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DFB 공진기의 광전소자에 응용

Application of DFB Resonators for Photonic Devices

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Abstract

The method of characteristic matrix used in the calculation of multi-layered optical coating is applied to the analysis of DFB waveguide with arbitrary structure of grating by piecewise approximation where the DFB structure is devided into many sections in which the gain and period, amplitude and phase of DFB structure are assumed to be constant in each section. The DFB structure with two section is analyzed using this method of C-matrix. The possible applications for photonic devices using DFB waveguides are proposed.

1. Introduction

Recently, there has been a great deal of applications of DFB(distributed feedback) or DBR(distributed Bragg reflector) waveguide resonators to laser diodes and waveguide filters. The structure of DFB resonator is similar in principle to the structure of multi-layer in optical coating devices. This similarity stems from the analogy between the variations of the effective index of refraction and period in the DFB waveguide resonator and the variation of the refractive index and the thickness of layers in the multi-layered optical coating. The main difference of the structure of DFB compared to that of optical coating is that the DFB structure has a small variation and a great number of corrugations of refractive index with accurate long-range ordering and that there are limitations in the control of DFB structure, especially in the fabrication of non-uniform structure of DFB where the period, phase, and amplitude change. These limitations are due to the holographic method of DFB fabrication[1] which is common method of fabrication. Currently various fabricating method is being developed including the electron beam writing,[2] direct mask method,[2] double exposure,[3] and spatial filtering of hologram,[4] and these limitations gradually disappear. These technical improvement will bring the versitility in the application of DFB structure and needs the theoretical treatment for the DFB with arbitrary structure .

There are three methods for the treatment of DFB structure. The first is a method using the characteristic matrix used in the design of optical coating[5] and not suitable for the analysis of DFB due to the huge number of corrugations in the DFB. The second is a method using differential equation[6] or the numerical repetition by integral forms of contradirectional coupled mode equations in its integral form[7] and not approroate for the complicated structure of DFB. These two methods require considerable computing and difficult to get the idea about the design for more complicated structure. The third method is C-matrix method developed recently which combines the first and the second method. In this method, an approximate solution of the differential equation is derived by deviding the waveguide into short segments in which the grating is assumed to be constant. The solution for an entire DFB structure can be found by the multiplying the C-matrix in its order corresponding to each segment of DFB.

Method of C-Matrix for the analysis of DFB was developed by Yamada[8] and independently by us.[9] C-matrix by Yamada uses coordinates for each segments of DFB which is sometimes confusing and inconvenient to match the phase relation between each segments of grating. We used the coordinate common to all segments of DFB which is more versatile and convenient. We also genalized the parameters of DFB structure including the gain or the absorption grating. DFB laser with gain grating[10] and DFB laser with absorption grating[11] was realized recently and our method of analysis for the DFB with gain or absorption grating can be applied to more complex structure of DFB which contains gain or absorption grating.

In this paper we developed the C-matrix using a coordinate common to each sections of grating . We apply this method to the design of waveguide Bragg filter and laser with two section. This approach can be a useful tool for the analysis of multi-electrode DFB laser which is studied extensively for the development of tunable or narrow linewidth laser. The possible applications for photonic devices using DFB waveguides are also proposed.

2. Derivation of characteristic matrix

The general wave equation for a waveguide with two dimensional guiding is

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}+\frac{\omega^2 N^2(x,y,z)}{c^2}\right]\Psi(x,y,z) = 0 \quad (1)$$

where z is the propagation direction, N(x,y,z) is the index of refraction. Assuming periodic variation along z in the real and imaginary part of the refractive index as shown in figure 1, N(x,y,z) can be expressed as

$$N(x,y,z) = n(x,y) + i n_i(x,y) + \Delta n_r(x,y)\cos(2\beta_{Br}z+\varphi_r) + i\Delta n_i(x,y)\cos(2\beta_{Bi}z+\varphi_i) \quad (2)$$

where $n(x,y)$ and $n_i(x,y)$ are the real and imaginary part of the refractive index constant along z, $\Delta n_r(x,y)$ and $\Delta n_i(x,y)$ are the amplitude of variation along z in the real and imaginary part of N(x,y), β_{Br} and

β_{Bi} are the real and imaginary parts of Bragg propagation constants defined as π/Λ_r and π/Λ_i where Λ_r and Λ_i are the period of variation along z in the real and imaginary part of N(x,y), and φ_r and φ_i are the phases of the periodic variations in the real and imaginary part of N(x,y). Assuming

$$n \gg n_i,\ \Delta n_r,\ \Delta n_i. \tag{3}$$

(1) can be written as

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}+\frac{\omega^2 n^2(x,y)}{c^2}+\frac{\omega^2\Delta n^2(x,y,z)}{c^2}\right]\Psi(x,y,z)=0 \tag{4}$$

where

$$\Delta n^2(x,y,z) = 2in_i(x,y)\,n(x,y) + 2\Delta n_r(x,y)\,n(x,y)\cos(2\beta_{Br}z+\varphi_r) + 2i\Delta n_i(x,y)\,n(x,y)\cos(2\beta_{Bi}z+\varphi_i). \tag{5}$$

We can expand the field $\Psi(x,y,z)$ in terms of eigenstates of the waveguide without the perturbation in the refractive index:

$$\Psi(x,y,z) = \sum_n A_n(z)\,\phi_n(x,y)\exp(i\beta_n z) \tag{6}$$

where ϕ_n satisfies

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\omega^2 n^2(x,y)}{c^2}-\beta_n^2\right]\phi_n(x,y) = 0. \tag{7}$$

Substituting of (6) and (7) into (4) converts (4) into a set of linear differential equations for the mode amplitudes $A_n(z)$.

$$\frac{dA_i(z)}{dz} = \frac{i}{2\beta_i}\sum_j K_{ij}A_j(z) \tag{8}$$

where the coupling coefficient K_{ij} is

$$K_{ij} = \frac{\omega^2}{c^2}\int \phi_i^*(x,y)\,\Delta n^2(x,y,z)\,\phi_j(x,y)\exp\{i(\beta_j-\beta_i)z\}\,dxdy. \tag{9}$$

In a single mode waveguide where only

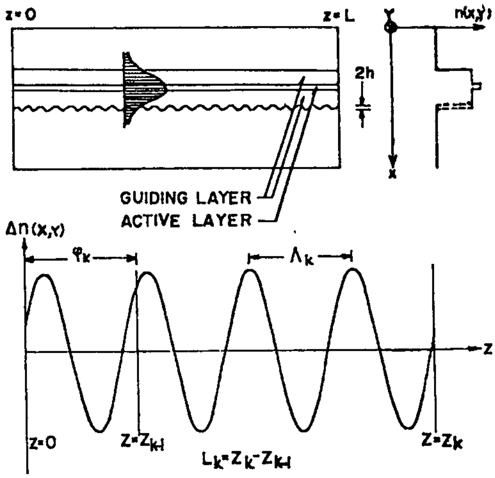


Fig.1 Structure of DFB waveguide resonator ; coordinate of k'th section of DFB structure.

two modes propagating forward and backward exist, (6) becomes

$$\Psi(x,y,z) = A(z)\,\phi(x,y)\exp(i\beta z) + B(z)\,\phi(x,y)\exp(-i\beta z) \tag{10}$$

where A(z) is the amplitude of the forward propagating wave and B(z) is the amplitude of the backward propagating wave.

Using (5),(9) and (10), (8) becomes

$$\frac{dA(z)}{dz} = \frac{g}{2}A(z) + i\kappa_r\exp(i\varphi_r)\exp(-2i\Delta\beta_r z)B(z) - \kappa_i\exp(i\varphi_i)\exp(-2i\Delta\beta_i z)B(z)$$

$$\frac{dB(z)}{dz} = -\frac{g}{2}B(z) - i\kappa_r\exp(-i\varphi_r)\exp(2i\Delta\beta_r z)A(z) + \kappa_i\exp(-i\varphi_i)\exp(2i\Delta\beta_i z)A(z) \tag{11}$$

where

$$\Delta\beta_r = \beta - \beta_{Br},\quad \Delta\beta_i = \beta - \beta_{Bi} \tag{12}$$

and

$$\frac{g}{2} = \frac{i}{2\beta}\frac{\omega^2}{c^2}\int\phi^*(x,y)\phi(x,y)2in_i(x,y)n(x,y)dxdy$$

$$\kappa_r = \frac{1}{2\beta}\frac{\omega^2}{c^2}\int\phi^*(x,y)\phi(x,y)\Delta n_r(x,y)n(x,y)dxdy$$

$$\kappa_i = \frac{1}{2\beta}\frac{\omega^2}{c^2}\int\phi^*(x,y)\phi(x,y)\Delta n_i(x,y)n(x,y)dxdy. \tag{13}$$

By the transformation of

$$A(z) = A(z)\exp(gz/2)$$
$$B(z) = B(z)\exp(-gz/2)$$
$$\beta' = \beta - ig/2 \qquad (14)$$

(10) and (11) can be rewritten as

$$\Psi(x,y,z) = A(z)\,\phi(x,y)\exp(i\beta' z) + B(z)\,\phi(x,y)\exp(-i\beta' z) \qquad (15)$$

$$\frac{dA(z)}{dz} = i\kappa_r\exp(i\varphi_r)\exp(-2i\Delta\beta_r' z)B(z) - \kappa_i\exp(i\varphi_i)\exp(-2i\Delta\beta_i' z)B(z)$$

$$\frac{dB(z)}{dz} = -i\kappa_r\exp(-i\varphi_r)\exp(2i\Delta\beta_r' z)A(z) + \kappa_i\exp(-i\varphi_i)\exp(2i\Delta\beta_i' z)A(z) \qquad (16)$$

where $\Delta\beta_r' = \beta' - \beta_{Br}$, and $\Delta\beta_i' = \beta' - \beta_{Bi}$.

In case of same period for the variation in the real and imaginary part of refractive index, the Bragg propagation constant $\beta_B = \beta_{Br} = \beta_{Bi}$ and (16) can be written as

$$\frac{dA(z)}{dz} = i\kappa_{AB}\exp(-2i\Delta\beta' z)\,B(z)$$

$$\frac{dB(z)}{dz} = -i\kappa_{BA}\exp(2i\Delta\beta' z)\,A(z) \qquad (17)$$

where

$$\Delta\beta' = \beta' - \beta_B$$
$$\kappa_{AB} = \kappa_r\exp(i\varphi_r) + i\kappa_i\exp(i\varphi_i)$$
$$\kappa_{BA} = \kappa_r\exp(-i\varphi_r) + i\kappa_i\exp(-i\varphi_i). \qquad (18)$$

The solution of (17) becomes

$$A(z) = C_1\exp(\Gamma_+ z) + C_2\exp(\Gamma_- z)$$
$$B(z) = \exp(2i\Delta\beta' z)/i\kappa_{AB}\,\{C_1\Gamma_+\exp(\Gamma_+ z) + C_2\Gamma_-\exp(\Gamma_- z)\} \qquad (19)$$

where

$$\Gamma_+ = -i\Delta\beta' + \sqrt{\kappa_{AB}\kappa_{BA} - \Delta\beta'^2}\,,$$

$$\Gamma_- = -i\Delta\beta' - \sqrt{\kappa_{AB}\kappa_{BA} - \Delta\beta'^2}. \qquad (20)$$

Using C_1 and C_2 determined from the boundary condition at $z=z_k$, $A(z)$ and $B(z)$ can be expressed as

$$\begin{bmatrix} A(z) \\ B(z) \end{bmatrix} = \begin{bmatrix} \exp(-i\Delta\beta' z) & 0 \\ 0 & \exp(i\Delta\beta' z) \end{bmatrix} \begin{bmatrix} F \end{bmatrix} \begin{bmatrix} \exp(i\Delta\beta' z_k) & 0 \\ 0 & \exp(-i\Delta\beta' z_k) \end{bmatrix} \begin{bmatrix} A(z_k) \\ B(z_k) \end{bmatrix} \qquad (21)$$

where the elements of matrix F are

$$F_{11} = \cosh\eta(z_k - z) - (i\Delta\beta'/\eta)\sinh\eta(z_k - z)$$
$$F_{12} = -(i\kappa_{AB}/\eta)\sinh\eta(z_k - z)$$
$$F_{21} = (i\kappa_{BA}/\eta)\sinh\eta(z_k - z)$$
$$F_{22} = \cosh\eta(z_k - z) + (i\Delta\beta'/\eta)\sinh\eta(z_k - z), \qquad (22)$$

and $\eta = \sqrt{\kappa_{AB}\kappa_{BA} - \Delta\beta'^2}$. The amplitudes of the fields, $A(z)$ and $B(z)$ become

$$\begin{bmatrix} A(z) \\ B(z) \end{bmatrix} = \begin{bmatrix} \exp(-i\Delta\beta z) & 0 \\ 0 & \exp(i\Delta\beta z) \end{bmatrix} \begin{bmatrix} F \end{bmatrix} \begin{bmatrix} \exp(i\Delta\beta z_k) & 0 \\ 0 & \exp(-i\Delta\beta z_k) \end{bmatrix} \begin{bmatrix} A(z_k) \\ B(z_k) \end{bmatrix} \qquad (23)$$

using (14) and (21), where $\Delta\beta = \beta - \beta_B$.

The forward propagating field $\Psi_A(x,y,z)$ and the backward propagating fields $\Psi_B(x,y,z)$ defined as (15) can be expressed as

$$\begin{bmatrix} \Psi_A(x,y,z) \\ \Psi_B(x,y,z) \end{bmatrix} = \phi(x,y)\begin{bmatrix} C \end{bmatrix} \begin{bmatrix} \Psi_A(x,y,z_k) \\ \Psi_B(x,y,z_k) \end{bmatrix} \qquad (24)$$

using (23). The matrix C will be called characteristic matrix in analogy with the terminology of optical thin film coating and related to the matrix F as

$$\begin{bmatrix} C \end{bmatrix} = \begin{bmatrix} P(z) \end{bmatrix} \begin{bmatrix} F \end{bmatrix} \begin{bmatrix} P^{-1}(z_k) \end{bmatrix} \qquad (25)$$

where the phase matrix P(z) is defined as

$$\begin{bmatrix} P(z) \end{bmatrix} = \begin{bmatrix} \exp(i\beta_B z) & 0 \\ 0 & \exp(-i\beta_B z) \end{bmatrix}. \qquad (26)$$

The C-matrix (characteristic matrix) is characterized by β_B, κ_r, κ_i, φ_r, φ_i, and g.

3. Application of C-matrix to the analysis of DFB waveguide with two segments

The method of characteristic matrix can be applied to the analysis of DFB waveguide with arbitrary structure by piecewise approximation. The DFB structure is devided into many sections in which the DFB parameters β_{Br}, β_{Bi}, κ_r, κ_i, φ_r, φ_i, and g are assumed to be constant as in figure 2. The field at $z=z_0$ can be calculated in terms of the field at $z=z_N$ and the C-matrix :

$$\begin{bmatrix}\Psi_A(x,y,z_0)\\ \Psi_B(x,y,z_0)\end{bmatrix} = \phi(x,y)\begin{bmatrix} C \end{bmatrix}\begin{bmatrix}\Psi_A(x,y,z_N)\\ \Psi_B(x,y,z_N)\end{bmatrix} \quad (27)$$

where

$$C = \prod_{k=1}^{N} C^k(\beta_B{}^k,\ \kappa_r{}^k,\ \kappa_i{}^k,\ \varphi_r{}^k,\ \varphi_i{}^k,\ g^k,\ L^k,\ z_k,\ z_{k-1}). \quad (28)$$

In the peacewise approximation of DFB, the derivation of C^k for each segment assumes L^k the length of the segment to be sufficiently longer than Λ^k the period of grating. This assumption is justified by considering the Fourier transformation of the DFB structure with the length of L^k. In our derivation of C-matrix, we considered only the first component of the Fourier transformation for the finite length of DFB. To apply this method to the case of rapidly varying structure of DFB, the structure of DFB needs to be devided in many short sections and the coupled mode equation with higher order terms in β_B must be considered. Here, only the DFB waveguide with slowly varying structure is considered.

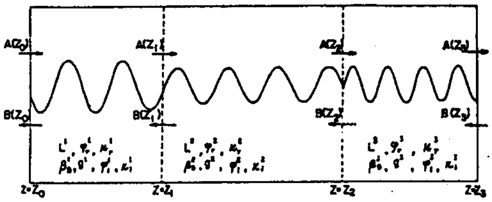


Fig.2 Fields in DFB structure with 3 section.

The reflection and transmission coefficients of the DFB with N section can be obtained by considering the boundary condition at the end of the DFB. Assuming the incident wave $A(z_0)$ only at $z=z_0$, the amplitude of the forward and backward wave at $z=z_0$ is $A(z_0)$ and $B(z_0)$, and the amplitude of the forward and backward wave at $z=z_N$ is $A(z_N)$ and 0. The reflection and transmission coefficient of the DFB structure with N section can be expressed as

$$R = r\exp(i\varphi_r) = B(z_0)/A(z_0)$$
$$T = t\exp(i\varphi_t) = A(z_N)/A(z_0) \quad (29)$$

where φ_r and φ_t are the phase change in the reflection and transmission, and r and t are the absolute value of the reflection and transmission coefficient. If we normalize the field at $z=z_N$, $A(z_N)$ is 1 and $B(z_N)$ is 0. Using this normalized fields, the field at $z=z_0$ can be determined from the C-matrix and the field at $z=z_N$.

The reflection and transmission coefficient R and T of a DFB structure with only one section can be expressed as

$$R = \frac{F_{21}}{F_{11}} = \frac{(i\kappa_{BA}/\eta)\sinh \eta L}{\cosh \eta L - (i\Delta\beta'/\eta)\sinh \eta L}$$
$$T = \frac{1}{F_{11}} = \frac{1}{\cosh \eta L - (i\Delta\beta'/\eta)\sinh \eta L} \quad (30)$$

by the equation (29). Assuming that the index

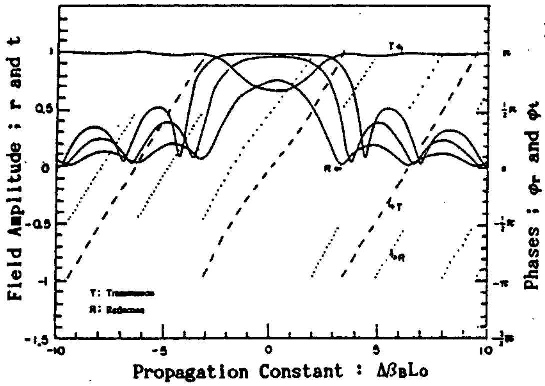


Fig.3 Spectrum of field amplitude and phases without gain ; The field amplitude of reflection is for κL_o = 1, 2, and 3 from lower to upper curve.

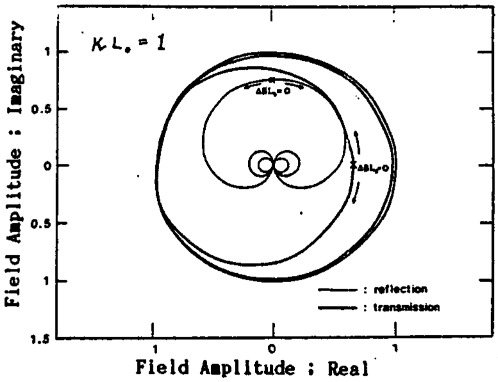


Fig.4 Field amplitude of r and t in complex plane ; The phase increases from 'X' for clockwise direction.

of refraction N(x,y,z) in (2) is real, the condition for the resonance mode of DFB resonator to be satisfied is $R(\Delta\beta) = 0$, which is similar to the case of Fabry-Perot resonator. This condition gives the resonance propagation constants of

$$\Delta\beta_n L_o = \pm\sqrt{(\kappa L_o)^2 + (n\pi)^2} \quad (n = \pm 1,\ 2,\ 3,\ \cdots) \qquad (31)$$

where $\Delta\beta_n$ is the deviation of the propagation constant from the Bragg propagation constant β_B in n'th resonance mode, L_o is the length of DFB waveguide, and $\kappa = \kappa_{BA} = \kappa_{AB}$. In the limit of weak coupling, $\kappa L_o \ll n\pi$ and the resonance propagation constants $\Delta\beta_n L_o$ approach $n\pi$ which is the resonance condition in case of Fabry-Perot resonator. On the other hand, in the limit of strong coupling the reflection bandwidth $\Delta\beta_R = \Delta\beta_1 - \Delta\beta_2$ approaches κ and the propagation within the reflection bandwidth becomes more reflective. The intensity of reflection and transmission of the DFB with one segment is shown in the figure 3.

The phases φ_r and φ_t of reflection and transmission in DFB structure are also shown in the figure 3. The phase of reflection at the Bragg propagation constant increases from $\pi/2$ as the propagation constant increases and jumps from $-\pi/2$ to $\pi/2$ at each resonance of propagation constant. The phase of transmission at the Bragg propagation constant is zero and increases as the propagation constant increases without jumps. The reflection and transmission coefficient in complex space in shown in the figure 4.

The phase condition of reflection for a resonance within the DFB with one section is π. This can be understood by considering the reflection at the middle position within the structure of DFB. The total phase of reflection from each side of DFB needs to be 2π. Actually the phase of reflection is $\pi/2$ and the total phase of reflection is π at $\Delta\beta_B L_o = 0$. If we introduce a phase slip of grating by $\Lambda/2$, the condition of resonance can be satisfied at $\Delta\beta_B L_o = 0$. The spectrum

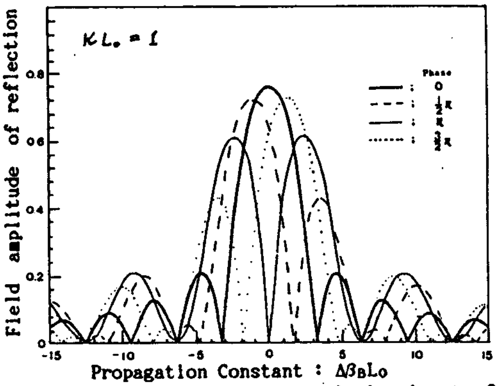


Fig.5 Reflection spectrum with the change of phase for 2 section DFB.

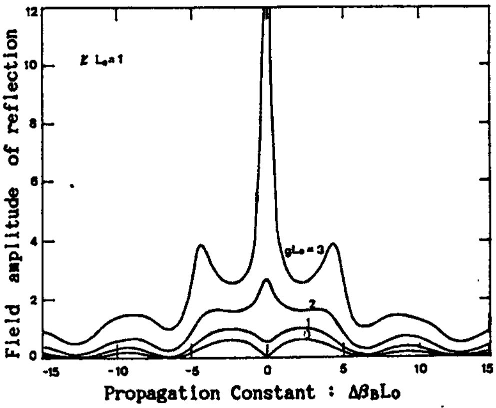


Fig.7 Reflection spectrum with the change of gain for the DFB with the phase shift of $\pi/2$.

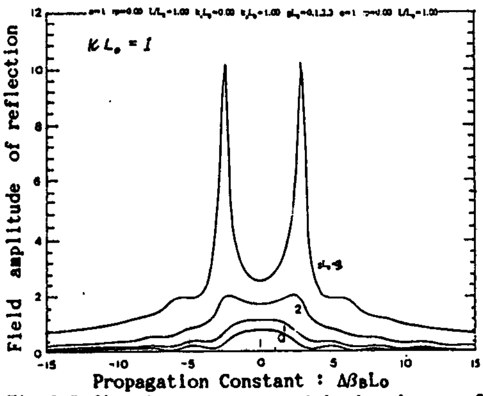


Fig.6 Reflection spectrum with the change of gain for the DFB without the phase shift.

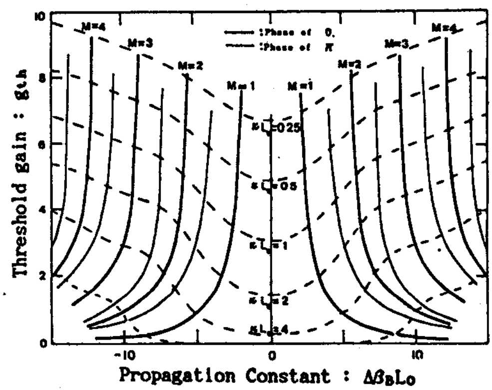


Fig.8 Threshold gain of DFB laser with the phase shift. ; M is the mode number.

of reflections with this change of phase is shown in Figure 5 and it shows the propagation constant of resonance as the relative phase between the two sections of DFB increases.

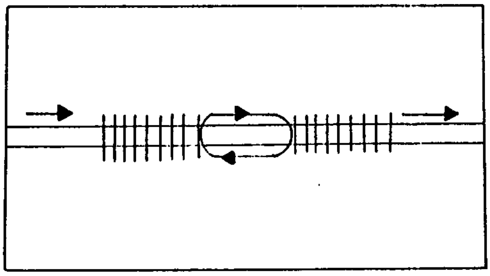
DFB structure becomes a laser resonator when we introduce gain within the waveguide. The reflection spectrum of a DFB resonator with one section and a DFB resonator with phase slip of $\Lambda/2$ is shown in the figure 6 and figure 7 where the reflection of resonance mode develops as the gain of the resonator increases.

The lasing condition that the reflection or the transmission occurs even without the incident light can be expressed as

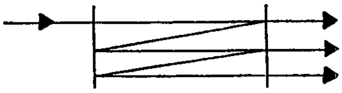
$$C_{11} = 0. \qquad (33)$$

The solution of this equation for a structure of DFB with a phase slip at the center is shown in the figure (8). We can minimize the requirement of gain for lasing and optimize the suppression of side modes. This method can be easily extended to more complex structure of DFB and can be used as a tool for the analysis and its design.

(a)

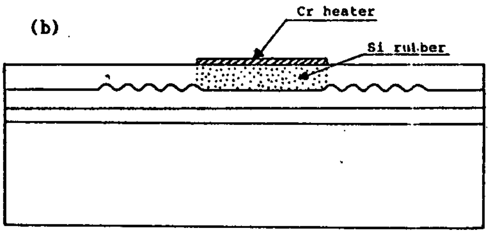


RESONATOR WITH BRAGG REFLECTOR



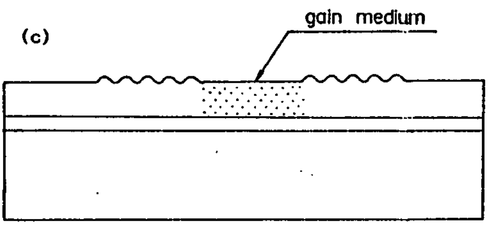
FABRY PEROT RESONATOR

(b)



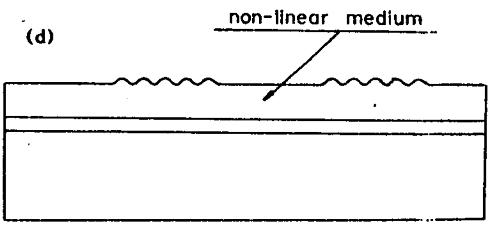
FREQUENCY TUNABLE
FABRY PEROT CAVITY

(c)



THIN FILM LASER

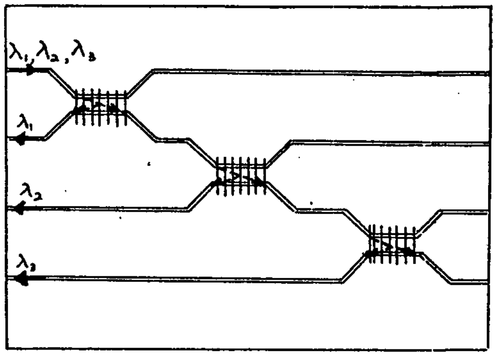
(d)



NON LINEAR FABRY PEROT CELL

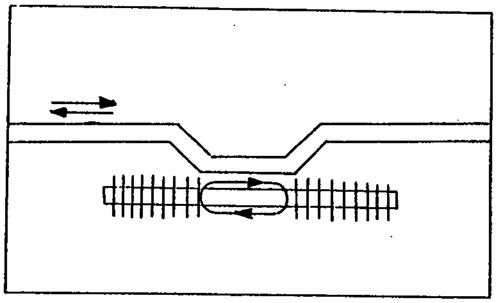
Fig.9 Applications of waveguide resonator of DBR type.

(a)



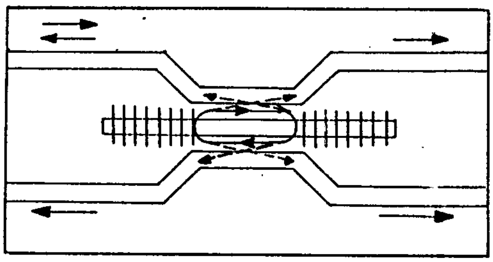
BRAGG REFLECTOR COUPLER
FOR WAVELENGTH DIVISION DEMULTIPLEXER

(b)



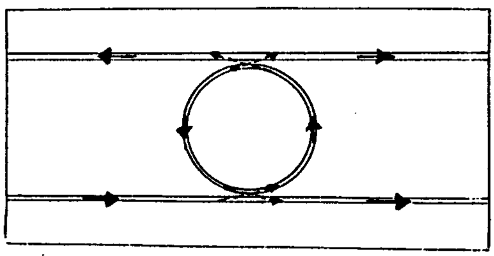
RESONANT OPTICAL REFLECTOR

(c)



RESONANT OPTICAL TRANSFORMER

(d)



RING CAVITY GYRO

Fig.10 Applications of DFB structure combined with directional coupler

4. Photonic devices using DFB or DBR structures and discussions

Some applications of DFB or DBR waveguide devices are illustrated in figure 9 and 10. In figure 9, some possible applications of waveguide resonator of DBR type are illustrated. The DBR type waveguide resonator can be used as a narrow linewidth optical filter[12] and this type of filter can be made tunable such as figure 9-(b). Waveguide laser using DFB or DBR resonator are used currently in the semiconductor laser for the optical communication[13] and the waveguide laser of dielectric media is also reported reacently.[14] This type of waveguide laser will not require the fiber coupling and can be integrated with other integrated optical devices. Resonator of nonlinear media[15] can be used for the nonlinear optical devices with materials such as leaded glasses,[16] superlattice of III-V semiconductors,[17] or polymers[17] having nonlinear optical properties.

The DFB structure can be combined with superlattice of III-V semiconductor.[17] structure confines light in some position of the resonator and the energy level of the light in the resonator is determined by the structure of DFB. The control of light by the DFB structure can be related to the control of electron by the superlattice. This concept may be useful for future application of superlattice devices.

In figure 10 the application of DFB structure combined with directional coupler are illustrated. Bragg reflector coupler combines Bragg reflection and directional coupler. WDM scheme using this kind of devices will provide very high degree of isolation between WDM channels due to the intrinsic property of Bragg reflector.

Resonant optical reflector can be used in a external waveguide laser for coherent communication and 130 kHz linewidth is already achieved.[17] Resonant optical transformer can also be applied to the WDM scheme. Finally the semiconductor laser using the resonant optical reflector can be used as a light source for ring cavity gyro with other integrated optical components.

References

[1] Hyung Jong Lee, Appl. Opt. 27(5), 1199(1988); W. W. Ng, IEEE Trans. Electron. Devices ED-25, 1193(1978).

[2] Toshiaki Suhara, IEEE J. Quant. Electron QE-22(6), 845(1986).

[3] Gerhard Heise, SPIE Vol. 651 Integrated Optical Circuit Engineering III, 87(1986).

[4] C. H. Henry, J-LT 9 1379(1989); Jewell J-LT 9, 1386(1989).

[5] Max Born and Emil Wolf, "Principles of Optics" 51p-70p, Pergamon Press, Oxford, 1975.

[6] H. Kogelnik, Bell Syst. Tech. J. 55,

109(1976).

[7] K. O. Hill, Appl. Opt. 13, 1853(1974); Appl. Opt. 13, 2886(1974).

[8] Makoto Yamada, Appl. Opt. 26, 3474(1987)

[9] Hyung Jong Lee, 56th Conf. of Kor. Phys. Soc. N-26, "New Approach for the Analysis of the Distributed Feedback Structure", Bulletin of the Kor. Phys. Soc. 6(1), April 1988.

[10] Y. Luo, IOOC' July 1989, Kobe Japan, 20PDB-2, "Gain-Coupled Semiconductor Laser Having Corrugated Active Layer", Technical Digest vol. 5, 40p.

[11] Hyung Jong Lee, (to be published); 5th Conf. on Waves and Lasers II-3, 1990 Feb. Seoul Nat'l Univ.

[12] C. H. Henry, H. J. Lee, IEEE J. QE 23(9), 1426(1987).

[13] N. A. Olsson, H. J. Lee, Appl. Phys. lett. 51(15), 1141(1987); Appl. Phys. Lett. 51(2), 92(1981).

[14] E. Lallier, "Nd:MgO:LiNbO$_3$ Waveguide Laser and Amplifier", Topical Meeting on Integrated Photonics Research TuJ5, Mar. 1990, Hilton Head, SC; Yoshinori Hibino, "Neodymiumdoped Silica Optical Waveguide Laser on Silicon Substrate" TuJ2.

[15] Herbert G. Winful, Appl. Phys. Lett. 40(4), 298(1982); 35(5), 379(1979).

[16] Stephen R. Friberg, IEEE J. QE 23(12), 2089(1987).

[17] See the proceedings of 'Topical Meeting on Integrated Photonics Research', Mar. 1990, Hilton Head, SC, USA.