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개별 탭 조절 등화기의 Wiener solution

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The Wiener Solution of the Individual Tap Controlled Equalizer

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ABSTRACT

The Wiener solution of the individual tap controlled TDL equalizer is derived and it is shown that the MSE can be written as a function of each single coefficient, holding all other coefficients constant.

I. Introduction

A new iterative procedure for adjusting the tap coefficients of an adaptive TDL equalizer was introduced[1]. Unlike the conventional LMS-TDL equalizer, the equalization process is obtained by applying the LMS algorithm to the individual tap coefficients.

Many applications using the proposed algorithm have been researched in equalization and DS jamming rejection[2][3][4][5].

In this paper, it is analyzed that the MSE of TDL equalizer is a parabolic curve of each single coefficient, holding all other coefficients constant and the individual optimum tap coefficients can be expressed by Wiener equation.

II. Problem statement

Consider the baseband PAM system depicted in Figure 1. consisting of a transmitter, a transmission path and a receiver (with a combined channel impulse response of $f(t)$) connected in cascade with a TDL equalizer.

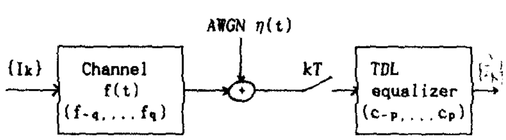

Fig. 1. Baseband PAM system with an adaptive equalizer

In general, the sampled impulse response $f(t)$ is given by a $(2q+1)$ component column vector

$$F^T = [f_{-q}, \dots, f_0, \dots, f_q]$$

where T denotes transpose. And the coefficients vector of the TDL equalizer is given by a $(2p+1)$ component column vector

$$C^T = [c_{-p}, \dots, c_0, \dots, c_p]$$

Considering Fig. 1, the output of the equalizer at time kT is given by

$$\hat{I}_k = \sum_{j=-(p+q)}^{p+q} \left(\sum_{i=-p}^{p} c_i f_{j-i}\right) I_{k-j} + \tilde{\eta}_k \qquad (1)$$

where I_k is the transmitted symbol and $\tilde{\eta}_k$ is the sample of additive noise at the equalizer output.

The error related this symbol becomes

$$e_k = \hat{I}_k - I_k \qquad (2)$$

and the MSE is

$$MSE = E\{ e_k^2 \} \qquad (3)$$

where $E\{.\}$ denotes the ensemble averaging operation.

III. Mean Square Error

From equation(3),

$$MSE = E\{(\sum_j \sum_i I_{k-j}\, c_i f_{j-i} + \tilde{\eta}_k - I_k)^2\} \qquad (4)$$

Taking into account that; (a) the transmitted symbols are zero mean and i.i.d., (b) I_k and $\tilde{\eta}_k$ are uncorrelated and $\tilde{\eta}_k$ is zero mean. Equation (5) can be written as

$$MSE = E\{(\sum_j \sum_i I_{k-j}\, c_i f_{j-i})^2\} + E\{\tilde{\eta}_k^2\} + E\{I_k^2\} - 2E\{I_k(\sum_j \sum_i I_{k-j}\, c_i f_{j-i})\} \qquad (5)$$

After representating MSE by MSE = MSE1+MSE2+MSE3+MSE4 where

$$MSE1 = E\{(\sum_j \sum_i I_{k-j}\, c_i f_{j-i})^2\} = \sum_m \sum_n \sum_j \sum_i E(I_{k-j} I_{k-n}) c_i c_m f_{j-i} f_{n-m} = \sigma_I^2 \sum_m c_m \sum_i c_i (\sum_j f_{j-i} f_{j-m}) \qquad (6)$$

$$\begin{aligned} MSE2 &= E\{\tilde{\eta}_k^2\} \\ &= \sum_i \sum_n E(\eta_k \eta_k) c_i c_n \\ &= \sigma_n^2 \sum c_i^2 \end{aligned} \quad (7)$$

$$MSE3 = E\{I_k^2\} = \sigma_I^2 \quad (8)$$

$$\begin{aligned} MSE4 &= -2E\{I_k(\sum_j \sum_i I_{k-j}\, c_i f_{j-i})\} \\ &= -2(\sum_j \sum_i E\{I_k I_{k-j}\}\, c_i f_{j-i}) \\ &= -2\, \sigma_I^2 \sum_i c_i f_{-i} \end{aligned} \quad (9)$$

where $\sigma_I^2 = E\{I_k^2\}$, $\sigma_n^2 = E(\eta_k \eta_k)$.
MSE normalized with σ_I^2 is denoted as ξ_n, then

$$\xi_n = \sum_m \sum_k c_k c_m (\sum_j f_{j-k} f_{j-m}) + \sigma_n^2/\sigma_s^2 \sum_k c_k^2 + 1 - 2 \sum_k c_k f_{-k} \quad (10)$$

IV. Parabolic function of individual tap coefficient

Equation (10) can be manapulated as

$$\left[\begin{array}{l} \sum_m \sum_{k \neq i} c_k c_m (\sum_j f_{j-k} f_{j-m}) \\ + c_i \sum_m c_m (\sum_j f_{j-k} f_{j-m}) \end{array} \right] + \left[\begin{array}{l} \sigma_n^2/\sigma_s^2 \sum_{k \neq i} c_k^2 \\ + \sigma_n^2/\sigma_s^2\, c_i^2 \end{array} \right] + 1 - \left[\begin{array}{l} 2 \sum_{k \neq i} c_k f_{-k} \\ + 2 c_i f_{-i} \end{array} \right] \quad (11)$$

$$= \left[\begin{array}{l} \sum_{m \neq i} \sum_{k \neq i} c_k c_m (\sum_j f_{j-k} f_{j-m}) \\ + c_i \sum_{k \neq i} c_k (\sum_j f_{j-k} f_{j-i}) \\ + c_i \sum_{m \neq i} c_m (\sum_j f_{j-i} f_{j-m}) \\ + c_i^2 \sum_j f_{j-i}^2 \end{array} \right] + \sigma_n^2/\sigma_s^2\, c_i^2 - 2 c_i f_{-i} + 1 + \sigma_n^2/\sigma_s^2 \sum_{k \neq i} c_k^2 - 2 \sum_{k \neq i} c_k f_{-k} \quad (12)$$

Arrangement of this equation in terms of c_i gives the following equation

$$\begin{aligned} \xi_n = {} & (\sum_j f_{j-i}^2 + \sigma_n^2/\sigma_s^2) c_i^2 \\ & + 2[(\sum_{k \neq i} c_k \sum_j f_{j-k} f_{j-i}) - f_{-i}] c_i \\ & + (\sum_{m \neq i} \sum_{k \neq i} c_k c_m (\sum_j f_{j-k} f_{j-m}) + \sigma_n^2/\sigma_s^2 \sum_{k \neq i} c_k^2 \\ & + 1 - 2 \sum_{k \neq i} c_k f_{-k} \end{aligned} \quad (13)$$

$$\xi_n(c_i) = A\, c_i^2 + 2B\, c_i + D \quad (14)$$

where

$$A = (\sum_j f_{j-i}^2 + \sigma_n^2/\sigma_s^2)$$

$$B = (\sum_{k \neq i} c_k \sum_j f_{j-k} f_{j-i}) - f_{-i}$$

$$D = \sum_{m \neq i} \sum_{k \neq i} c_k c_m (\sum_j f_{j-k} f_{j-m}) + \sigma_n^2/\sigma_s^2 \sum_{k \neq i} c_k^2 + 1 - 2 \sum_{k \neq i} c_k f_{-k}$$

Equation (14) shows that $\xi_n(c_i)$ is a parabolic function in terms of c_i.
That is, when tap coefficient c_i is changed while others are kept constant, the locus of $\xi_n(c_i)$ becomes a parabolic function in terms of c_i.

V. The Wiener solution of optimum tap coefficients

In this section, the tap coefficients that minimize ξ_n are obtained, and that the optimum coefficients have the Wiener solution is shown.
The set of (2p+1) simultaneous linear equations for the tap coefficients that minimizes MSE can be obtained from

$$\frac{\delta \xi_n}{\delta c_i} = 0 \quad (15)$$

From equation(13), equation(15) becomes

$$\begin{aligned} \frac{\delta \xi_n}{\delta c_i} &= 2 \sum_{k \neq i} c_k (\sum_j f_{j-k} f_{j-i}) + 2 c_i \sum_j f_{j-i}^2 + 2 c_i\, \sigma_n^2/\sigma_s^2 - 2 f_{-i} \\ &= 2 \sum_k c_k \sum_j f_{j-k} f_{j-i} + 2 c_i\, \sigma_n^2/\sigma_s^2 - 2 f_{-i} \\ &= 2 \sum_k c_k [\sum_j f_{j-k} f_{j-i} + \sigma_n^2/\sigma_s^2\, \delta(k-i)] - 2 f_{-i} \\ &= 0 \end{aligned}$$

The set of equation for solving the optimum tap coefficients is found as

$$\sum_k \hat{c}_k [\sum_j f_{j-k} f_{j-i} + \sigma_n^2/\sigma_s^2\, \delta(k-i)] = 2 f_{-i}, \quad i = (-p, \ldots, p) \quad (16)$$

Equation (16) can be expressed as a matrix form.

$$R_f \triangleq \begin{bmatrix} \sum_j f_{j+p}^2 + \sigma_n^2/\sigma_s^2 & \sum_j f_{j+p-1} f_{j+p} & \cdots & \sum_j f_{j-p} f_{j+p} \\ \sum_j f_{j-p} f_{j+p-1} & \sum_j f_{j+p-1}^2 + \sigma_n^2/\sigma_s^2 & & \vdots \\ \vdots & \ddots \quad \sum f_j^2 + \sigma_n^2/\sigma_s^2 & \ddots & \\ \sum_j f_{j-p} f_{j+p} & \cdots & & \sum_j f_{j-p}^2 + \sigma_n^2/\sigma_s^2 \end{bmatrix} \quad (17)$$

$$\overset{*}{C} = \begin{bmatrix} c_{-p} \\ c_{-p+1} \\ \vdots \\ c_0 \\ \vdots \\ c_p \end{bmatrix}^*, \qquad P_f = \begin{bmatrix} f_p \\ f_{p-1} \\ \vdots \\ f_0 \\ \vdots \\ f_{-p} \end{bmatrix} \tag{18}$$

$$R_f \cdot \overset{*}{C} = P_f \tag{19}$$

In order to prove that equation(19) satisfies the Wiener solution, the input vector of the equalizer at time kT is defined as

$$X^T(k) = [x_{k+p}, \cdots, X_k, \cdots, X_{k-p}]$$

$$\text{where} \quad x_{k+p} = \sum_j f_j I_{k+p-j}$$

The auto-correlation matrix of the input vector X(k) is defined as Rx and the cross-correlation vector between the I_k and the input vector X(k) is defined as Px. Then the Wiener solution of the optimum coefficients becomes the equation(20)[6].

$$\overset{*}{C} = Rx^{-1} \cdot Px \tag{20}$$

In the matrix **Rx, the** (1,1) element is

$$E[x_{k+p}x_{k+p}] = E[(\sum_j f_j I_{k+p-j})(\sum_j f_j I_{k+p-j})] + E[\eta_{k+p}^2]$$

$$(\text{Let } p-j \triangleq -i, \text{ then } \sum_j f_j I_{k+p-j} \equiv \sum_i f_{p+i} I_{k-i})$$

$$= E[(\sum_j f_{j+p} I_{k-j})(\sum_i f_{i+p} I_{k-i})] + E[\eta_{k+p}^2]$$

$$= \sum_{j}\sum_{i} f_{j+p} f_{i+p} E[I_{k-j} I_{k-i}] + \sigma_n^2$$

$$= \sigma_I^2 \left(\sum_j f_{j+p}^2 + \sigma_n^2/\sigma_I^2 \right) \tag{21}$$

And the first element of vector Px is

$$E[I_k x_{k+p}] = E[(\sum_j I_k f_j I_{k+p-j})]$$

$$= E[(\sum_j I_k f_{j+p} I_{k-j})]$$

$$= \sum_j f_{j+p} \sigma_I^2 \delta(j)$$

$$= \sigma_I^2 f_p \tag{22}$$

From the equations (17) and (21),

$$R_f = Rx \tag{23}$$

From the equations (18) and (22),

$$P_f = Px \tag{24}$$

As a result, Equation (19) satisfies the Wiener solution

$$\overset{*}{C} = Rx^{-1}\, Px \tag{25}$$

VI. Conclusion

In this paper, the Wiener solution of the individual tap controlled TDL equalizer is derived and it is shown that the MSE can be written as a function of each single coefficient, holding all other coefficients constant.

Measuring the slope of the MSE curve at the initial value of each single coefficient, a new value of coefficient can be chosen. Its optimal coefficient can be reached by repeating this procedure. All other optimum coefficients can be also acquired by this method. Therefore, the tap coefficients of TDL equalizer can be individualy adjusted from this analysis.

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