\theta J - L_2\omega_2J]i_M \\ &\quad +u \\ L_\sigma(1+\frac{M}{L_2})\dot{i}_1 &= -[(1+\frac{M}{L_2})RI_2 + \frac{L_\sigma M}{L_2}(\theta-\omega_2)J - L_\sigma\theta J]i_1 \\ &\quad +[\frac{M}{L_2}RI_2 - M\omega_2J]i_M + u \end{aligned} \quad (19)$$

$$I\dot{\omega}_2 = (L_\sigma {i_1}^T + M{i_M}^T)Ji_1 - T_L$$

Since for induction motors the relationships

$$L_\sigma \ll M \approx L_2 \quad (20)$$

hold, the dissipation inductance $L_\sigma = \mu$ may be handled as a small papameter and the first equation in (20) represent the fast motion. Zeroing μ should be justified by the analysis of asymptotic stability of the first equation under the assumption that the rest of the state vector components are constant. It may be represented as

$$2\mu\begin{bmatrix} \dot{i}_\alpha \\ \dot{i}_\beta \end{bmatrix} = -\begin{bmatrix} C & -D \\ D & C \end{bmatrix}\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} + \begin{bmatrix} M \\ N \end{bmatrix} \quad (21)$$

where

$$\begin{aligned} C &= 2R \\ D &= \dot{\theta}\mu + (\dot{\theta} - \omega_2)\mu \\ M &= u + i_M R \\ N &= -i_M \omega_2 L_2 \end{aligned}$$

or as equation with respect to deviation

$$\begin{aligned} \dot{\delta i} &= -\tfrac{1}{2\mu} A_1 \delta i \\ i &= i_0 + \delta i \\ i_0 &= A_1^{-1} B \\ A_1 &= \begin{bmatrix} C & -D \\ D & C \end{bmatrix} \\ B &= \begin{bmatrix} M \\ N \end{bmatrix} \end{aligned} \quad (22)$$

Time derivative of positive definite Lyapunov function

$$v = \frac{1}{2}\delta i^T \delta i \quad (23)$$

on the system (22) trajectories

$$\dot{v} = -\frac{1}{2\mu}\delta i^T \frac{A_1 + A_1^T}{2}\delta i$$

$$\frac{A_1 + A_1^T}{2} = \begin{bmatrix} C & 0 \\ 0 & C \end{bmatrix} \quad (24)$$

is negative definite. Hence the fast motion is asymptotically stable which enables reduction of the system order finding the stator current from the first equation of (19) with $L_\sigma = 0$ as a function of $u, i_M, \dot{\theta}, \omega_2$ and then substituting it into the rest equation.

The above procedure results in motion equations in the coordinate system rotating with the vector i_M:

$$\begin{aligned} &2M\frac{d|i_M|}{dt} = |u|\cos\varphi - |i_M|R \\ &0 = |u|\sin\varphi - 2\dot{\theta}|i_M|M + |i_M|\omega_2 M \\ &I\frac{d\omega_2}{dt} = \frac{M}{2R}|i_M|(|u|\sin\varphi - |i_M|\omega_2 M) - T_L \end{aligned} \quad (25)$$

where φ is angle between the vectors u and i_M.

The angle φ is handled as a control action: jumpwise increment of φ by π may lead to change of sign of the right-hand side in equation of the mechanical motion and as a result to accelaration or decelaration of the motor shaft rotation. Inversion of the voltage phase in correspondence with

$$\varphi = \alpha_u(t) - \frac{\pi}{2}(1 - \text{sgn } \sigma), \quad \sigma = \omega_0(t) - \omega_2 \quad (26)$$

($\alpha_u(t)$ - continuous time function depending on the motor state) similarly to the discontinuous control (18) makes the sgns of σ and $\dot{\sigma}$ opposite and due to origination of sliding mode the rotor speed tracks the reference input.

4 Combined Control Algorithm

For the above control algorithms it is assumed that the motor slip or the phase of the voltage are discontinuous funtions of the control error. Futher the system combining both control methods will be studied analytically.

The vectors u and i_M are presented in Fig.2.

Let the value of the input voltage rotation speed is formed in correspondence with the slip control algorithm.

$$\omega_1 = \omega_2 + s_{cr}\text{sgn } \sigma \quad (27)$$

Taking into account the phase control algorithm (26) and relationships

$$\alpha_u = \theta + \varphi, \quad \alpha_u = \int_0^t \omega_1 dt - \frac{\pi}{2}(1 - \text{sgn } \sigma) \quad (28)$$

obtain

$$\theta_1 = \theta + \varphi = \int_0^t (\omega_2 + s_{cr}\text{sgn } \sigma)dt - \frac{\pi}{2}(1 - \text{sgn } \sigma) \quad (29)$$

Let $\varphi^* = \varphi - \frac{\pi}{2}\text{sgn } \sigma$, then $\frac{\pi}{2}\text{sgn } \sigma + \varphi^* = \varphi$, $\dot{\theta} = \omega_2 + s_{cr}\text{sgn } \sigma - \dot{\varphi}^*$. Substitution φ and $\dot{\theta}$ into (25) results in

$$\begin{aligned} \frac{d|i_M|}{dt} &= -\frac{|u|}{2M}\sin\varphi^*\text{sgn } \sigma - \frac{|i_M|}{2M}R \\ \frac{d\varphi^*}{dt} &= s_{cr}\text{sgn } \sigma + \frac{w_2}{2} - \frac{|u|}{2M|i_M|}\cos\varphi^*\text{sgn } \sigma \\ I\frac{d\omega_2}{dt} &= \frac{M}{2R}|i_M||u|\cos\varphi^*\text{sgn } \sigma - \frac{M^2}{2R}\omega_2|i_M|^2 - T_L \end{aligned} \quad (30)$$

The condition of sliding mode to exist (s and $\dot{s}$ should have opposite sgns) may be derived from (30):

$$\frac{M}{2RI}|i_M||u|\cos\varphi^* > \left|\dot{\omega}_0 + \frac{M^2}{2RI}\omega_0 + \frac{T_L}{I}\right| \quad (31)$$

Origination of the sliding mode means that the control mismatch is steered to zero. To analyze the behaviour of the rest two coordinates φ^* and i_M, the equation $\sigma = 0$ should be solved with respect to sgnσ and the solution refered to as equivalent control is to be substituted into the first two equation with $\sigma = 0$[4]. Applying the conventional linearization method it can be shown that for time invariant load torques the motion equation has the only equilibrium point which is asymptotically stable.

5 Implementation

The small time constants having been neglected in the reduced order models may result in oscillatory component in system coordinates since switching in control excites the unmodelled dynamics. This phenomenon — so-called chattering — is eliminated in the sliding mode control systems with asymptotic observers [5]. In practical applications the observers were designed under the assumption that the load torque varies much slower than the motor state variables.

In the system with an observer the estimate $\hat{\omega}_2$ of the rotor speed is used in the switching function

$$\sigma = \omega_0(t) - \hat{\omega}_2 \quad (32)$$

while the observer is governed by equations

$$\begin{aligned} I\frac{d\hat{\omega}_2}{dt} &= (P_1\text{sgn } \sigma - P_2\hat{\omega}_2 - \hat{T}_L) + l_1(\omega_2 - \hat{\omega}_2) \\ \frac{d\hat{T}_L}{dt} &= l_2(\omega_2 - \hat{\omega}_2) \end{aligned} \quad (33)$$

The experimental set up was designed, assembled and tested. Its blockdiagram is presented in Fig. 3. Figure 4 shows the transient processes for periodical reference input with the amplitude equal to 0.3 of the nominal rotor speed.

6 Conclusion

The developed control algorithms are considerably simpler than conventional ones associated with the field oriented approach since for their implementation only output variable is needed and the control actions are sign functions of the mismatch.

The experiments demonstrated high dynamic properties of the system. Phase control in the combined systems enables reduction of the voltage amplitude at low speed modes automatically which prevents the motor under control from overheating.

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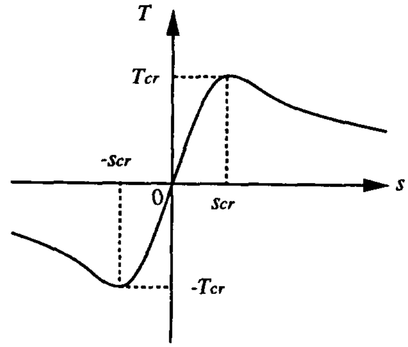

Figure 1. Torque-Slip Characteristics

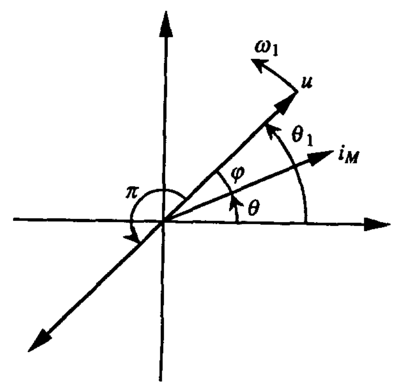

Figure 2. Vector diagram of magnetized current and input voltage

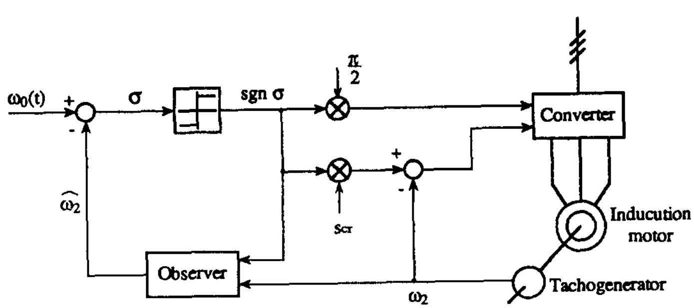

Figure 3. Blockdiagram of control system

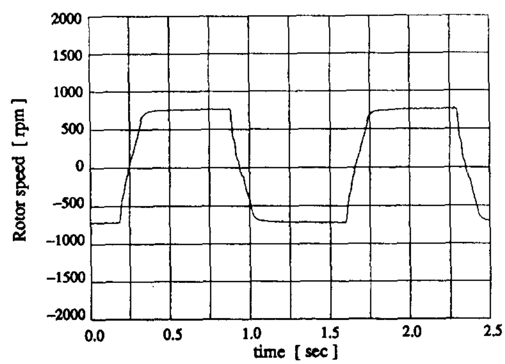

Figure 4. Time response of rotor speed