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Identification of continuous systems in the presence of input-output measurement noises

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Abstract

The problem of identification of continuous systems is considered when both the discrete input and output measurements are contaminated by white noises. Using a predesigned digital low-pass filter, a discrete-time estimation model is constructed easily without direct approximations of system signal derivatives from sampled data. If the pass-band of the filter is designed so that it includes the main frequencies of both the system input and output signals in some range, the noise effects are sufficiently reduced, accurate estimates can be obtained by least squares(LS) algorithm in the presence of low measurement noises. Two classes of filters(infinite impulse response(IIR) filter and finite impulse response(FIR) filter) are employed. The former requires less computational burden and memory than the latter while the latter is suitable for the bias compensated least squares(BCLS) method, which compensates the bias of the LS estimate by the estimates of the input-output noise variances and thus yields unbiased estimates in the presence of high noises.

1 Introduction

Recently, the direct approaches to continuous system parameter estimation purely using digital computers have received more and more attention due to the rapid development of digital computers.

A major difficulty of identification of continuous-time models is that the derivatives of the system input-output signals are not measured directly and the differentiations may accentuate the noise effects. Therefore an important problem is how to handle the time derivatives[8]. In this report we present a digital low-pass filtering approach to direct recursive identification of linear SISO continuous systems. It is assumed that both the system input and output sampled data are contaminated by white noises. The approach includes the following steps:

1: Find a low-pass digital filter to prefilter the noise accentuating signal derivatives.

2: Construct a discrete-time estimation model with continuous system parameters.

3: Use a recursive identification algorithm to estimate the system parameters from filtered input-output sampled data.

In the case where the input is exactly known and only the output is corrupted by a measurement noise, it is well-known that the pass-band of the filters should be chosen such that it matches that of the system under study as closely[5,6,10]. And when the input measurement is assumed to be noise-free and only the output measurement is corrupted by a high measurement noise, the bootstrap method gives excellent results.

However, in most practical situtations, it may not be possible to avoid the noise when measuring the input signal. In this case, the bootstrap method may give erroneous results. So far, although some works have discussed the identification of discrete-time systems in the presence of input noise[1,2,7,9], we have not found such works for identification of continuous systems.

It will be found that in the presence of input measurement noise, it is not appropriate to let the pass-band of the pre-filters match that of the continuous system under study as suggested in some previous works. Our simulation results will show that when both the input and output signals are corrupted by measurement noises, the pass-band of the digital low-pass filters should be chosen such that it includes the main frequencies of both the system input and output signals in some range. Since most physical systems are low-pass systems, we emphasize that the selection of the pass-band of the pre-filters should be based on the frequencies of the input signals. To the limit of our knowledge, this aspect has not received so much attention in the literature.

Clearly, the digital low-pass filters employed in continuous system identification can be obtained using the existing digital filter design techniques[3] to have excellent filtering effects. Two classes of filters(FIR filter and IIR filter) are employed. The IIR filters require less computational burden and memory than the FIR filters while the FIR filters are suitable for the BCLS method which yields unbiased estimates in the presence of high input-output measurement noises[1,9].

2 Statement of the problem

Consider the following SISO continuous system

$$\begin{aligned} A(p)x(t) &= B(p)u(t) \\ A(p) &= \sum_{i=0}^{n} a_i p^{n-i} \quad (a_0 = 1) \\ B(p) &= \sum_{i=1}^{n} b_i p^{n-i} \end{aligned} \tag{1}$$

where p is differential operator, $u(t)$, $x(t)$ are the real input and the real output, and n is the known system order.

Our goal is to identify the system parameters from the noisy sampled input-output data:

$$\begin{aligned} y(k) &= x(k) + e(k) \\ w(k) &= u(k) + v(k) \end{aligned} \tag{2}$$

where k denotes the sampling time instants $t = kT(k = 0, 1, \cdots, N)$ for convenience of notation. and T is the sampling period. $v(k)$ and $e(k)$ are white noises such that

$$\begin{gathered} E[e(k)] = 0,\ E[e(k)^2] = \sigma_e^2 \\ E[v(k)] = 0,\ E[v(k)^2] = \sigma_v^2 \\ E[e(k)v(k)] = 0,\ E[u(k)v(k)] = 0,\ E[u(k)e(k)] = 0 \end{gathered} \tag{3}$$

Since differential operations may accentuate the measurement noise effects, it is inappropriate to identify the parameters using direct approximations of differentiations. Our objective here is to introduce a digital low-pass filter which would reduce the noise effects sufficiently. Then we can obtain a discrete-time estimation model with continuous system parameters.

3 Discrete-time estimation models

In this section, we describe the design techniques of the two classes of digital filters and the discrete-time estimation models derived by the pre-designed filters.

3.1 FIR filtering approach

It is known that the differential operator in (1) can be replaced by the bilinear transformation:

$$\sum_{i=0}^{n} a_i\left(\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}\right)^{n-i} x(k) = \sum_{i=1}^{n} b_i\left(\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}\right)^{n-i} u(k) \quad (4)$$

Introduce a low-pass digital filter $F(z^{-1})$ as

$$F(z^{-1}) = Q_F(z^{-1})\left(\frac{1+z^{-1}}{2}\right)^n \quad (5)$$

where $Q_F(z^{-1})$ is a kind of FIR filter.

Many types of FIR digital filters can be applied to $Q_F(z^{-1})$. For simplicity, we consider a desired ideal low-pass filter which has the specification:

$$H_d(\omega) = \begin{cases} 1 & |\omega| \le \omega_{dc} \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

Various design techniques[3] can be used to design FIR filters. The Hamming window method for the ideal low-pass filter is outlined here:

1: Select an appropriate integer M, sampling interval T, and the desired cut-off frequency ω_{dc}.

2: Perform the inverse Fourier transform to have the coefficients c_m of $H_1(z^{-1})$:

$$\begin{aligned} H_1(z^{-1}) &= \sum_{m=-M}^{M} c_m z^{-m} \\ c_m &= \frac{T}{2\pi}\int_{-\pi/T}^{\pi/T} H_d(\omega) e^{jm\omega T}\, d\omega = \frac{\sin(mT\omega_{dc})}{\pi m} \end{aligned} \quad (7)$$

3: Multiply c_m by the Hamming window function:

$$w_m = \begin{cases} 0.54 + 0.46\cos(\pi m/M) & |m| \le M \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

and then obtain

$$H_1'(z^{-1}) = \sum_{m=-M}^{M} c_m' z^{-m}, \quad c_m' = c_m\, w_m \quad (9)$$

4: Obtain the causal filter $Q_F(z^{-1})$ by delaying the entire sequence by M sampling intervals:

$$Q_F(z^{-1}) = \sum_{m=0}^{2M} q_m z^{-m} \quad (q_m = c'_{m-M}) \quad (10)$$

Multiplying both sides of (4) by the pre-designed $F(z^{-1})$ and using (2), we have

$$\sum_{i=0}^{n} a_i \xi_{Fiy}(k) = \sum_{i=1}^{n} b_i \xi_{Fiw}(k) + r_F(k) \quad (11)$$

where

$$\begin{aligned} \xi_{Fif}(k) &= Q_F(z^{-1})\left(\tfrac{T}{2}\right)^i (1+z^{-1})^i (1-z^{-1})^{n-i} f(k) \\ &= \sum_{j=0}^{2M+n} f_j^i z^{-j} f(k) \\ r_F(k) &= \sum_{i=0}^{n} a_i \xi_{Fie}(k) - \sum_{i=1}^{n} b_i \xi_{Fiv}(k) \\ &= \sum_{j=0}^{2M+n} \alpha_j z^{-j} e(k) - \sum_{j=0}^{2M+n} \beta_j z^{-j} v(k) \\ & \alpha_j = \sum_{i=0}^{n} f_j^i a_i, \quad \beta_j = \sum_{i=1}^{n} f_j^i b_i \end{aligned} \quad (12)$$

3.2 IIR filtering approach

There are many design methods for IIR digital filters. One of the most popular formulations is to use the large body of knowledge of the continuous-time or analog filters such as Butterworth filter, Chebyshev filter, inverse Chebyshev filter or the analog state variable filters[5,10]. When a continuous filter is designed, we can transform it in some manner such as the bilinear transformation to obtain the IIR digital filter.

In this report, we choose an mth$(m \ge n)$ order Butterworth filter $F_I(p)$:

$$F_I(p) = \frac{1}{(p/\omega_c)^m + c_1(p/\omega_c)^{m-1} + c_2(p/\omega_c)^{m-2} + \cdots + c_m} \quad (13)$$

where $c_i(i = 1, 2, \cdots, m)$ are the coefficients which let the Butterworth filter whose magnitude response is maximally flat for $\omega \le \omega_c$ and declines monotonically for $\omega > \omega_c$, has the basic form for the 'amplitude-squared function' as

$$|F_I(\omega)|^2 = F_I(p)F_I(-p)\,|_{p=j\omega} = \frac{1}{1+(\omega/\omega_c)^{2m}} \quad (14)$$

where ω_c is the cut-off frequency for which

$$|F_I(\omega)|^2 \le \frac{1}{2}, \qquad |\omega| \ge \omega_c \quad (15)$$

Multiplying both sides of the system (1) by the pre-designed $F_I(p)$, we have

$$F_I(p)p^n x(t) + \sum_{i=1}^{n} a_i F_I(p) p^{n-i} x(t) = \sum_{i=1}^{n} b_i F_I(p) p^{n-i} u(t) \quad (16)$$

Discretizing it by the bilinear transformation and using (2), we obtain

$$\sum_{i=0}^{n} a_i \xi_{Iiy}(k) = \sum_{i=1}^{n} b_i \xi_{Iiw}(k) + r_I(k) \quad (17)$$

where

$$\begin{aligned} r_I(k) &= \sum_{i=0}^{n} a_i \xi_{Iie}(k) - \sum_{i=1}^{n} b_i \xi_{Iiv}(k) \\ \xi_{Iif}(k) &= Q_I(z^{-1})\left(\frac{T}{2}\right)^i (1+z^{-1})^i (1-z^{-1})^{n-i} f(k) \\ Q_I(z^{-1}) &= \frac{\left(\frac{T}{2}\right)^{m-n}(1+z^{-1})^{m-n}}{\left(\frac{1-z^{-1}}{\omega_c}\right)^m + \sum_{i=1}^{m} c_i \left(\frac{1-z^{-1}}{\omega_c}\right)^{m-i}\left(\frac{T}{2}\right)^i (1+z^{-1})^i} \end{aligned} \quad (18)$$

Both types of digital filters are useful. The IIR filters have simple design methods using the bilinear transformation. And the IIR filters can produce desired amplitude response with significantly few coefficients than nonrecursive

FIR filters. Therefore, usually, the IIR filters are more convenient for the LS or the bootstrap algorithms[5,6]. However, as shown later, the FIR filters are suitable for the so-called BCLS method which yields unbiased estimates in the presence of high input-output measurement noises. The pass-band of the filters should be chosen to reduce the noise effects. It can be shown that some well-known methods can be viewed as either the FIR or the IIR filtering approach[5,6].

4 LS method

When the digital low-pass filters have been designed, we have the discrete-time estimation model of (11) for the FIR filtering approach, or the model of (17) for the IIR filtering approach. Both can be written in vector form:

$$\begin{aligned} \xi_{0y}(k) &= \mathbf{z}^T(k)\theta + r(k) \\ \mathbf{z}^T(k) &= [-\xi_{1y}(k), \cdots, -\xi_{ny}(k), \xi_{1w}(k), \cdots, \xi_{nw}(k)] \\ \theta^T &= [a_1, \cdots, a_n, b_1, \cdots, b_n] \end{aligned} \tag{19}$$

where

$$\begin{aligned} \xi_{iy}(k) = \xi_{Fiy}(k), \xi_{iw}(k) = \xi_{Fiw}(k), r(k) = r_F(k) \text{ (FIR filter)} \\ \xi_{iy}(k) = \xi_{Iiy}(k), \xi_{iw}(k) = \xi_{Iiw}(k), r(k) = r_I(k) \text{ (IIR filter)} \end{aligned} \tag{20}$$

We can estimate the continuous system parameters by the following LS method:

$$\hat{\theta} = [\sum_{k=k_0}^{N} \mathbf{z}(k)\mathbf{z}^T(k)]^{-1} \cdot [\sum_{k=k_0}^{N} \mathbf{z}(k)\xi_{0y}(k)] \tag{21}$$

When the noise effects can not be neglected, it is well-known that the LS estimate is asymptotically biased in general. For the case where both the input and output signals are corrupted by low measurement noises, if the pass-band of the pre-designed digital low-pass filter includes both the system input and output signals in some range, the LS method is still efficient in the presence of low measurement noises. When the discrete input-output measurements are corrupted by high white noises, we will extend the BCLS method[1,4,9] which yields unbiased parameter estimates of discrete-time systems, to the case of continuous systems in the next section.

5 BCLS method for the FIR filtering approach

In this section, we extend the BCLS method to the problem of identification of continuous system in the presence of input-output measurement noises by making use of the FIR filtering approach. The IIR filtering approach while requiring less computational effort and memory, is not suitable since it is not easy to express the correlations of the outputs of the IIR filters.

5.1 Expression of the bias

Consider the discrete model derived by the FIR filter:

$$\begin{aligned} \xi_{F0y}(k) &= \mathbf{z}^T(k)\theta + r_F(k) \\ \mathbf{z}^T(k) &= [-\xi_{F1y}(k), \cdots, -\xi_{Fny}(k), \xi_{F1w}(k), \cdots, \xi_{Fnw}(k)] \\ \theta^T &= [a_1, \cdots, a_n, b_1, \cdots, b_n] = [\mathbf{a}^T, \mathbf{b}^T] \end{aligned} \tag{22}$$

The LS estimate is

$$\hat{\theta} = [\sum_{k=k_0}^{N} \mathbf{z}(k)\mathbf{z}^T(k)]^{-1} \cdot [\sum_{k=k_0}^{N} \mathbf{z}(k)\xi_{F0y}(k)] \tag{23}$$

It can be shown that

$$\operatorname*{plim}_{N\to\infty} \hat{\theta} = \theta + NP(N)[\operatorname*{plim}_{N\to\infty} \frac{1}{N} \sum_{k=k_0}^{N} \mathbf{z}(k) r_F(k)] \tag{24}$$

where

$$P(N) = [\operatorname*{plim}_{N\to\infty} \sum_{k=k_0}^{N} \mathbf{z}(k)\mathbf{z}^T(k)]^{-1} \tag{25}$$

With straight calculations, we have

$$[\operatorname*{plim}_{N\to\infty} \frac{1}{N} \sum_{k=k_0}^{N} \mathbf{z}(k) r_F(k)] = -\mathbf{D}\mathbf{H}(\theta) \tag{26}$$

where

$$\begin{aligned} \mathbf{H}(\theta) &= [\mathbf{h}^e(\mathbf{a}), \mathbf{h}^v(\mathbf{b})]^T \\ \mathbf{h}^e(\mathbf{a}) = [h_1^e(\mathbf{a}), \cdots, h_n^e(\mathbf{a})], &\quad \mathbf{h}^v(\mathbf{b}) = [h_1^v(\mathbf{b}), \cdots, h_n^v(\mathbf{b})] \\ h_i^e(\mathbf{a}) = \sum_{j=0}^{2M+n} f_j^i \alpha_j(\mathbf{a}), &\quad h_i^v(\mathbf{b}) = \sum_{j=0}^{2M+n} f_j^i \beta_j(\mathbf{b}) \end{aligned} \tag{27}$$

and the $2n \times 2n$ matrix $\mathbf{D}$ is of the form

$$\mathbf{D} = \begin{bmatrix} \sigma_e^2 \mathbf{I}_n & \mathbf{0}_n \\ \mathbf{0}_n & \sigma_v^2 \mathbf{I}_n \end{bmatrix} \tag{28}$$

where $\mathbf{I}_n$, $\mathbf{0}_n$ are an identity matrix and a zero matrix of order n respectively.

Unfortunately, for the IIR filtering approach, it is difficult to express the bias explicitly by σ_e^2 and σ_v^2, since, usually, expressions of the correlations of the outputs of the IIR filters are not so simple.

5.2 BCLS method

From (24) and (26), we have

$$\theta = \operatorname*{plim}_{N\to\infty} \hat{\theta} + NP(N)\mathbf{D}\mathbf{H}(\theta) \tag{29}$$

which implies that an unbiased estimate of the unknown parameters can be obtained by substracting an estimate of the bias. If we have the estimates of σ_e^2 and σ_v^2, the BCLS algorithm is given by

$$\hat{\theta}_{BC}(N) = \hat{\theta}(N) + NP(N)\hat{\mathbf{D}}\mathbf{H}[\hat{\theta}_{BC}(N-1)] \tag{30}$$

and $\hat{\theta}(N)$, $P(N)$ are obtained by

$$\begin{aligned} \hat{\theta}(N) &= \hat{\theta}(N-1) + L(N)[\xi_{F0y}(N) - \mathbf{z}^T(N)\hat{\theta}(N-1)] \\ L(N) &= \frac{P(N-1)\mathbf{z}(N)}{\rho(N) + \mathbf{z}^T(N)P(N-1)\mathbf{z}(N)} \\ P(N) &= \frac{1}{\rho(N)}[P(N-1) - \frac{P(N-1)\mathbf{z}(N)\mathbf{z}^T(N)P(N-1)}{\rho(N) + \mathbf{z}^T(N)P(N-1)\mathbf{z}(N)}] \end{aligned} \tag{31}$$

where $\rho(N)$ is forgetting factor and is chosen to be

$$\rho(N) = (1-0.01)\rho(N-1) + 0.01 \qquad \rho(0) = 0.95 \tag{32}$$

The BCLS method requires the estimates of σ_v^2 and σ_e^2. In the next subsection, we will show the method to find $\hat{\sigma}_e^2$ and $\hat{\sigma}_v^2$.

5.3 Estimation of σ_e^2 and σ_v^2

The equation error $\hat{r}_F(k)$ for the LS estimate $\hat{\theta}(N)$ is

$$\hat{r}_F(k) = \xi_{F0y}(k) - \mathbf{z}^T(k)\hat{\theta}(N) \tag{33}$$

Using (22), we have

$$\hat{r}_F(k) = \mathbf{z}^T(k)[\theta - \hat{\theta}(N)] + r_F(k) \tag{34}$$

From (23) and (33), we have

$$\sum_{k=k_0}^{N} \mathbf{z}(k)\hat{r}_F(k) = \mathbf{0} \tag{35}$$

Using (34) and (35), we have the sum of squared residuals:

$$\begin{aligned} g(N) &= \sum_{k=k_0}^{N} \hat{r}_F(k)\hat{r}_F(k) \\ &= \sum_{k=k_0}^{N} r_F^2(k) + \sum_{k=k_0}^{N} \mathbf{z}^T(k) r_F(k)(\theta - \hat{\theta}(N)) \end{aligned} \tag{36}$$

Since

$$\operatorname*{plim}_{N\to\infty} \frac{1}{N} r_F^2(k) = E[r_F^2(k)] = \sum_{j=0}^{2M+n} \alpha_j^2(\mathbf{a})\sigma_e^2 + \sum_{j=0}^{2M+n} \beta_j^2(\mathbf{b})\sigma_v^2 \tag{37}$$

and

$$\operatorname*{plim}_{N\to\infty} \frac{1}{N}\mathbf{z}^T(k) r_F(k) = E[\mathbf{z}^T(k) r_F(k)] = -\mathbf{H}^T(\theta)\mathbf{D} \tag{38}$$

then the following result can be obtained:

$$\begin{aligned} \operatorname*{plim}_{N\to\infty} \frac{g(N)}{N} &= \sum_{j=0}^{2M+n} \alpha_j^2(\mathbf{a})\sigma_e^2 + \sum_{j=0}^{2M+n} \beta_j^2(\mathbf{b})\sigma_v^2 \\ &\quad -\mathbf{h}^e(\mathbf{a})[\mathbf{a} - \hat{\mathbf{a}}(N)]\sigma_e^2 - \mathbf{h}^v(\mathbf{b})[\mathbf{b} - \hat{\mathbf{b}}(N)]\sigma_v^2 \end{aligned} \tag{39}$$

Similar to the above discussions, we deifine the instrumental variable estimte $\bar{\theta}(N)$ by

$$\bar{\theta}(N) = [\sum_{k=k_0}^{N} \mathbf{z}(k)\mathbf{z}^T(k-L)]^{-1} \cdot [\sum_{k=k_0}^{N} \mathbf{z}(k)\xi_{F0y}(k-L)] \tag{40}$$

where L is a natural number. The recursive form is

$$\begin{aligned} &\bar{\theta}(N) = \\ &\bar{\theta}(N-1) + \bar{L}(N)[\xi_{F0y}(N-L) - \mathbf{z}^T(N-L)\bar{\theta}(N-1)] \\ &\bar{L}(N) = \frac{\bar{P}(N-1)\mathbf{z}(N)}{\rho(N) + \mathbf{z}^T(N-L)\bar{P}(N-1)\mathbf{z}(N)} \\ &\bar{P}(N) = \\ &\frac{1}{\rho(N)}[\bar{P}(N-1) - \frac{\bar{P}(N-1)\mathbf{z}(N)\mathbf{z}^T(N-L)\bar{P}(N-1)}{\rho(N) + \mathbf{z}^T(N-L)\bar{P}(N-1)\mathbf{z}(N)}] \end{aligned} \tag{41}$$

The equation error for $\bar{\theta}(N)$ is defined by

$$\bar{r}_F(k) = \xi_{F0y}(k) - \mathbf{z}^T(k)\bar{\theta}(N) \tag{42}$$

and it can also be shown that

$$\begin{aligned} &\bar{r}_F(k) = \mathbf{z}^T(k)[\theta - \bar{\theta}(N)] + r_F(k) \\ &\sum_{k=k_0}^{N} \mathbf{z}(k)\bar{r}_F(k-L) = \mathbf{0} \end{aligned} \tag{43}$$

Hence we have

$$\begin{aligned} f(N) &= \sum_{k=k_0}^{N} \bar{r}_F(k)\bar{r}_F(k-L) \\ &= \sum_{k=k_0}^{N} r_F(k) r_F(k-L) \\ &\quad + \sum_{k=k_0}^{N} \mathbf{z}^T(k-L) r_F(k)(\theta - \bar{\theta}(N)) \end{aligned} \tag{44}$$

and thus

$$\begin{aligned} \operatorname*{plim}_{N\to\infty} \frac{f(N)}{N} &= \sum_{j=0}^{2M+n-L} \alpha_j(\mathbf{a})\alpha_{j+L}(\mathbf{a})\sigma_e^2 \\ &\quad + \sum_{j=0}^{2M+n-L} \beta_j(\mathbf{b})\beta_{j+L}(\mathbf{b})\sigma_v^2 \\ &\quad -\bar{\mathbf{h}}^e(\mathbf{a})[\mathbf{a} - \bar{\mathbf{a}}(N)]\sigma_e^2 - \bar{\mathbf{h}}^v(\mathbf{b})[\mathbf{b} - \bar{\mathbf{b}}(N)]\sigma_v^2 \end{aligned} \tag{45}$$

where

$$\begin{aligned} &\bar{\mathbf{h}}^e(\mathbf{a}) = [\bar{h}_1^e(\mathbf{a}), \cdots, \bar{h}_n^e(\mathbf{a})], \quad \bar{\mathbf{h}}^v(\mathbf{b}) = [\bar{h}_1^v(\mathbf{b}), \cdots, \bar{h}_n^v(\mathbf{b})] \\ &\bar{h}_i^e(\mathbf{a}) = \sum_{j=0}^{2M+n-L} f_j^i \alpha_{j+L}(\mathbf{a}), \quad \bar{h}_i^v(\mathbf{b}) = \sum_{j=0}^{2M+n-L} f_j^i \beta_{j+L}(\mathbf{b}) \end{aligned} \tag{46}$$

Then the estimates of the unknown variances σ_e^2 and σ_v^2 are given by the solution of the following equation:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \sigma_e^2 \\ \sigma_v^2 \end{bmatrix} = \mathbf{A} \begin{bmatrix} \sigma_e^2 \\ \sigma_v^2 \end{bmatrix} = \frac{1}{N} \begin{bmatrix} g(N) \\ f(N) \end{bmatrix} \tag{47}$$

where

$$\begin{aligned} a_{11} &= \sum_{j=0}^{2M+n} \alpha_j^2(\hat{\mathbf{a}}_{BC}(N-1)) \\ &\quad -\mathbf{h}^e(\hat{\mathbf{a}}_{BC}(N-1))\ [\hat{\mathbf{a}}_{BC}(N-1) - \hat{\mathbf{a}}(N)] \\ a_{12} &= \sum_{j=0}^{2M+n} \beta_j^2(\hat{\mathbf{b}}_{BC}(N-1)) \\ &\quad -\mathbf{h}^v(\hat{\mathbf{b}}_{BC}(N-1))\ [\hat{\mathbf{b}}_{BC}(N-1) - \hat{\mathbf{b}}(N)] \\ a_{21} &= \sum_{j=0}^{2M+n-L} \alpha_j(\hat{\mathbf{a}}_{BC}(N-1))\ \alpha_{j+L}(\hat{\mathbf{a}}_{BC}(N-1)) \\ &\quad -\bar{\mathbf{h}}^e(\hat{\mathbf{a}}_{BC}(N-1))\ [\hat{\mathbf{a}}_{BC}(N-1) - \bar{\mathbf{a}}(N)] \\ a_{22} &= \sum_{j=0}^{2M+n-L} \beta_j(\hat{\mathbf{b}}_{BC}(N-1))\ \beta_{j+L}(\hat{\mathbf{b}}_{BC}(N-1)) \\ &\quad -\bar{\mathbf{h}}^v(\hat{\mathbf{b}}_{BC}(N-1))\ [\hat{\mathbf{b}}_{BC}(N-1) - \bar{\mathbf{b}}(N)] \end{aligned} \tag{48}$$

It should be noted that the delay L should be chosen such that $\mathbf{A}$ and $[\sum_{k=k_0}^{N} \mathbf{z}(k)\mathbf{z}^T(k-L)]$ are nonsingular. To our experiences, the estimation results are not sensitive to L.

Now we will consider the methods to calculate $g(N)$ and $f(N)$. It can be shown that

$$\begin{aligned} &\sum_{k=k_0}^{N} \begin{bmatrix} -\xi_{F0y}(k) \\ \mathbf{z}(k) \end{bmatrix} [-\xi_{F0y}(k),\ \mathbf{z}^T(k)] \begin{bmatrix} 1 \\ \hat{\theta}(N) \end{bmatrix} \\ &= \sum_{k=k_0}^{N} \begin{bmatrix} -\xi_{F0y}(k) \\ \mathbf{z}(k) \end{bmatrix} [-\hat{r}_F(k)] = \begin{bmatrix} g(N) \\ \mathbf{0} \end{bmatrix} \\ &\sum_{k=k_0}^{N} \begin{bmatrix} -\xi_{F0y}(k) \\ \mathbf{z}(k) \end{bmatrix} [-\xi_{F0y}(k-L),\ \mathbf{z}^T(k-L)] \begin{bmatrix} 1 \\ \bar{\theta}(N) \end{bmatrix} \\ &= \sum_{k=k_0}^{N} \begin{bmatrix} -\xi_{F0y}(k) \\ \mathbf{z}(k) \end{bmatrix} [-\bar{r}_F(k-L)] = \begin{bmatrix} f(N) \\ \mathbf{0} \end{bmatrix} \end{aligned} \tag{49}$$

Hence we can express $g(N)$, $f(N)$ as

$$\begin{aligned} &g(N) = \sum_{k=k_0}^{N} \xi_{F0y}^2(k) - \sum_{k=k_0}^{N} \xi_{F0y}(k)\mathbf{z}^T(k)\hat{\theta}(N) \\ &f(N) = \sum_{k=k_0}^{N} \xi_{F0y}(k)\xi_{F0y}(k-L) - \sum_{k=k_0}^{N} \xi_{F0y}(k)\mathbf{z}^T(k-L)\bar{\theta}(N) \end{aligned} \tag{50}$$

Based on the above discussions, we can summarize the on-line BCLS algorithm as:

1: Calculate the LS estimate $\hat{\theta}(N)$ and the IV estimate $\bar{\theta}(N)$ by (31) and (41) respectively.

2: Calculate $g(N)$ and $f(N)$ by (50).

3: Solve (47) to have the variance estimates $\hat{\sigma}_e^2$ and $\hat{\sigma}_v^2$.

4: Compensate the bias of the LS estimate $\hat{\theta}(N)$ by (30).

5: Return to 1 untill convergence

6 Illustrative examples

Consider a second-order system described by

$$\begin{aligned} &\ddot{x}(t) + a_1\dot{x}(t) + a_2x(t) = b_1\dot{u}(t) + b_2u(t) \\ &a_1 = 3.0,\ a_2 = 4.0,\ b_1 = 0.0,\ b_2 = 4.0 \end{aligned} \tag{51}$$

The input $u(t)$ is the output of a second-order continuous-time Butterworth input filter driven by a stationary random signal $\zeta(t)$:

$$u(t) = L(p)\zeta(t) = \frac{1}{(p/\omega_c)^2 + \sqrt{2}(p/\omega_c) + 1}\,\zeta(t), \quad \omega_c = 4.0 \tag{52}$$

The sampling interval is taken to be $T = 0.04$, and $\sigma_u = 2.38, \sigma_x = 0.7$. The LS estimates for the case of low measurement noises where $\sigma_v = 0.24, \sigma_e = 0.07$(N/S ratio$\approx$ 0.1) are shown in Table 1 for the FIR filters($M = 25$), and Table 2 for the IIR filters($m = 2$). In Table 1, ω_{ac} denotes the actual cut-off frequency of the FIR filter $F(z^{-1})$ defined in equation(5) which lets $|F(\omega)|^2 \leq 1/2$, for $|\omega| \geq \omega_{ac}$.

Each of the tables includes the mean and standard deviation of the estimates obtained from Monte-Carlo simulation of 20 experiments. 10000 samples are taken for each experiment. And in each table, $\Delta\|\theta\| = \|\theta - \hat{\theta}\|$.

The frequency responses of the system, the digital prefilters used in Tables 1~2 and the input filter $L(p)$ in (52) are shown in Figs.1~2. It is clear that accurate estimates can be obtained if the pass-band of the pre-filters includes that of the low-pass input filter $L(p)$ in equation(52) in some range. Therefore, for the case of low input-output measurement noises, if the pass-band of the digital low-pass filters is chosen such that it *includes the main frequencies of both the real system input-output signals in some range*, the noise effects are sufficiently reduced, and thus the LS estimates are still acceptable. For the case where the input is noise-free and only the output is corrupted by a measurement noise, it is known that the pass-band of the filters should be chosen such that it matches that of the system under study as closely. This suggestion is not appropriate in the presence of input measurement noise.

The LS estimates and the BCLS estimates($L = 5$) for the FIR filters are shown in Table 3 and Table 4 when $\sigma_v = 0.60, \sigma_e = 0.17$(N/S ratio$\approx$ 0.25). In the presence of high input-output measurement noises, it is difficult to obtain accurate estimates with the LS method. However, the BCLS method is very efficient in this case.

7 Conclusion

In this report, the digital filtering approach to recursive identification of continuous systems from noisy sampled input-output data have been discussed. Using a pre-designed digital low-pass filter, a discrete-time estimation model with continuous system parameters is constructed easily. And it was emphasized that in the presence of input measurement noise, the pass-band of the filters should be chosen such that it includes the main frequencies of the real system input-output signals in some range to reduce the noise effects.

Two classes of filters(FIR filter and IIR filter) have been applied. Both classes of filters have excellent noise reducing effects. Usually, the IIR filters require less computational burden and memory than the FIR filters and are more convenient. However, it has been shown that for the discrete-time model derived by the FIR filters, the bias of the LS estimates can be expressed explicitly by σ_e^2 and σ_v^2. Hence the FIR filtering approach is suitable for the BCLS method which yields unbiased estimates in the presence of high measurement noises.

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Table 1: LS estimates(FIR filter, N/S ratio≈ 0.1)

ω_{dc} (ω_{ac})	$\hat{a}_1$ (3.0)	$\hat{a}_2$ (4.0)	$\hat{b}_1$ (0.0)	$\hat{b}_1$ (4.0)	$\Delta\|\theta\|$
14.0 (12.47)	2.892 ±0.041	4.089 ±0.030	0.009 ±0.006	4.012 ±0.034	0.141
12.0 (10.57)	2.918 ±0.037	4.052 ±0.027	0.009 ±0.006	4.004 ±0.030	0.097
10.0 (8.62)	2.930 ±0.030	4.020 ±0.025	0.010 ±0.005	3.989 ±0.025	0.074
8.0 (6.69)	2.934 ±0.025	3.993 ±0.023	0.011 ±0.004	3.972 ±0.021	0.073
7.0 (5.69)	2.934 ±0.024	3.980 ±0.023	0.013 ±0.004	3.963 ±0.020	0.080
5.0 (3.78)	2.920 ±0.024	3.945 ±0.023	0.018 ±0.005	3.933 ±0.022	0.120
4.0 (3.01)	2.9018 ±0.026	3.914 ±0.0245	0.025 ±0.006	3.904 ±0.026	0.163

Table 2: LS estimates(IIR filter, N/S ratio≈ 0.1)

ω_c	$\hat{a}_1$ (3.0)	$\hat{a}_2$ (4.0)	$\hat{b}_1$ (0.0)	$\hat{b}_1$ (4.0)	$\Delta\|\theta\|$
10.0	2.857 ±0.039	4.100 ±0.029	0.010 ±0.006	3.988 ±0.030	0.176
8.0	2.906 ±0.035	4.050 ±0.027	0.009 ±0.005	3.990 ±0.026	0.107
6.0	2.936 ±0.030	4.013 ±0.025	0.009 ±0.004	3.984 ±0.022	0.068
4.0	2.944 ±0.024	3.979 ±0.023	0.011 ±0.004	3.965 ±0.019	0.070
3.0	2.930 ±0.024	3.950 ±0.022	0.015 ±0.004	3.938 ±0.020	0.108
2.0	2.875 ±0.030	3.866 ±0.024	0.030 ±0.005	3.852 ±0.027	0.238

Table 3: LS estimates(FIR filter, N/S ratio≈ 0.25)

ω_{dc} (ω_{ac})	$\hat{a}_1$ (3.0)	$\hat{a}_2$ (4.0)	$\hat{b}_1$ (0.0)	$\hat{b}_1$ (4.0)	$\Delta\|\theta\|$
12.0 (10.57)	2.363 ±0.072	4.219 ±0.063	0.041 ±0.0131	3.703 ±0.067	0.737
10.0 (8.62)	2.477 ±0.062	4.042 ±0.057	0.045 ±0.011	3.697 ±0.056	0.608
7.0 (5.69)	2.570 ±0.051	3.840 ±0.053	0.056 ±0.010	3.649 ±0.043	0.581
5.0 (3.78)	2.536 ±0.049	3.672 ±0.053	0.083 ±0.012	3.535 ±0.044	0.738
4.0 (3.01)	2.471 ±0.051	3.545 ±0.053	0.110 ±0.012	3.420 ±0.050	0.914

Table 4: BCLS estimates(FIR filter, N/S ratio≈ 0.25)

ω_{dc} (ω_{ac})	$\hat{a}_1$ (3.0)	$\hat{a}_2$ (4.0)	$\hat{b}_1$ (0.0)	$\hat{b}_1$ (4.0)	$\Delta\|\theta\|$
12.0 (10.57)	3.076 ±0.096	4.030 ±0.072	0.001 ±0.015	4.094 ±0.077	0.124
10.0 (8.62)	3.052 ±0.077	4.024 ±0.067	0.002 ±0.012	4.072 ±0.061	0.092
7.0 (5.69)	3.034 ±0.061	4.023 ±0.070	0.002 ±0.010	4.053 ±0.051	0.067
5.0 (3.78)	3.037 ±0.062	4.031 ±0.062	-0.000 ±0.013	4.057 ±0.061	0.075
4.0 (3.01)	3.039 ±0.071	4.035 ±0.066	-0.001 ±0.016	4.061 ±0.072	0.081

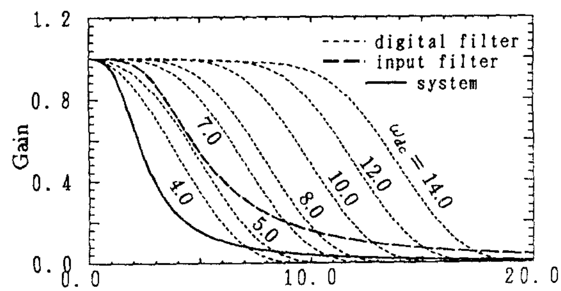

Figure 1. Frequency responses of the FIR filters

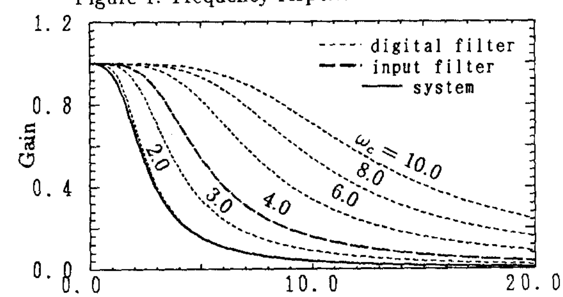

Figure 2. Frequency responses of the IIR filters