CONTROL OF A BATCH REACTOR BY LEARNING OPERATION

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ABSTRACT

The iterative learning control synthesized in the frequency domain has been utilized for temperature control of a batch reactor. For this purpose, a feedback-assisted generalized learning control scheme was constructed first, and the convergence and robustness analyses were conducted in the frequency domain. The feedback-assisted learning operation was then implemented in a bench scale batch reactor where reaction heat is simulated using an electric heater. As a result, progressive reduction of temperature control error could be obviously observed as batch operation is repeated.

I. Introduction

The batch reactor, especially due to its flexibility, is the most widely used type of reactors in the chemical and the related industries. Due to significant nonlinearity and large variation of operating conditions, however, control of a batch reactor has long been considered as a difficult and challengeable research subject in chemical engineering. Many of the industrial reactors still have recourse to open-loop operation and/or conventional cascade feedback control, in some case with gain scheduling. Advanced control methods such as adaptive or nonlinear controls have been attempted but still not in extensive use because of required adaptation transients or difficulty in process modeling. Based on the above considerations, in the present work a new batch control method has been attempted utilizing the iterative learning control. The iterative learning control algorithm produces an improved open-loop control signals from the past operation records such that the tracking error is reduced as the operation iterates, i.e., learning through experience. It was originally developed by Arimoto et. al. [1] as a teaching mechanism of robot manipulators which must track some predefined reference trajectories precisely. After the pioneering work by Arimoto, many different learning control methods have been proposed [2-6], including the first order learning, the second order learning and the PID-type learning algorithms and so on. Most of the works, however, have been focused on finding open-loop control signals. When there occur unpredicted external disturbances, the algorithms may fail to work.

Purposing for batch reactor control where various disturbances are introduced in an unexpected manner, in this work, the existing open-loop learning algorithm is modified first by combining feedback control loop for disturbance rejection. Formulating a general form of the feedback-assisted learning algorithm, various learning operation hints were then deduced through convergence and robustness analyses in frequency domain. Finally the proposed learning algorithm was implemented to a simulated bench scale batch reactor for experimental evaluation. Tracking as well as regulatory controls have been attempted when there is significant reaction heat.

II. Frequency Domain Design of Feedback-Assisted Learning Operation

II-1. Learning Operation combined with Feedback Control

Consider a SISO linear process shown in Fig. 1, subjected to initial perturbation and external disturbance d.

Dynamics of the process can be written in the transformed domain as follows:

$$y = \frac{B}{A} u + \frac{B}{A} d + \frac{w}{A} \quad (1)$$

where y, u, d and w denote the output, input, external disturbance and initial perturbation, respectively. Variables in eq. (1) are functions of s or z according as the process is described in continuous-time or discrete-time, respectively. In continuous-time case, the initial perturbation w is expressed by

$$w = s^{n-1}y(0) + s^{n-2}y'(0) + \dots y^{(n-1)}(0) \qquad (2)$$

Let the subscript k denote the k-th operation. The control input calculated by the first-order linear iterative learning operation can be expressed in the generic form as follows:

$$u_k = u_{k-1} + \frac{C}{E} y_d - \frac{D}{F} y_{k-1} \qquad (3)$$

where y_d is the reference trajectory. If we combine a feedback control with the learning operation for disturbance rejection, then we have the overall control loop as shown in Fig. 2. In Fig. 2, for the feedback controller to be realizable (causal), the following inequality should hold.

$$\deg R \geq \deg S,\ \deg T \qquad (4)$$

Casaulity is not required for the learning block because calculation is performed with data records from a past run.

Through block diagram analysis, it can be easily derived that the updating equation of y is as follows:

$$\left[1 + \frac{B}{A}\frac{S}{R} \right] y_k = \left[1 - \frac{B}{A}\frac{D}{F} \right] y_{k-1} + \frac{B}{A} \left[\frac{C}{E} + \frac{T}{R} \right] y_d$$

$$+ \left[\frac{B}{A} \Delta d_k + \frac{\Delta w_k}{A} \right] \quad \text{where } \Delta w_k = w_k - w_{k-1} \qquad (5)$$

$$\Delta d_k = d_k - d_{k-1}$$

II-2. Conditions for Convergence

The objective of the learning operation is to drive y_k to converge to the reference trajectory y_d as k increases. To find the convergence condition, we assume that Δw_k and Δd_k in eq. (5) are null. Under these assumptions, it can be easily seen that

$$\|y_k - y_d\| \longrightarrow 0 \quad \text{as } k \longrightarrow \infty \qquad (6)$$

when the following two conditions are satisfied.

$$\alpha = \left\| 1 - \frac{B}{A}\frac{D}{F} \Big/ 1 + \frac{B}{A}\frac{S}{R} \right\| < 1 \qquad (7)$$

$$\frac{S - T}{R} = \frac{C}{E} - \frac{D}{F} \qquad (8)$$

Equation (7) is the condition for the sequence $\{y_k\}$ to have a limit. Equation (8) is the condition for y_d to be the limit. In eqs. (6) and (7), $\| \ \|$ denotes an appropriate norm for the function space concerned. In continuous-time case, signals are defined over a time segment $[0,T]$ and thus have only discrete frequency components at $\omega_k = (2\pi/T)k$, $k=0,1,2,\dots$ Likewise, discrete-time signals are defined over a finite discrete time sequence $\{0,h,2h,..Nh\}$, where $Nh=T$ and h is the sampling interval, and thus have frequency components at $\omega_k = (2\pi/N)k$, $k = 0, 1, 2, \dots, N-1$. Bearing these in mind, one of the useful but consistent norm definition will be

$$\|f\| = \sup_k |f(\omega_k)|,$$

$$\omega_k = \begin{cases} (2\pi/T)k & 0,1,2,.. \quad \text{for continuous-time} \\ (2\pi/N)k & 0,1,..,N-1 \quad \text{for discrete-time} \end{cases} \qquad (9)$$

Based on this norm definition, the condition (7) can be rewritten as

$$\left| 1 - \frac{B(s)D(s)}{A(s)F(s)} \Big/ 1 + \frac{B(s)S(s)}{A(s)R(s)} \right|_{s=j(2\pi/T)k} < 1,$$

$$k = 0, 1, 2, \dots \qquad (10)$$

for continuous-time case and

$$\left| 1 - \frac{B(z)D(z)}{A(z)F(z)} \Big/ 1 + \frac{B(z)S(z)}{A(z)R(z)} \right|_{z=e^{j(2\pi/N)k}} < 1,$$

$$k = 0, 1, 2, \dots, N-1 \qquad (11)$$

for discrete-time case, respectively.

II-3. Design of the Controller Blocks

When the plant model B/A is exactly known, maximum convergence rate is obtained when

$$\frac{D}{F} = \frac{A}{B} \qquad (12)$$

The condition (8), in this case, becomes

$$\frac{S - T}{R} = \frac{C}{E} - \frac{A}{D} \qquad (13)$$

From the causality condition (4) and the fact $\deg A \geq \deg B$, an obvious way to satisfy eq. (13) is

$$S = T \quad \text{and} \quad \frac{C}{E} = \frac{A}{B} \qquad (14)$$

In practical situations, only an approximate process model is available. Model error, though small or large, is inevitably introduced. Let B'/A' be the nominal process model. Then the learning block will be changed to

$$\bar{u}_k = u_{k-1} + \frac{A'}{B'} \left[y_d - y_{k-1} \right] \qquad (15)$$

Figure 3 shows the learning control scheme combined with feedback loop proposed through this analysis. Loop analysis shows the iterative updating procedure of y for the proposed learning control scheme is

$$\left[1 + \frac{B}{A}\frac{S}{R} \right] y_k = \left[1 - \frac{B}{A}\frac{A'}{B'} \right] y_{k-1} + \frac{B}{A} \left[\frac{A'}{B'} + \frac{S}{R} \right] y_d \quad (16)$$

Equation (16) guarantees that the limit of the sequence $\{y_k\}$ becomes y_d once the sequence converges.

II-4. Effect of Model Error on Convergence Condition

Let's express the model error by an additive term as follows:

$$\frac{B}{A} = \frac{B'}{A'}\left[1 + \Delta \right] \quad (17)$$

Substitution of eqs. (12) and (17) into eq. (7) while replacing A/B in eq. (12) with A'/B' gives

$$\| \Delta / 1 + \frac{B}{A}\frac{S}{R} \| < 1 \quad \text{or} \quad \| \Delta \| < \| 1 + \frac{B}{A}\frac{S}{R} \| \quad (18)$$

Equation (18) tells the maximum allowable model error for convergence is bounded by $\|1+BS/AR\|$. To investigate how the error bound varies with frequencies, let's confine our concerns to the continuous-time case.

Frequency characteristics of a typical open-loop gain BS/AR with an integral model in Rand the corresponding model error bound are shown in Fig. 4. Although only the discrete frequency components at $\omega_k=(2\pi/T)k$, k=0,1,2,... should be considered in this figure, we neglect this fact and consider the whole continuous frequency spectum for simplicity of analysis. From this figure, we can see that large model error is allowed in low frequency range. This says in turn fast convergence can be achieved for low frequency signals. As the frequncy increases, however, the open-loop gain gets small making tolerable model error bound approach to 1. Especially around the crossover frequency, the error bound has the minimum value about $|1 - 1/GM|$, where GM is the gain margin of the loop. Although relative model error of about 100% is tolerable at high frequencies, it can be thought to be stringent in practice because precise modeling of high frequency behaviors is usually not so easy while the low frequency behaviors can be accurately characterized.

II-5. Effect of Disturbance and Initial Perturbation

Disturbances including initial perturbation may seriously deteriorate learning operation. From eq. (5), we see that their effect on y_k is according to $((B/A)\Delta d_k+\Delta w_k/A)/(1+BS/AR)$.

An important aspect noticed in eq. (5) is that disturbance acts as a difference term in two subsequent iterations. In the linear description of a process as in eq. (1), the disturbance term d may be considered to include unmodelled part of the process such as higher order dynamics and/or nonlinear residual term as well as true external disturbance. In the former case, once the learning algorithm starts to converge, the disturbance appears similarly at each iteration. Accordingly the disturbances are self-eliminated and exert only a minor effect on y_k. This tells that the learning control scheme may work well for a large class of nonlinear processes, too.

Occurence of initial perturbation and external disturbance does not change the convergence condition. A well-designed feedback control loop can properly suppress their effects.

II-6. Design Examples

In this section, we present some design examples. Through these examples, we want to draw design and operation hints for learning operation. Only the continuous-time cases are considered for demonstration.

[Example 1] Continuous-time first-order system

Assume that $B/A = K/(\tau s+1)$ and $B'/A' = K'/(\tau' s+1)$. The model error is $\Delta = \frac{K}{K'}\frac{\tau' s+1}{\tau s+1} - 1$. Here $\frac{K}{K'}\frac{\tau' s+1}{\tau s+1}$ is a linear fractional(Mobius) transformation and maps $s=j\omega$, $\omega \in (-\infty, \infty)$ on the circle as shown in Fig. 5.

Convergence is gauranteed if the image is contained in the circle with center at (1,0) and radius 1/GM. If the open-loop has no crossover frequency or only the learning block is used without the feedback loop, the convergence region will be the open unit disk with center at (1,0). Notice that this condition is ony the sufficient one. At low frequency range, modulus of the map can exceed the disk: see Fig. 4.

From Fig. 5., we can draw simple modeling hints: Overestimation or underestimation of both K and τ is safer than overestimation of K with underestimation of τ or vice versa.

[Example 2] When the order of the process transfer function is not exactly known.

Assume that the process is of second-order while it is modeled as first-order, i.e., $B/A = K/(\tau^2 s^2+2\zeta\tau s+1)$ and $B'/A' = K'/(\tau' s+1)$. The model error will be $\Delta = \frac{BA'}{AB'}-1 = \frac{K}{K'}\frac{\tau' s+1}{\tau^2 s^2+2\zeta\tau s+1} - 1$. In this case mapping of $s=j\omega$, $\omega = (-\infty,\infty)$ by Δ is not as simple as in Example 1. However, we know that $\left.\frac{BA'}{AB'}\right|_{s=j\omega}$ starts at K/K' when $\omega = 0$, approaches to the origin as ω goes to infinity while the image stays only in first and/or fourth quadrants. Since the allowable model error bound is a disk of center at (1,0) with varying diameter as in Fig. 4(b), this loop becomes conditionally convergent depending on the parameters, especially on ζ.

If the order of B/A is larger than that of B'/A' by more than one, $\left.\frac{BA'}{AB'}\right|_{s=j\omega}$ intrudes the left-half complex plane and the convergence is violated.

In case that the order of the process transfer function is overestimated, e.g., B'/A' is of second-order while B/A is of first-order, the loop cannot satisfy the convergence condition because $\left.\frac{BA'}{AB'}\right|_{s=j\omega}$ goes to infinity as ω increases.

From the above considerations, we can notice that overestimation of the process order should be avoided. Underestimation of the process order may be allowed but the order difference should be less than two.

[Example 3] Process with time-delay

Assume that $B/A = K e^{-ds}/(\tau s+1)$ and $B'/A' = K' e^{-d's}/(\tau' s+1)$. Then the model error will be $\Delta = \frac{BA'}{AB'}-1 = \frac{K}{K'}\frac{\tau' s+1}{\tau s+1} e^{-(d-d')s}-1$. Map of $\left.\frac{BA'}{AB'}\right|_{s=j\omega}$ will look like Figs. 6(a) to 6(d). We can see that the learninig operation can never satify the convergence condition as far as time-delay error exists. We can see, however, that convergence violation is less serious when $(K\tau'/K'\tau) < (K/K')$ compared to the case when $(K\tau'/K'\tau) > -(K/K')$. Especially, in the former case, convergence violation is minimized if $\tau'=0$.

Actually convergence violation means that only the signal components over a certain frequency diverge as iteration continues while the remaining signal components still converges. From this consideration and by noting that the process signals are usually composed of low frequency components, we can draw an important operation hint.

Stop learning if high frequency signals start to grow and take $\bar{u}$ at the last iteration as feedforward compensation signal.

Compared to the feedback-only-control, this method would still improve the control performance significantly.

III. Experimental Evaluation

III-1. Experimental Apparatus and Conditions

Figure 7 shows the schematic diagram of the experimental setup. The reactor is equipped with a cooling coil and an electric heater. Reaction heat is simulated by an electric heater and the reactor temperature is controlled by the cooling water control valve. Temperature tracking as well as regulation have been attemped in the experiments. In both cases, severe reaction heat is applied in a predefined pattern.

In the respective experiment, PID control is conducted at the first batch and then the learning mode is introduced in the subsequent batches. The parameters for the PID and the learning block were determined based on the open-loop transfer function of the reactor. In Table 1, the learning block equation used in the experiments are given along with the open-loop transfer function and physical dimensions of the reactor system. The steady state gain, however, changes significantly with valve position due to the implemented characteristics of the linear control valve.

III-2. Results and Discussions

Results of the regulatory experiment and the reaction heat applied during the run are shown in Figs. 8, 9 and 10, respectively. As can be seen in Fig. 8, at the first batch the PID controller can not effectively reject the heat disturbance and relatively large temperature excursion results. After learning algorithm starts to work, it is clearly seen temperature control error becomes reduced more and more as batch is repeated. The control signals as shown in Fig. 9, however, get more oscillatory as learning iterates. It is thought to come from the derivative action of the learning algorithm.

In Figs. 11 to 13, results and heat input for the tracking experiments are summerized. The PID control in the first batch produces oscillatory response fighting against the heat input. In addition, delayed responses are are observed in the ramps, a well known property of the PID controller. As the feedforward signal by the learning mode is injected in the second batch, the response is greatly improved producing better tracking, less oscillation and smaller control error. As the batch continues, however, larger control error occurs. The experiment has been tried several times, but the similar results have been obtained every time. Especially, in the third and fourth batches, large deviation results just after the initial ramp and around the end of the cycle time. After 1,200 sec, the reactor temperature seems to follow the set point change very nicely but stops tracking with large deviation, redering the post-correction to the PID mode. Currently, it is not clear what is the main cause of this performance degradation, but the follwing two different causes can be conceivable. One is violation of the convergence condition and the other is accumulation of disturbance effects, see eq. (5). In practice, however, the learning operation need not be iterated endlessly. Once a satisfactory control performce is obtained, it is enough to stop the learning mode and take the control signal as the feedforward compensating input for the succeeding operations. Figure 13 shows the control inputs for the repeatitive learnings. As was in Fig. 13, the control input becomes more oscillatory as learning is repeated.

IV. Conclusions

The existing firtst-order learning control is revised by combining feedback control loop for utilization in a chemical batch reactor. Some of the important consequences obtained by anlayzing the feedback- assisted learning algorithm in the frequency domain and through experimental evaluation in a bench-scale batch reactor are as follows:

1. The best convergence for the learning operation can be obtained when the inverse process transfer function is operated on the previous control error.
2. When there is error in delay estimation, convergence is always violated. The violation, however, occurs at high frequency range so the learning operation still may help improve the control performance if the iteration continues only until a clue of performance degradation starts to appear.
3. The feedback-assisted learning control is proved to be useful in improving tracking as well as regulation performance of batch reactor temperature. As the iteration continues, however, control signals gets oscilltory.

References

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Table 1. Experimental System and Conditions

Reactor Vol.: 3.5 l	Control Valve: linear, Cv=0.5
Initial Steady States :	
Regulatory Control	8 % valve open, 1.0 Kw
Tracking Control	20 % valve open, 1.3 Kw
Open Loop T.F. at 8 % valve open, 1.0 Kw	$\frac{0.17(C/\%)}{6(min)s+1} e^{-1.1(min)s}$
Learning Algorithm	
Regulatory Control	$\bar{u}_{k+1}(q) = u_k(q) + 100(e_k(q+3) - 0.97\, e_k(q+2))$
Tracking Control	$\bar{u}_{k+1}(q) = u_k(q) + 20\,(e_k(q+3) - 0.95\, e_k(q+2))$
Sampling Interval: 30 sec	

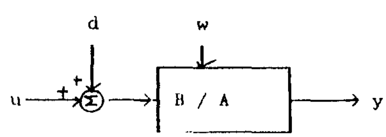

Fig. 1 . SISO linear process.

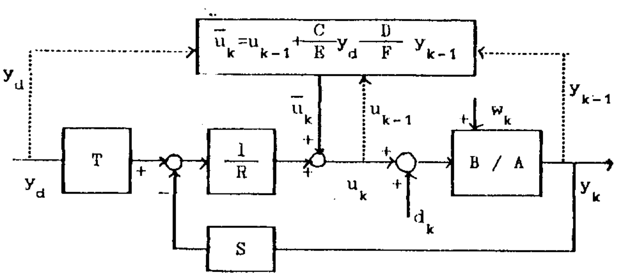

Fig. 2. Block diagram of feedback-assisted learning control loop.

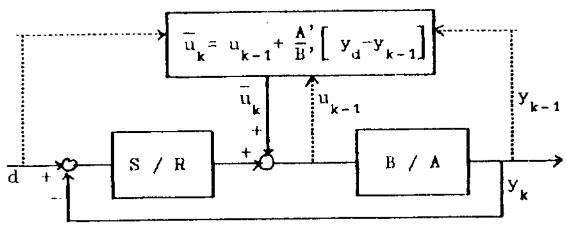

Fig. 3. Porposed learning control scheme.

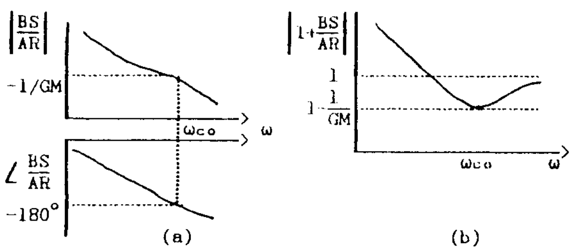

Fig. 4. Bode plot of a typical open-loop transfer function (a) and allowable model error bound for convergence (b).

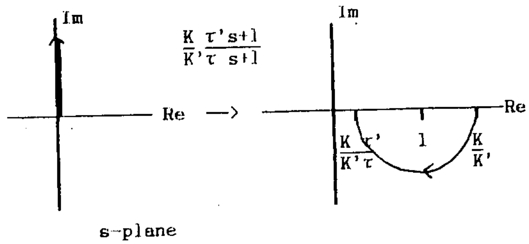

Fig. 5. Mapping of s=jω by $\frac{K}{K'}\frac{\tau s+1}{\tau' s+1}$.

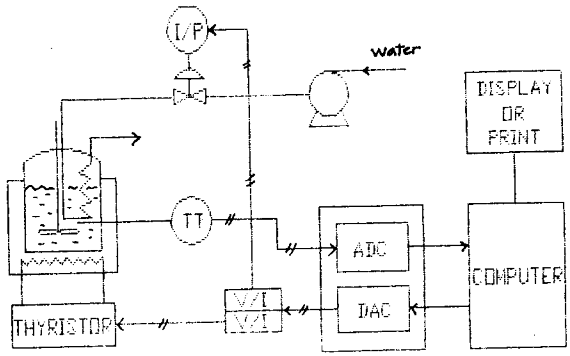

Fig. 7. Schematic diagram of the experimental system.

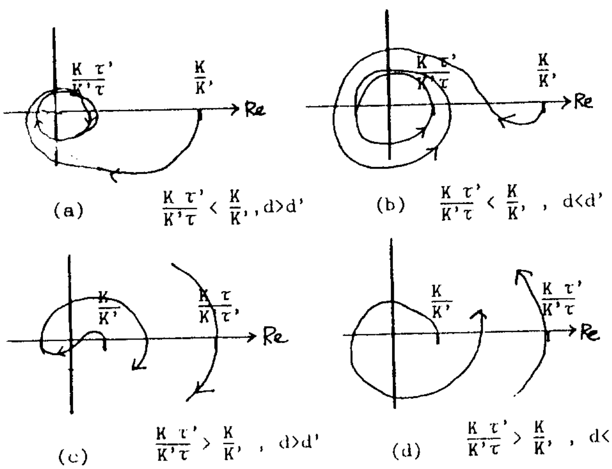

Fig. 6. Mapping of s=jω by $\frac{K}{K'}\frac{\tau' s+1}{\tau s+1} e^{-(d-d')s}$

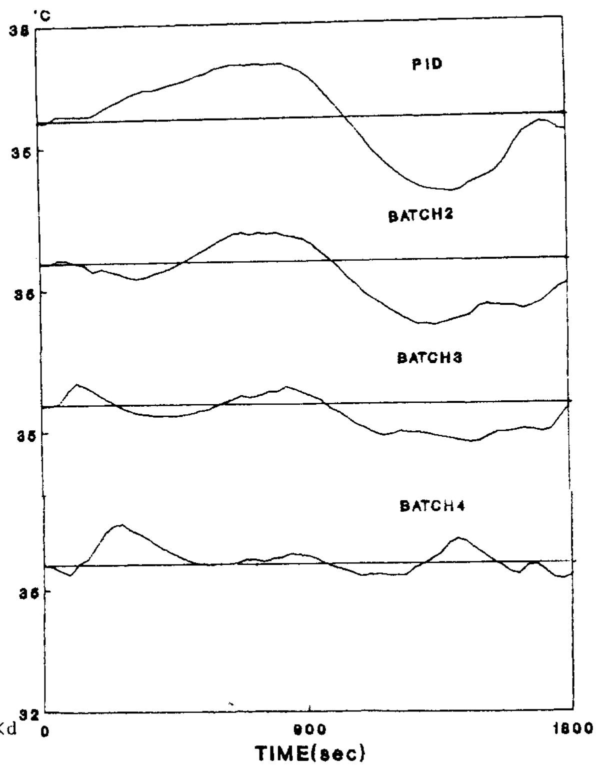

Fig. 8. Reaction temperature changes(regulation).

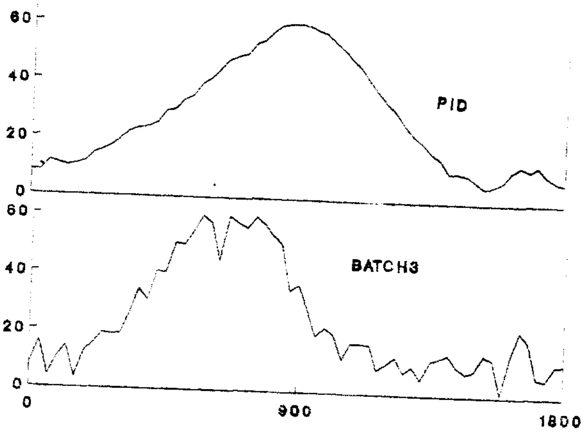

Fig. 9. Control input changes (regulation).

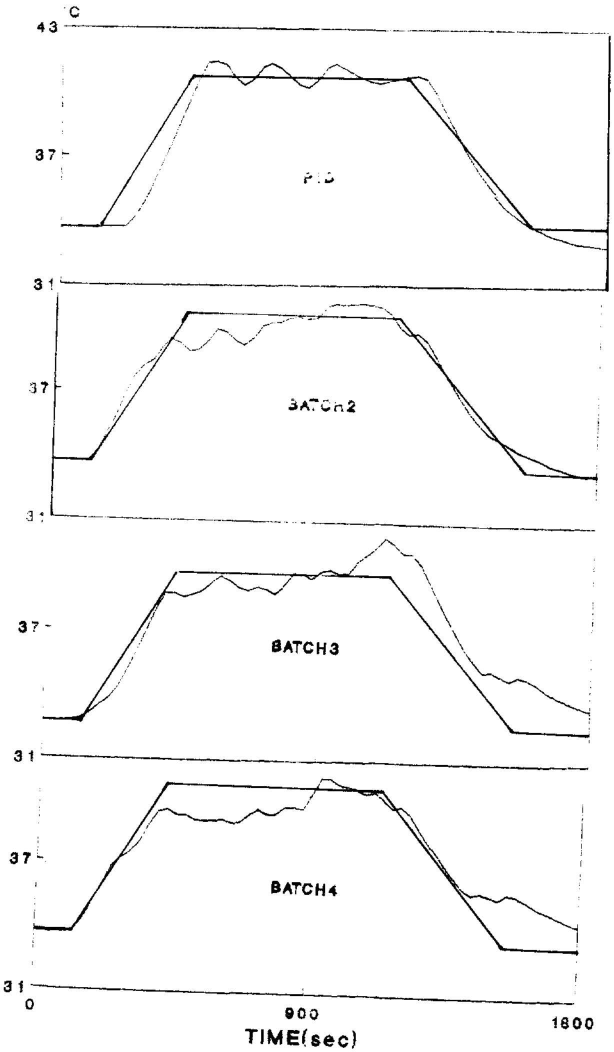

Fig.11. Reaction temperature changes(tracking).

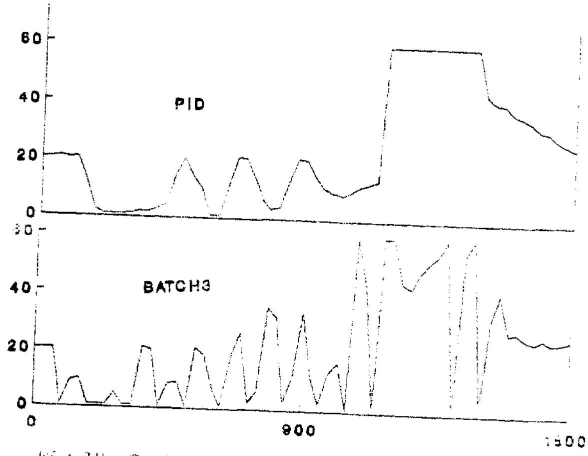

Fig.12. Control input changes (tracking).

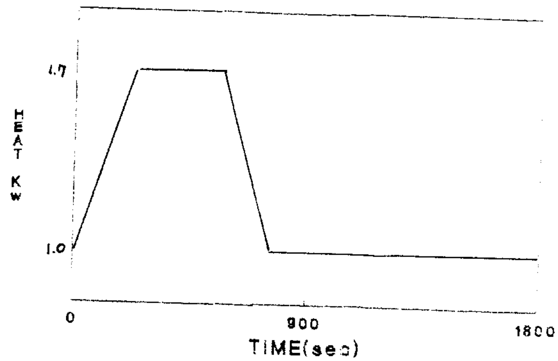

Fig.10. Simulated reaction heat (regulation).

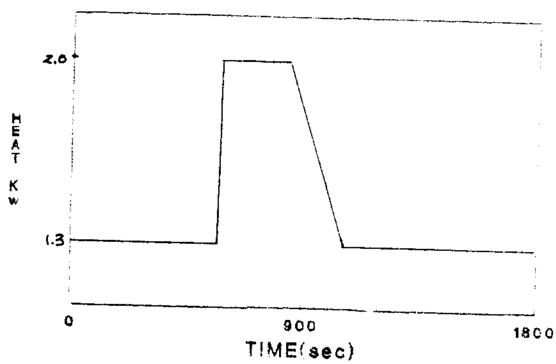

Fig.13. Simulated reaction heat (tracking).