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A Robust Control System Design by a Parameter Space Approach Based on Sign Definite Condition

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ABSTRACT

A parameter space approach for robust control system design is developed by reducing several design specifications to sign definite conditions. It is shown that the gain and phase margin constraints for the parametric perturbed plant hold if and only if the four Kharitonov systems satisfy the margins. On pole location, it is shown that D-stability of convex combinations $(1-t)p(s)+tq(s)$ can be determined by the coefficients corresponding to $p(s)$ and $q(s)$ based on the sign definite condition. We show a method of PI-type robust control system design as a useful example.

1. INTRODUCTION

Multi-objective problem, that is, to keep several specifications simultaneously, is a main concern in the control system design. However, the problem is generally hard to solve analytically and a parameter space design method is one of the useful tools to treat this kind of problem. Therefore, we have proposed a method of control system design by a parameter space approach based on sign definite condition with numerical calculation and symbolic manipulation [1]. By using the sign definite condition, several important design specifications can be treated uniformly. The design scheme is as follows:

1. determine the structure of the controller and select the parameters, e.g., PI-compensator $(K+Ts)/s$, K and T are the parameters.
2. reduce the specifications to the corresponding sign definite conditions.
3. compute the admissible regions in the parameter space for each specification.
4. superpose the regions and take the parameters in the intersections.

In the specifications, robustness have been paid lots of attention in recent years. It have been shown that the following specifications, which are often used as indices of robustness of systems, can be reduced to the sign definite conditions for fixed plant [1] : gain margin, phase margin, H_∞ norm constraints, and frequency restricted norm constraints. For the plant with parametric perturbation, H_∞ norm constraints and frequency restricted norm constraints have been investigated [2] [4] and these constraints have also been reduced to sign definite conditions [4].

In this paper, we investigate the gain and phase margins and pole location for the parametric perturbed plant. By using our design method, these constrains can be solved for the parametric perturbed plants with other specifications.

2. SIGN DEFINITE CONDITION

Definition 1 *A function $f(x) : \mathbf{R} \mapsto \mathbf{R}$ is sign definite in the interval $x \in [a,b], a < b$, denote $f(x) \in \mathrm{N}_0[a,b]$ hereafter, if $f(x)$ preserves the sign in the interval, or does not cross zero in the interval.*

Remark: The sign definite condition $f(x) \in \mathrm{N}_0[a,b]$ can be transformed to the condition $f(z) \in \mathrm{N}_0[0,\infty]$ by the bilinear transformation $z=-(x-a)/(x-b)$. Then, if $f(x)$ is polynomial in x, the sign definite condition can readily checked by the following Routh-Hurwitz type criterion:

Lemma 1 [7] *An n-th order polynomial $f(x)$ with real coefficients is sign definite in $x \in [0,\infty]$ if and only if*

$$V[f(x)] = n \tag{1}$$

holds, where V is the number of sign changes of the most left column of the Modified Routh Array defined by

$$\begin{array}{llll} (-1)^n f_n & (-1)^{n-1} f_{n-1} & \cdots & -f_1 \quad f_0 \\ (-1)^n n f_n & (-1)^{n-1}(n-1) f_{n-1} & \cdots & -f_1 \\ \cdot & \cdot & \cdot & \\ \vdots & & & \\ f_0 & & & \end{array} \tag{2}$$

Note that there exists Hurwitz type criterion [7].

It have been shown that H_∞ norm constraint can be directly reduced to the sign definite condition[4]. The following lemma is useful in order to reduce a constraint to a sign definite condition and it will be used in Sections 3. and 4..

Lemma 2 *Consider simultaneous equation*

$$\begin{cases} f_1(\omega,t) &= 0 \\ f_2(\omega,t) &= 0 \end{cases} \quad (3)$$

where $f_1(\omega,t)$ and $f_2(\omega,t)$ are polynomial in ω and t. Then, this does not hold in $\omega \in \mathbf{R}, t \in [a,b]$ if and only if

$$f(t) \in \mathbf{N}_0[a,b] \quad (4)$$

or

$$\begin{array}{c} {}^{\exists}t_0 \in [a,b] \ s.t. \\ f(t_0) = 0 \ \ \text{and} \ \ g(\omega,t)|_{t=t_0} \in \mathbf{N}_0[-\infty,\infty] \end{array} \quad (5)$$

where $f(t)$ and $g(\omega,t)$ are defined as follows: without loss of generality, we can assume the order of $f_1(\omega,t)$ corresponding to ω is equal to or greater than the order of $f_2(\omega,t)$. By using the Euclidean algorithm, we can eliminate one of the variables and obtain $f(t)$, that is,

$$\begin{array}{rcl} f_1(\omega,t) &=& q_1(\omega,t)f_2(\omega,t) + f_3(\omega,t) \\ f_2(\omega,t) &=& q_2(\omega,t)f_3(\omega,t) + f_4(\omega,t) \\ &\vdots& \\ f_k(\omega,t) &=& g(\omega,t)f_{k+1}(\omega,t) + f(t) \end{array} \quad (6)$$

We denote $f(t) := \mathbf{Euc}[f_1(\omega,t), f_2(\omega,t), \omega]$ hereafter. If the order of $g(\omega,t)$ w.r.t ω is odd, the sign definite condition (5) is not satisfied for every t. Then, we have more simple condition:

Corollay 1 *If the order of $g(\omega,t)$ w.r.t ω is odd, where $g(\omega,t)$ is defined in the Lemma 2, then the simultaneous equation defined by (3) dose not hold in $\omega \in \mathbf{R}, t \in [a,b]$ if and only if*

$$f(t) \in \mathbf{N}_0[a,b] \quad (7)$$

holds.

Corollay 2 *If the parity of the order corresponding to $f_1(\omega,t)$ and $f_2(\omega,t)$ w.r.t ω is different, that is, the orders are even and odd or odd and even, the simultaeious equation (3) does not hold in $\omega \in \mathbf{R}$, $t \in [a,b]$ iff*

$$f(t) \in \mathbf{N}_0[a,b] \quad (8)$$

holds.

proof: we can say from the definition of $g(\omega,t)$ shown in (6) that if the parity of the order of $f_1(\omega,t)$ and $f_2(\omega,t)$ w.r.t. ω are not identical, the order of $g(\omega,t)$ is odd in general. Then, Corollary 1 leads to Corollary 2

Note that if there exist $f_1^*(\omega,t)$ and $f_2^*(\omega,t)$ such that the parity of the orders are distinct each other and the order of $g^*(\omega,t)$ w.r.t. ω is even. Then, with small perturbations ϵ_1 and ϵ_2, we can find out $\tilde{f}_1(\omega,t) := f_1^* + \epsilon_1$ and $\tilde{f}_2(\omega,t) := f_2^* + \epsilon_2$ such that the order of $\tilde{g}(\omega,t)$ w.r.t. ω is odd. From the continuity, this implies that $g^*(\omega,t)$ have real roots in ω since the order of $\tilde{g}(\omega,t)$ w.r.t. ω is odd.

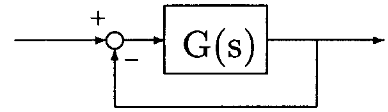

Figure 1: A Unity Feedback System (1)

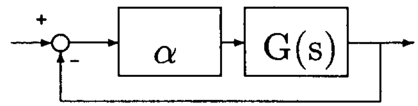

Figure 2: A Unity Feedback System (2)

3. GAIN AND PHASE MARGINS

3.1 STABILITY MARGINS FOR FIXED SYSTEMS

We first define stability margins for the sake of clarify of the discussion, Consider the open loop system $G(s)$ shown in Fig. 1 and suppose that the closed loop system is stable. Then, the gain margin and phase margin are defined as follows:

Definition 2 (Gain Margin) *The open loop system $G(s)$ shown in Fig. 1 holds gain margin (γ_m, γ^M) if the closed loop system shown in Fig. 2 is stable for all $\alpha \in (\gamma_m, \gamma^M)$ and $\alpha G(s)$ at $\alpha = \gamma_m$ and γ^M reaches stability horizons, that is,*

$$\begin{array}{rcl} \gamma_m &:=& \min\{\gamma | q_\gamma(s) \text{ has no zeros} \\ && \qquad \text{in open right half plane}\} \\ \gamma^M &:=& \max\{\gamma | q_\gamma(s) \text{ has no zeros} \\ && \qquad \text{in open right half plane}\} \end{array}$$

where $q_\gamma(s)$ is the characteristic polynomial of the closed loop system shown in Fig.2 defined by

$$q_\gamma(s) := \gamma n(s) + d(s) \quad (9)$$

In the same manner, we define phase margin.

Definition 3 (Phase Margin) *The open loop system $G(s)$ shown in Fig.1 holds phase margin $0 \le \phi < 2\pi$ if the closed loop system shown in Fig.2 is stable for all $\alpha := e^{-j\theta}, \theta \in [0,\phi)$, and $q_\phi(s)$ at $\alpha = e^{-j\theta}, \theta = \phi$ reaches stability horizon, that is,*

$$\phi := \max\{\theta | q_\phi(s) \text{ has no zeros in open right half plane}\}$$

where $q_\phi(s)$ is the characteristic polynomial of the closed loop system shown in Fig. 2 defined by

$$q_\phi(s) := e^{j\theta}n(s) + d(s) \quad (10)$$

Gain margin and phase margin constraints can be reduced to sign definite condition as follows [1]: Consider a rational function $G(s)$ and decompose $G(j\omega)$ as

$$G(j\omega) =: \frac{g_r(\omega) + jg_j(\omega)}{d(\omega)} \quad (11)$$

where $g_r(\omega), g_j(\omega)$ and $d(\omega)$ are polynomials in ω. Using these notations, we obtain the following theorems.

Theorem 1 *$G(s)$ holds gain margin (γ_m, γ^M) iff the simultaneous equation defined by*

$$\begin{cases} f_1(\omega, t) &:= g_r(\omega) - d(\omega)t = 0 \\ f_2(\omega) &:= g_j(\omega) = 0 \end{cases} \tag{12}$$

is not satisfied in $\omega \in \mathbf{R}, t \in [-1/\gamma_m, -1/\gamma^M]$.

Note that since the parity of the orders of $f_1(\omega, t)$ and $f_2(\omega)$ w.r.t ω are different from each other, from the Corollary 2, the gain margin constraint can be checked by only one sign definite condition

$$f_g(t) \in \mathbf{N}_0[-1/\gamma_m, -1/\gamma^M] \tag{13}$$

where

$$f_g(t) := \mathbf{Euc}[f_1(x, \omega), f_2(\omega), \omega] \tag{14}$$

Theorem 2 *$G(s)$ holds the phase margin ϕ iff the simultaneous equation defined by*

$$\begin{cases} f_1(\omega) &:= g_r^2(\omega) + g_j^2(\omega) - d^2(\omega) = 0 \\ f_2(\omega, t) &:= g_r(\omega) - d(\omega)t = 0 \end{cases} \tag{15}$$

is not satisfied in $\omega \in \mathbf{R}, t \in [-1, \cos(-\pi + \phi)]$.

From the Lemma 2, we can see that the phase margin constraint can be reduced to the sign definite conditions.

3.2 STABILITY MARGIN BOUNDS FOR INTERVAL SYSTEMS

Again consider the unity feedback system shown in Fig. 1, where the open loop system $G(s)$ is the interval rational function defined by

$$G(s) := \frac{n(s)}{d(s)} = \frac{\sum b_i s^i}{\sum a_i s^i}, \quad a_i \in [\underline{a}_i, \overline{a}_i], b_k : \text{fixed} \tag{16}$$

Then, we obtain the following Theorem:

Theorem 3 *The open loop system with parametric perturbation defined by (16) holds gain margin (γ_m, γ^M) and phase margin ϕ if and only if the fixed 4 plants*

$$G_i(s) := \frac{n(s)}{k_i(s)} \quad i = 1 \sim 4 \tag{17}$$

hold the gain margin (γ_m, γ^M) and the phase margin ϕ, where $k_i(s)$ represent the four Kharitonov polynomials associated with $d(s)$ denoted by

$$\begin{aligned} k_1(s) &:= \underline{a}_0 + \overline{a}_1 s + \overline{a}_2 s^2 + \underline{a}_3 s^3 + \cdots \\ k_2(s) &:= \underline{a}_0 + \underline{a}_1 s + \overline{a}_2 s^2 + \overline{a}_3 s^3 + \cdots \\ k_3(s) &:= \overline{a}_0 + \overline{a}_1 s + \underline{a}_2 s^2 + \underline{a}_3 s^3 + \cdots \\ k_4(s) &:= \overline{a}_0 + \underline{a}_1 s + \underline{a}_2 s^2 + \overline{a}_3 s^3 + \cdots \end{aligned} \tag{18}$$

proof: Though we will only prove for the phase margin constraint, the proof for the gain margin constraint is carried out in the same way.

Necessity is trivial. By using contradiction, we will proof the sufficiency. Assume that the four Kharitonov systems (17) hold phase margin ϕ and there exists a system $G^*(s) := n(s)/d^*(s)$ of which phase margin is ϕ^* s.t. $\phi^* < \phi$.

By following the definition of the phase margin, this implies that the characteristic polynomial of the closed loop system associated with the four rotated Kharitonov plants defined by

$$q_i(s) := e^{j\phi^*} n(s) + k_i(s) \tag{19}$$

is Hurwitz and the characteristic polynomial of the closed loop system associated with $e^{j\phi^*} G^*(s)$ defined by

$$q^*(s) := e^{j\phi^*} n(s) + d^*(s) \tag{20}$$

is not Hurwitz. Since this characteristic polynomial of the closed loop system associated with the rotated system $e^{j\phi^*} G(s)$ is the interval polynomial with complex coefficients, where the imaginary part is fixed, this contradicts Kharitonov theorem with complex coefficient (see Appendix).

4. POLE LOCATION

In control system degin, it is useful to locate the roots of the characteristic polynomial in a specified region. In this section, we will show that the pole location can be reduced to sign definite conditions.

4.1 POLE LOCATION FOR FIXED PLANT

We here focus on the domain $\mathcal{D} \in \mathbf{C}$ of which the complementary set $\overline{\mathcal{D}} := \mathbf{C} - \mathcal{D}$ can be expressed as

$$\overline{\mathcal{D}} := \{x(\omega, t) + jy(\omega, t) \in \mathbf{C} | \omega \in \mathbf{R}, t \in [\underline{t}, \overline{t}]\} \tag{21}$$

where $x(\omega, t)$ and $y(\omega, t)$ are rationals in ω and t. A wedge shape region and inside of a circle are in the class of the domain $\mathcal{D}$, since for the wedge shape region, $x + jy \in \overline{\mathcal{D}}$ can be expressed as

$$\begin{cases} x &= \omega \\ y &= m(\omega - t) \end{cases} \quad \omega \in \mathbf{R}, \ t \in [b, \infty] \tag{22}$$

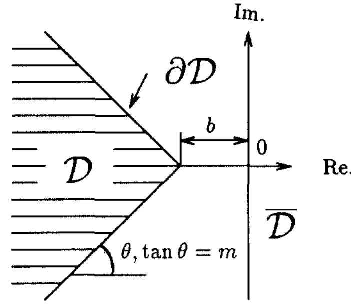

Figure 3: a wedge shape

For a circle, $x + jy \in \overline{\mathcal{D}}$ can be expressed as

$$\begin{cases} x &= \dfrac{r(t^2 + \omega^2 - 1)}{(t-1)^2 + \omega^2} + b \\ y &= \dfrac{-2r\omega}{(t-1)^2 + \omega^2} \end{cases} \quad \omega \in \mathbf{R}, \ t \in [0, \infty] \tag{23}$$

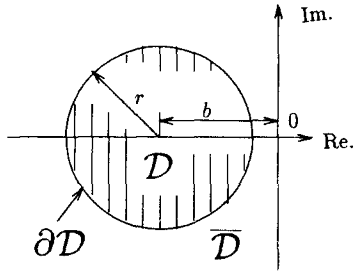

Figure 4: inside of a circle

Let us consider how to assign all roots of a polynomial $p(s)$ in the specified region $\mathcal{D} \in \mathbf{C}$. This is equivalent to

$$p(s) \neq 0 \quad {}^{\forall} s \in \overline{\mathcal{D}} \tag{24}$$

where $\overline{\mathcal{D}}$ denotes the complementary set of $\mathcal{D}$ in $\mathbf{C}$. In this case, $p(s)$ is called D-stable. Then, the pole location problem is stated as follows:

$$\left\{ \begin{array}{l} p_r(x,y) = 0 \\ p_j(x,y) = 0 \end{array} \right. , \quad p(s) =: p_r(x,y) + p_j(x,y) \tag{25}$$

do not hold in $s = x + \jmath y \in \overline{\mathcal{D}}$.

Then, we obtain the following theorem:

Theorem 4 *A polynomial $p(s)$ has all roots in $\mathcal{D}$ of which complementary set $\overline{\mathcal{D}}$ is defined by (21) iff*

$$\left\{ \begin{array}{lcl} P_r(\omega,t) &=& 0 \\ P_j(\omega,t) &=& 0 \end{array} \right. \text{where} \left\{ \begin{array}{lcl} P_r(\omega,t) &:=& p_r(x(\omega,t), y(\omega,t)) \\ P_j(\omega,t) &:=& p_j(x,y) \end{array} \right. \tag{26}$$

do not satisfied in $\omega \in \mathbf{R}, t \in [\underline{t}, \overline{t}]$, where $p_r(x,y)$ and $p_j(x,y)$ are defined by (25).

Applying Lemma 2 to the condition (26) in Theorem 4 the pole location constraint for fixed plant can be checked by sign definite conditions.

For example, consider to determine whether the all roots of polynomial $p(s)$ defined by

$$p(s) := 2 + 4s + 3s^2 + s^3 \tag{27}$$

are in the wedge shape region $\mathcal{D}$ defined by (22) with $m = 1.5$ and $b = -0.2$. Based on sign definite condition, this can be check as follows:

Decompose

$$\begin{array}{rcl} p(s|s \in \overline{\mathcal{D}}) &=& p(\omega + m(\omega - t)\jmath); \omega \in \mathbf{R}, t \in [b, \infty] \\ &=& Pr(\omega,t) + \jmath Pj(\omega,t) \end{array} \tag{28}$$

where

$$\begin{array}{rcl} Pr(\omega,t) &:=& -5.75\omega^3 + (-3.75 - 9t)\omega^2 \\ && +(4 - 9t - 3t^2)\omega + 2 - 3t^2 \\ Pj(\omega,t) &:=& 1.125\omega^3 + (9 - 3.75t)\omega^2 \\ && +(6 + 6t - 4.5t^2)\omega + 4t - t^3 \end{array} \tag{29}$$

By following Theorem 4, all roots are located in the region $\mathcal{D}$ iff

$$\left\{ \begin{array}{lcl} Pr(\omega,t) &=& 0 \\ Pj(\omega,t) &=& 0 \end{array} \right. \tag{30}$$

do not hold in $\omega \in \mathbf{R}, t \in [b, \infty]$. With Euclidean algorithm, we can eliminate ω from $Pr(\omega,t)$ and $Pj(\omega,t)$ and obtain

$$f_p(t) := \mathbf{Euc}[P_r(\omega,t), P_j(\omega,t), \omega] \tag{31}$$

With the linear transformation $x = t - b$, we obtain

$$\tilde{f}_p(x) := f_p(x + b) \tag{32}$$

of which the numerator is 15th order polynomial in x.

Since V[numerator of $\tilde{f}_p(x)$] $= 15$, Lemma 1 leads $\tilde{f}_p(x) \in \mathbf{N}_0[0,\infty]$ and this implies that $f_p(t) \in \mathbf{N}_0[b,\infty]$. Hence, we conclude that all roots of (27) are in the wedge shape region $\mathcal{D}$. See Fig. 5

4.2 POLE LOCATION FOR INTERVAL SYSTEMS

On the pole location for the parametric perturbed system of which coefficients are polytope in the coefficient space, it is important to check the D-stability of a convex combination of polynomials [8]. In this subsection, it is shown that the D-stability condition can also be reduced to the sign definite condition. Assume that the boundary $\partial\mathcal{D}$ can be expressed by one parameter ω as

$$\partial\mathcal{D} := \eta(\omega) + \jmath\xi(\omega), \quad \omega \in \mathbf{R} \tag{33}$$

Consider the convex combination associated with two polynomials $p(s)$ and $q(s)$ defined by

$$f(s) := (1-t)p(s) + tq(s); \quad t \in [0,1] \tag{34}$$

and decompose $f(s)$ at $s = \eta(\omega) + \jmath\xi(\omega) \in \partial\mathcal{D}$ as

$$\begin{array}{rcl} f(\eta(\omega) + \jmath\xi(\omega)) &=& \{(1-t)p_r(\omega) + tq_r(\omega)\} \\ && +\jmath\{(1-t)p_j(\omega) + tq_j(\omega)\} \\ &=:& f_r(x,t) + \jmath f_j(x,t) \end{array} \tag{35}$$

where

$$\begin{array}{rcl} p(\eta(\omega) + \jmath\xi(\omega)) &=:& p_r(\omega) + \jmath p_r(\omega) \\ q(\eta(\omega) + \jmath\xi(\omega)) &=:& q_r(\omega) + \jmath q_j(\omega) \end{array} \tag{36}$$

Then, by following the continuity of the roots of the polynomial w.r.t. its coefficients, we obtain the following theorem.

Theorem 5 *Consider the domain $\mathcal{D}$ defined by (33). Then, the polynomial $f(s)$ defined by (34) is $\mathcal{D}$-stable iff the following two condition are satisfied:*

1) $p(s)$ or $q(s)$ is $\mathcal{D}-$stable

2) $\left\{ \begin{array}{lcl} f_r(\omega,t) &=& 0 \\ f_j(\omega,t) &=& 0 \end{array} \right.$ do not hold for $\omega \in \mathbf{R}, t \in [0,1]$

where $f_r(\omega,t)$ and $f_j(\omega,t)$ are defined by (35).

By applying Theorem 4 and Lemma 2, Theorem 5 can be checked by sign definite conditions.

For example, consider to determine whether the all roots of the convex combination $f_1(s) = (1-t)p_1(s) + tp_1(s), t \in [0,1]$, where

$$\begin{array}{rcl} p_1(s) &:=& 2 + 4s + 3s^2 + s^3 \\ q_1(s) &:=& 12.75 + 16.25s + 7.s^2 + s^3 \end{array} \tag{37}$$

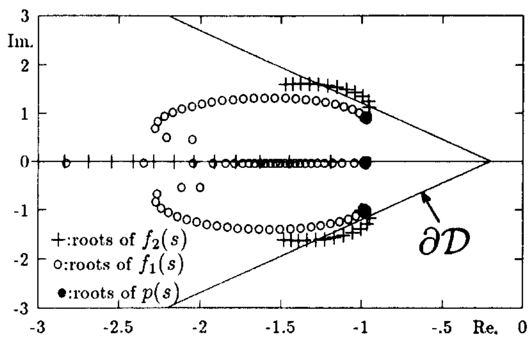

Figure 5: pole locations

are in the wedge shape region $\mathcal{D}$ defined by (22) with $m = 1.5$ and $b = -0.2$.. From the example in the subsection 4.1, we see that $p_1(s)$ is $\mathcal{D}$-stable. Since the boundary is expressed as

$$\partial\mathcal{D} := x + jm(x+b), \quad x \in \mathbf{R} \tag{38}$$

then, we obtain

$$f_1(x + jm(x+b)) = f_r(\omega, t) + jf_j(\omega, t) \tag{39}$$

where

$$\begin{aligned} f_r(\omega,t) &:= -5.75\omega^3 + (-6.45 - 5t)\omega^2 \\ &\quad +(1.03 + 8.65t)\omega + 1.73 + 10.39t \\ f_j(\omega,t) &:= 1.125\omega^3 + (7.875 + 12t)\omega^2 \\ &\quad +(7.395 + 20.775t)\omega + 1.173 + 3.675t \end{aligned} \tag{40}$$

With Euclidean algorithm, set

$$f_p(t) := \mathbf{Euc}[f_r(\omega,t), f_j(\omega,t), \omega] \tag{41}$$

and define

$$\bar{f}_p(x) := f_p(\frac{1}{(x+1)}) \tag{42}$$

Since the order of the numerator of $\bar{f}_p(x)$ is 11 and V[numerator of $\bar{f}_p(x)$] = 11, Lemma 1 leads that $\bar{f}_p(x) \in \mathbf{N}_0[0,\infty]$ and this implies that $f_p(t) \in \mathbf{N}_0[0,1]$. Then, from Lemma 2, we see that

$$\begin{cases} f_r(\omega,t) = 0 \\ f_j(\omega,t) = 0 \end{cases} \tag{43}$$

do not hold for any $\omega \in \mathbf{R}$ and $t \in [0,1]$ Hence, we conclude that all roots of $f_1(s)$ are in the wedge shape region $\mathcal{D}$ for every $t \in [0,1]$. See Fig. 5

By following same scheme, we can find out that the convex combination $f_2(s) = (1-t)p_2(s) + tp_2(s), t \in [0,1]$, where

$$\begin{aligned} p_2(s) &:= p_1(s) \\ q_2(s) &:= 14.43 + 13.81s + 6.s^2 + s^3 \end{aligned} \tag{44}$$

is not $\mathcal{D}$-stable. (see Fig. 5)

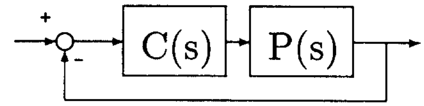

Figure 6: a PI type unity feedback system

5. DESIGN EXAMPLE

In this section, we give an design example for a PI-type feedback control system shown in Fig. 6 with parametric perturbed plant $P(s)$ and PI-type controller $C(s)$, where

$$\begin{aligned} P(s) &= \frac{1}{s-\alpha}; \quad \alpha \in [\underline{\alpha}, \overline{\alpha}] \\ C(s) &= \frac{K_I + K_P s}{s} \end{aligned} \tag{45}$$

Our aim is to obtain the parameters K_I and K_P which satisfy the following robust stability property under parametric perturbation $\alpha \in [\underline{\alpha}, \overline{\alpha}]$.

1. the phase margin is lager than ϕ.
2. the complementary sensitivity function $T(s)$ satisfies the following norm constraints:

$$\|T(s)\|_{[\omega_t,\infty]} < \gamma_1 \;,\; \|T(s)\|_{[0,\omega_t]} < \gamma_2 \tag{46}$$

where

$$T(s) := \frac{P(s)C(s)}{1+P(s)C(s)} = \frac{K_I + K_P s}{s^2 + (K_P - \alpha)s + K_I} \tag{47}$$

and $\underline{\alpha} = 1, \overline{\alpha} = 4.5, \omega_t = 20, \gamma_1 = -10\text{dB}, \gamma_2 = 15\text{dB}, \phi = \pi/6$.

There are only two Kharitonov systems

$$G_1(s) := \frac{K_I + K_P s}{s^2 - \underline{\alpha} s} \quad G_2(s) := \frac{K_I + K_P s}{s^2 - \overline{\alpha} s} \tag{48}$$

associated with the open loop system. From Theorem 3, phase margin constraint is satisfied under parametric perturbation iff the two Kharitonov system defined by (48) hold the constraint.

From [4], the two norm constraints (46) can be decomposed as sign definite condition.

By using symbolic manipulations and numerical calculations, we obtain admissible region shown in Fig. 7 By taking parameters K_P and K_I in the intersection, e.g. $K_P = 6.2, K_I = 7.0$, we can satisfy all all constraints. (see Fig. 8, 9)

6. CONCLUSION

we have investigated gain and phase margin constraints for the parametric perturbed system and it have been shown that the stability margins are bounded by the Kharitonov systems. Pole location is also investigated and it have been shown that this constraint can be reduced to sign

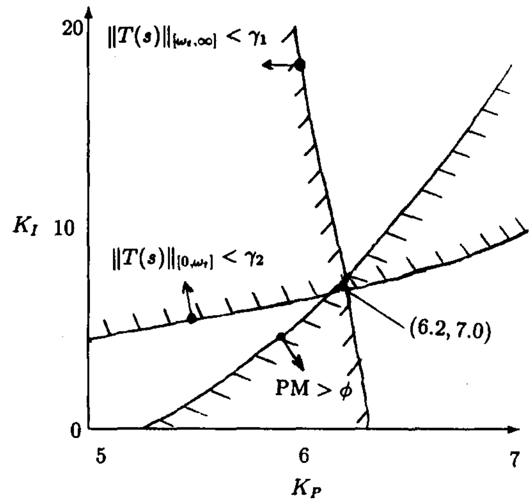

Figure 7: admissible parameter space

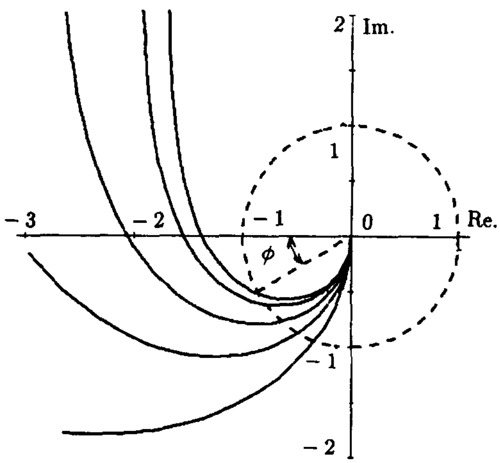

Figure 8: nyquist plots of $C(\jmath\omega)P(\jmath\omega)$ for $\alpha = 1, 2, 3, 4, 4.5$

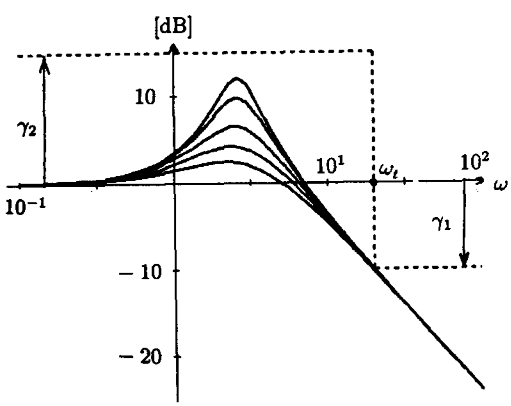

Figure 9: gain plots of $T(\jmath\omega)$ for $\alpha = 1, 2, 3, 4, 4.5$

definite conditions. In addition, it have been shown that the D-stability condition of the convex combination corresponding to two polynomial can be reduced to a sign definite condition. Combining the parameter space design method, we can satisfy the stability margin constraint for the parametric perturbed plant by a PI-type compensator with other specifications.

Finally, we wish to express our appreciation to Prof. S.P.Bhattacharyya for his useful comments on our results.

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APPENDIX

Theorem 6 [Kharitonov thorem with complex coefficients]
Let $\mathcal{P} := \{p(s)\}$ is a interval polynomial with complex coefficients, where

$$p(s) := \sum_{k=0}^{n}(a_k + \jmath b_k)s^k \tag{49}$$

$$a_k \in [\underline{a}_k, \bar{a}_k] \quad b_k \in [\underline{b}_k, \bar{b}_k] \tag{50}$$

Then, the all member of $\mathcal{P}$ are strictly Hurwitz if and only if the following eight complex case Kharitonov polynomials $kci(s), i = 1 \sim 8$ associated with $\mathcal{P}$ are strictly Hurwitz

proof: see e.g. [6].

Remark: If the all complex part b_k are fixed, the interval polynomial with complex coefficients defined by (49) is urwitz if and only if the four polynomials corresponding to real coefficients case are Hurwitz.