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# 규격부재로 이루어진 대형 철골구조물의 최적설계를 위한 알고리즘

## An Optimal Design Algorithm for The Large-Scale Structures with Discrete Steel Sections

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ABSTRACT

An optimization method has been developed to find the minimum weight design of steel building structures which consist of the commercially available discrete sections. In this study, an emphasis was particularly placed on the practical applicability of optimization algorithm in engineering practice. The structure is optimized through element optimization under the element level constraints first and then, if there is any violation of structural level constraints, it is adequately compensated by the constraint error correction vector obtained through the sensitivity analysis. A scaling procedure is introduced for the problems of large violated displacement constraint. The oscillation control in the objective function is also discussed. By dividing the available H-sections into two groups based on their section characteristics, much improved relationships between section variables were obtained and used efficiently in searching the optimum section in the section table.

## 1.0 INTRODUCTION

The optimum structural design has been one of the areas that attracted many engineers' attention. Since the introduction of mathematical programming technique by L.A. Schmit[1], a number of other structural optimization schemes have been proposed for different optimization problems by many investigators[2,3,4]. Fully stressed design[2], linear programming[5], sequential unconstrained minimization techniques[6] and gradient projection method[3] are typical direct methods. In spite of quite effective method for small-scale structures, the increased number of design variables and the computational cost may limit the practical applicability of the direct method to large-scale structures[7]. The optimal criteria technique[7,8,9], which is a typical indirect method based on the optimization necessary conditions(e.g. Kuhn-Tucker condition) has been developed for large-scale structures. The methods which utilize the substructuring technique[10], the multilevel decomposition of constraints[11], dual method[12], and the artificial intelligence approaches[13] are also given attention lately.

In most previous investigations, however, the solutions(optimal designs) were based on the assumption that are design variables continuous. Since most steel building frames are composed of members of commercially available rolled sections due to the economical efficiency and standardization, the continuous solution schemes can not be applied directly to such structures. Therefore, in order to apply the continuous optimal solutions obtained by one of the aforementioned optimization methods to practical design problems, the structure has to be either constructed with members machined precisely to the optimal sizes obtained, or simply fabricated with commercially available sections of the next large sizes. The former in many cases incurs a large increase of the production cost to an uneconomical extent and the latter may lead to a non-optimal structure[14].

Lately, the discrete optimization techniques which produce the optimal solutions in the specified member sizes for practical applications[14,15] have drawn a lot of investigator's attention. Many such techniques, however, are not widely used in practical problems yet since the stability, convergence and efficiency of the methods can not always be guaranteed when they are applied to a structural optimization problem with a large number of variables.

In this paper, an optimization algorithm based on both the optimal criteria method and the gradient projection method defined in the state space were developed for steel building structures. In this algorithm, the design constraints are decomposed into two levels, namely, element(component) level and structural level constraints. The optimal solution is first sought in the problem formulated with the element level constraints. Then, the violated structural level constraints, if any, are corrected effectively by the method explained in the next sections. Also, the following points are particularly emphasized in this study. 1) The optimal member sizes should be found in the commercially available

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section table. 2) The algorithm should be able to introduce various practical design routines and design codes as design constraints. 3) The method should be stable and effective for the problems of large-scale structures such as high-rise buildings with many design variables(members).

## 2.0 OPTIMUM DESIGN WITH DISCRETE MEMBER SIZES

The optimum structural design problem in this paper may be defined as finding a design variable vector **b** that minimizes the objective function(the total weight of the structure) in the following form

$$\Psi_o = \sum_{i=1}^{NE} W_i L_i A_i(b) \tag{1}$$

satisfying

$$\mathbf{h}(b,u) = \mathbf{K}(b)\mathbf{U} - \mathbf{P}(b) = 0 \tag{2}$$

subjected to element level constraints

$$\Psi^s(b,u) \le 0 \tag{3}$$

$$\Psi^v(b) \le 0 \tag{4}$$

$$\mathbf{b} \in \mathbf{T} \tag{5}$$

and structural level constraints

$$\Psi^d(u) \le 0 \tag{6}$$

where **b** = design variable vector, $NE$ = total number of elements, $W_i$, $L_i$ and $A_i$ = weight density, length, and cross sectional area of i-th element, respectively, $\mathbf{h}(b,u)$ = state equation(equilibrium equation in finite element method), **U** = state variable vector(n nodal displacements), $u$ = elements of **U**. **K** = (n x n) structural stiffness matrix, **P** = load vector(n nodal loads), $\Psi^s$ = stress constraints, $\Psi^d$ = constraints on the nodal displacements, $\Psi^v$ = design variable constraints, such as predetermined flange widths and beam depths, etc., and **T** = section table. The design variable vector **b** in this study must be found in **T**. It is noted that the element level constraints are mainly the constraints on stress and design variables while the structural level constraints are such as the maximum structural displacements and natural frequencies.

With constraints decomposed as above, it is now possible to maximize the combined merits of the optimal criteria method and those of the gradient projection method. While the former is used quite efficiently for the structural optimizations under the stress and design variable constraints, the latter is useful in deciding the manner how the current member sizes should be changed to obtain the desired structural behavior, i.e. deciding the member whose property change may have the largest influence on the structural behavior.

## 3.0 TRANSFORMATION OF DISCRETE VARIABLES

A sensitivity analysis is necessary to identify the members of which the structural behavior is most sensitive to the property changes. At least the first derivatives of stiffness matrix are necessary accomplish the sensitivity analysis. Therefore, it is desirable that the discrete design variables of H–sections(or WF sections) in the table are transformed into a continuous form so that the variables may become differentiable.

The section properties of rolled section can be characterized by four variables, i.e. the thicknesses of web and flange, and the depth and width of section. However, these four variables can not be used independently in defining the sensitivity vector, since such independently defined four variables may not define a unique section in the section table. Thus, a number of ways to establish the relationship between the design variable and the desired section property were suggested to get rid of aforementioned difficulties[8,16,17].

In this paper, the relationships between any two section variables are established by regression for H–sections in the metric measuring system. In Figs. 1 and 2, for example, cross sectional areas and section modulus of commercially available H–sections are plotted with major axis moments of inertia of sections, respectively. In these figures, it is clearly seen that instead of a single curve as used in many previous investigations[8,16,17], two different curves used in this study can represent more suitably the two different groups of sections, i.e. the sections suitable for columns(HC) which have the similar of width and depth dimensions, and those for beams(HB) which have larger values of major axis moments of inertia than the columns of similar cross sectional areas. Typical relationships between variables used in this study are given in Table 1. The coefficients of determination in Table 1 which indicate the fitness of obtained relationships, are very close to 1.0(mostly 0.99) and can be accepted as a very good fitting. Once any one of the variables is known, e.g. the required moment of inertia, then the sequential number(N) of sections in the section table can be immediately obtained by Eqs. (7.c) or (8.c) and other variables such as sectional areas(A), section modulus(Z), radius of gyration(R), and etc, can be immediately retrieved from the table as needed.

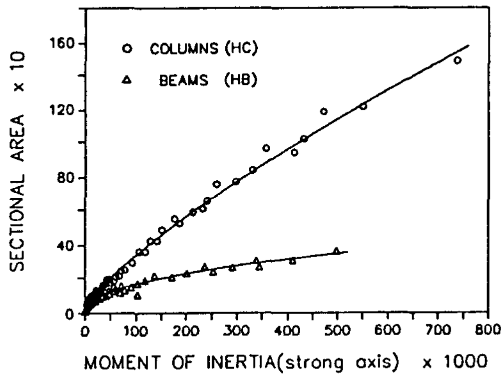


Fig. 1. Relation between cross sectional area and moment of inertia

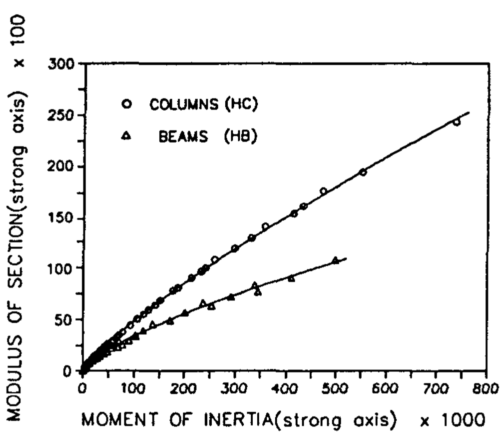


Fig. 2. Relation between modulus of section and moment of inertia

Table 1. Relationships between variables

| GROUP | RELATIONSHIPS | COEFFICIENT OF DETERMINATION | |
|---|---|---|---|
| HC | $A = 0.035 I^{.79} + 27.7$ | 0.99 | (7.a) |
| | $Z = 0.500 I^{.80} - 76.5$ | 0.99 | (7.b) |
| | $N = 20.212 I^{.12} - 54.4$ | 0.99 | (7.c) |
| | $= -238.464 A^{-.11} + 152.5$ | 0.99 | |
| HB | $A = 0.945 I^{.45} + 1.3$ | 0.97 | (8.a) |
| | $Z = 1.099 I^{.70} - 28.8$ | 0.99 | (8.b) |
| | $N = 7.869 I^{.17} + 29.0$ | 0.99 | (8.c) |
| | $= 7.796 Z^{.24} + 29.8$ | 0.99 | |
| RANGE OF DESIGN VARIABLE ($cm^4$) | | | |
| HC GROUP | $2880 \leq I \leq 737000$ | | |
| HB GROUP | $187 \leq I \leq 498000$ | | |

Table 2. Available sections

| HC Group(columns) | | | | HB Group(beams) | | | |
|---|---|---|---|---|---|---|---|
| No | I($cm^4$) | A($cm^2$) | Z($cm^3$) | No | I($cm^4$) | A($cm^2$) | Z($cm^3$) |
| 1 | 2880.0 | 51.21 | 330.0 | 51 | 187.0 | 11.85 | 37.5 |
| 2 | 4720.0 | 63.53 | 472.0 | 52 | 413.0 | 16.84 | 66.1 |
| 3 | 4980.0 | 71.53 | 498.0 | 53 | 666.0 | 17.85 | 88.8 |
| 4 | 6530.0 | 83.69 | 628.0 | 54 | 1020.0 | 26.84 | 138.0 |
| 5 | 8790.0 | 82.06 | 720.0 | 55 | 1210.0 | 23.04 | 139.0 |
| 6 | 9930.0 | 84.70 | 801.0 | 56 | 1530.0 | 29.65 | 181.0 |
| 7 | 10800.0 | 92.18 | 867.0 | 57 | 1580.0 | 23.18 | 160.0 |
| 8 | 11500.0 | 104.70 | 919.0 | 58 | 1840.0 | 27.16 | 184.0 |
| 9 | 16900.0 | 107.70 | 1150.0 | 59 | 2690.0 | 39.01 | 277.0 |
| 10 | 18800.0 | 110.80 | 1270.0 | 60 | 3540.0 | 32.68 | 285.0 |
| 11 | 20400.0 | 119.80 | 1360.0 | 61 | 4050.0 | 37.66 | 324.0 |
| 12 | 21500.0 | 134.80 | 1440.0 | 62 | 6120.0 | 56.24 | 502.0 |
| 13 | 23400.0 | 134.80 | 1540.0 | 63 | 6320.0 | 40.83 | 424.0 |
| 14 | 28200.0 | 135.30 | 1670.0 | 64 | 7210.0 | 46.78 | 481.0 |
| 15 | 33300.0 | 146.00 | 1940.0 | 65 | 11100.0 | 52.68 | 641.0 |
| 16 | 35300.0 | 166.60 | 2050.0 | 66 | 11300.0 | 72.38 | 771.0 |
| 17 | 40300.0 | 173.90 | 2300.0 | 67 | 13300.0 | 83.36 | 893.0 |
| 18 | 42800.0 | 198.40 | 2450.0 | 68 | 13600.0 | 63.14 | 775.0 |
| 19 | 47600.0 | 202.00 | 2670.0 | 69 | 18500.0 | 88.15 | 1100.0 |
| 20 | 49000.0 | 178.50 | 2520.0 | 70 | 20000.0 | 72.16 | 1010.0 |
| 21 | 56100.0 | 186.80 | 2850.0 | 71 | 21700.0 | 101.50 | 1280.0 |
| 22 | 59700.0 | 241.40 | 3030.0 | 72 | 23700.0 | 84.12 | 1190.0 |
| 23 | 66600.0 | 218.70 | 3330.0 | 73 | 28700.0 | 84.30 | 1290.0 |
| 24 | 70900.0 | 250.70 | 3540.0 | 74 | 33500.0 | 96.76 | 1490.0 |
| 25 | 78000.0 | 254.90 | 3840.0 | 75 | 33700.0 | 120.10 | 1740.0 |
| 26 | 92800.0 | 295.40 | 4480.0 | 76 | 38700.0 | 136.00 | 1980.0 |
| 27 | 107000.0 | 361.80 | 5120.0 | 77 | 41900.0 | 101.30 | 1690.0 |
| 28 | 119000.0 | 360.70 | 5570.0 | 78 | 46700.0 | 135.00 | 2160.0 |
| 29 | 129000.0 | 424.90 | 6030.0 | 79 | 47800.0 | 114.20 | 1910.0 |
| 30 | 142000.0 | 423.30 | 6470.0 | 80 | 56100.0 | 157.40 | 2550.0 |
| 31 | 152000.0 | 489.00 | 6950.0 | 81 | 56500.0 | 131.30 | 2230.0 |
| 32 | 177000.0 | 554.10 | 7900.0 | 82 | 60400.0 | 145.50 | 2500.0 |
| 33 | 187000.0 | 528.60 | 8170.0 | 83 | 68700.0 | 120.50 | 2310.0 |
| 34 | 214000.0 | 593.70 | 9130.0 | 84 | 71000.0 | 163.50 | 2910.0 |
| 35 | 233000.0 | 612.00 | 9740.0 | 85 | 77600.0 | 134.40 | 2590.0 |
| 36 | 242000.0 | 659.80 | 10100.0 | 86 | 90400.0 | 152.50 | 2980.0 |
| 37 | 260000.0 | 755.40 | 10900.0 | 87 | 102999.0 | 107.70 | 3380.0 |
| 38 | 298000.0 | 770.10 | 12000.0 | 88 | 103000.0 | 174.50 | 3530.0 |
| 39 | 331000.0 | 838.70 | 13000.0 | 89 | 118000.0 | 192.50 | 4020.0 |
| 40 | 358000.0 | 965.70 | 14100.0 | 90 | 137000.0 | 222.40 | 4620.0 |
| 41 | 414000.0 | 942.90 | 15400.0 | 91 | 172000.0 | 211.50 | 4980.0 |
| 42 | 433000.0 | 1024.00 | 16100.0 | 92 | 201000.0 | 235.50 | 5760.0 |
| 43 | 472000.0 | 1185.00 | 17600.0 | 93 | 237000.0 | 273.60 | 6700.0 |
| 44 | 551000.0 | 1214.00 | 19400.0 | 94 | 254000.0 | 243.40 | 6410.0 |
| 45 | 737000.0 | 1488.00 | 24300.0 | 95 | 292000.0 | 267.40 | 7290.0 |
| 46 | - | - | - | 96 | 339000.0 | 307.60 | 8400.0 |
| 47 | - | - | - | 97 | 345000.0 | 270.90 | 7760.0 |
| 48 | - | - | - | 98 | 411000.0 | 309.80 | 9140.0 |
| 49 | - | - | - | 99 | 498000.0 | 364.00 | 10900.0 |
| 50 | - | - | - | 100 | - | - | - |

For an effective search for the discrete solution, the tabulated sections are arranged in the ascending order of moment of inertia and stored in the data-base of the program. A part of the data-base is given in Table 2 to show its contents.

## 4.0 DECOMPOSITION METHOD

### 4.1 Structural Optimization through Element Optimization

The following stress ratio recursion formula, which is frequently used for stress constrained problems is used in this study.

$$b_i^{v+1} = \alpha \left[ \frac{\max(\sigma_{ij})}{\bar{\sigma}_i} b_i^v \right] \quad (9)$$

where $v$ = number of iterations, $\sigma_{ij}$ = calculated stress, $\bar{\sigma}_i$ = allowable stress in i-th member. $\alpha$ = relaxation parameter(1.0-1.01) used to speed up the convergence under the stress constraints only and j = load case. This formula is known to give stable solutions[8].

The basic procedures of optimization in this study are as following ; The variables constrained, such as cross sectional areas(A), section modulus(Z) and major axis moments of inertia(I) are transformed into section identification numbers(N) as needed by the appropriate relationships in Table 1. With the given loads, the continuous optimum solution is basically obtained at first in terms of section modulus for beam or cross sectional areas for columns in accordance with Eq. (9) and then, transformed into the section identification numbers by Eqs. (7.c) or (8.c). However, while the combined stresses are mainly induced, the basic continuous solution is obtained as the form of moment of inertia and then, it is transformed into the section identification numbers. Using the continuous solution obtained as a starting point, a search technique is used to find the discrete optimum solution near the continuous solution.

Fig. 3 shows the path for searching discrete solution in the tabulated sections. The point A on the curve which is the continuous solution obtained by Eqs. (7.c) or (8.c) in Table 1, is generally not a tabulated section number. Therefore, the initial section member in the table should be determined by the nearest tabulated section(point B in Fig. 3) to point A by truncation or rounding off. Starting from point B, the search continues through the path B → C → D → E → F → E until the point E is finally obtained as the discrete optimum solution. In case column members are subjected to the considerable bending moments in addition to axial forces, after obtaining optimum section from HC, it is desirable to check if there is more economical section in HB group for given design conditions. After the first solutions for entire members are obtained, the reanalysis of structure and the redesign of members are repeated until the prescribed stop criteria is satisfied.

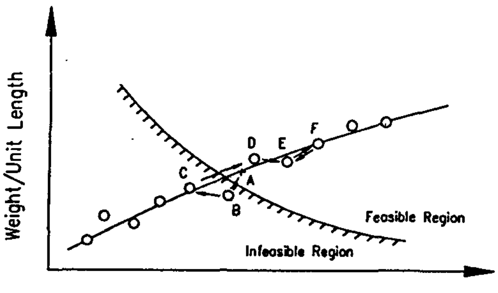


Fig. 3. Path of searching discrete optimum size in the section table

## 4.2 Oscillation Problems in Element Optimization

In discrete optimization, designers are cautioned to pay special attention to the problem of oscillation. Some members in a structure are alternately designed with feasible and infeasible sections as the solutions are nearing to converged sections. Thus the solutions continually oscillate between the feasible and the infeasible regions and never reach optimum values. This phenomenon is defined as the local oscillation and these oscillating members are called rascal members in this study. On the other hand, when the objective function is nearing a converged optimum, there may be another type of oscillation. The objective function can oscillate without rascal members during iterations. This is defined as the global oscillation phenomenon in this paper. More detailed discussions on the oscillation and its control are found in Ref. [18].

## 4.3 Structural Level Constraints

In this paper, the optimization of entire structure under the structural displacement constraints is considered as a problem of correcting violated displacement constraints effectively after the optimization under the stress constraints is obtained. If there are any components of the displacements that exceed the prescribed limits, they can be corrected by the error correction vector obtained by both the sensitivity analysis of the gradient projection method and Kuhn-Tucker necessary condition for the optimum design. The correction vector is given by the following equation[3].

$$\Delta \mathbf{b} = \mathbf{l}\,\frac{\Delta\Psi}{\mathbf{l}^T\mathbf{l}} \tag{10}$$

where $\Delta\Psi$ = amount of violated displacement constraints and $\mathbf{l}$ = sensitivity vector indicates the effect that a certain design change will have on the constraint functions. The vector is obtained by the following equation.

$$\mathbf{l} = \frac{\partial\Psi}{\partial b} - \frac{\partial}{\partial b}[\mathbf{K}(b)\mathbf{U} - \mathbf{P}(b)]^T \boldsymbol{\lambda} \tag{11}$$

where $\boldsymbol{\lambda}$ is defined as adjoint variable and obtained by the following equation.

$$\mathbf{K}(b)\boldsymbol{\lambda} = \frac{\partial\Psi}{\partial u} \tag{12}$$

where $\mathbf{K}$ = total stiffness matrix of the entire structure and $\frac{\partial\Psi}{\partial u}$ = derivative of the violated displacement constraints with respect to each displacement component.

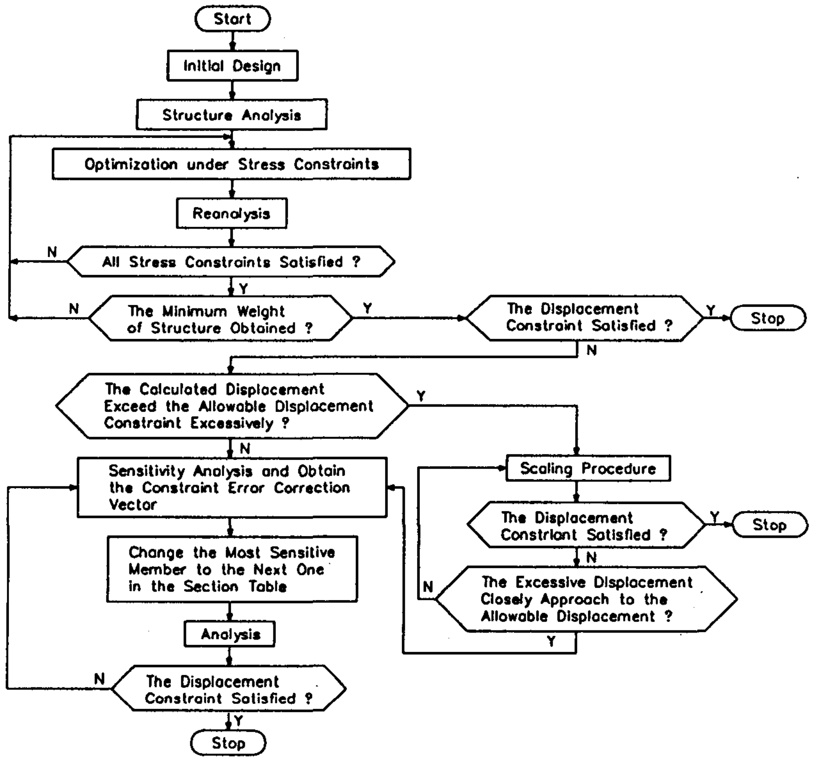


Fig. 4. Flow chart of the algorithm in this study

The constraint error correction vector obtained by above equations is the magnitude of design change needed to compensate the violated displacement constraint. The magnitudes of elements in this vector arranged in the descending order tells which design variables(or members) have more significant effects in correcting the violated constraints and which ones have less. This characteristics can be used in searching the most effective member in the entire structure to correct the current violated displacement. The member(or a group of members) identified to be most sensitive is replaced by the member of the next or a few step larger variable in the tabulated sections to obtain the maximum correction in violated constraints with minimum change(increase) of the section properties. It may be more effective to replace a certain number of members at the same time instead of successively replacing the most sensitive member only. This process is repeated until the displacement constraints are fully satisfied and the final optimum design is determined. The efficiency of this correction technique in the discrete variables has been shown in the test examples in the next section of this paper.

4.4 Scaling Procedure

If the magnitude of violated displacement is very large as compared with the prescribed displacement limit, the correction by the above procedures, i.e. replacing the most sensitive member by the next section in the section table, may result in a very slow convergence and the optimal distribution of stiffness in the structure may be disturbed. Even the violation of stress constraints as a result of the stiffness disturbance may occur. Therefore, the scaling procedure is introduced to expedite the convergence while retaining the optimal stiffness distribution in this study. The detailed explanations are found in Ref. [19].

On the other hand, if the lateral response of a building is too large to be controlled in ordinary ways, the following ways can be considered : 1) the overall increase of lateral displacement resistance capacity of the structure by the use of more efficient structural systems for a given building shape, and 2) the use of more efficient building shapes. Fig. 5 shows the flow chart for the algorithm used in this paper under the assumption that the structural system and building shape are already selected.

## 5.0 CONCLUSIONS

In this paper, a new algorithm that is practically applicable to the optimum design problems of large steel building structures has been developed. The numerical examples presented in this paper show that the structures under the various constraints such as the stress, design variable and displacement constraints, can be effectively optimized by using the search algorithm. Decomposing the imposed constraints into two levels(categories), i.e. the element and structural level constraints, and applying them at the different stages of optimization made the optimization algorithm in this study very effective.

The identification of the most critical member in the structure by a sensitivity analysis and replacing it by the next large section in the section table is an effective and practical way of optimization under the structural level constraints. If the magnitudes of the violated constraints are too large to be corrected by aforementioned manner, the use of scaling techniques prior to the sensitivity analysis is necessary.

The oscillation of solutions between feasible and infeasible regions near the converged value can be controlled either by terminating the iteration when the minimum objective function is repeatedly met in global oscillation problems or by imposing lower limit which some selected members(rascal members) should be designed with. Dividing the commercially available H–sections into two groups(namely, beams(HB) and columns(HC)) based on their section properties, the relationships between the section properties could be better expressed than a single curve. Any further study in connection with this work should consider the total optimization approach in which the configuration(or shape) optimization is also included in addition to the structural member optimization.

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