ON THE MANIPULABILITY MEASURE OF DUAL ARM

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Abstract

The concept of the manipulability measure of the robotic mechanism is extended to the dual arm holding a single object. This is a measure of manipulating ability of the dual arm forming a closed kinematic chain in positioning and orienting the object. Dual arm manipulability measure is defined and compared to the single arm manipulability measure, and some properties are investigated.

I. Introduction

Dual arm cooperative manipulation of objects can perform many tasks that would be impossible for a single robot, and it provides flexibility and versatility in task execution . Dual arm cooperation can perform parts assembly, without the mechanical aids such as fixtures or jigs, and it can handle heavy or voluminous objects , whose weight exceeds the load capacity of the individual arms, or whose size precludes the single arm grip of the objects .

Applications for cooperating robots may be grouped into two catergories. In the first catergory, all robot arms are in rigid contact with the object. The second category comprises those assembly tasks where each arm is holding a seperate object. In this case, unlike the first case, robot arms do not form a complete closed kinematic loop. The problem considered in this paper deals only with the first case.

Mainpulability measure has been used for joint configuration optimization of redundant arms. Reasearch on redundant arms has been focused on how to obtain optimal inverse kinematics or inverse Jacobians from the underspecified set of kinematic equations [1],[3] - [7]. Kinematic equations are compensated with additional constraints obtained from certain performance index criteria. Approaches to the solution can be classified according to the additional constraints used: i) Jacobian pseudo-inverse solution where minimum norm constraint of the joint velocity vector is used[2], ii) The modified Jacobian pseudo-inverse solution where Jacobian null space solution is added to the Jacobian pseudo-inverse solution to minimize certain performance index[4], iii) The extended Jacobian solution in which extra constraints are obtained from the conditions of the optimal joint configurations and added to the forward kinematics to form the extended Jacobian[5] .

Works on the use of the manipulability measure has been largely concentrated on a single robot, and less attention has been paid to the dual robot problem. Dual arm manipulability has been studied in literatures such as S. Lee[8], and Tao and Luh[9]. In [8], dual arm manipulability is defined as the approximate representation of the volume of intersection between two individual manipulability ellipsoids. Task oriented dual manipulability is also defined in [8], and used to optimize the dual redundant arm configuration by closely matching the dual arm manipulability ellipsoid with the desired motion manipulability ellipsoid. In [9], a different definition of the dual arm manipulability measure is given and used for the coordination of the dual arm.

This paper presents a new definition of the dual arm manipulability measures. We show that, in the case of two arms tightly holding a single object, the definition of the single arm manipulability measure can be directly extended to the dual arm case, and the manipulability ellipsoid and the force ellipsoid exhibit the same inverse relation as in the single arm case. This paper is organized as follows. Manipulability measure definition used for a single robot are reviewed in section II. A new definition of the dual arm manipulability ellipsoid is given in section III, and in section IV, the corresponding force ellipsoid is presented . In section V, some properties of the dual arm manipulability measures are investigated and some examples are presented for illustrations in section VI. Conclusions are drawn in section VII.

II. Review of Manipulability and Force Ellipsoids

The unit (hyper) sphere defined at the origin of the joint velocity space can be mapped to the (hyper) ellipsoid in the Cartesian velocity space by Jacobian transformation. This ellipsoid is called the manipulability ellipsoid[2][10]. The manipulability ellipsoid describes the characteristics of the feasible motion in the Cartesian space corresponding to all unit norm joint velocities.

The manipulability ellipsoid can be mathematically defined as follows. Assuming that an n degree of freedom arm is working in an m dimensional task space, where $m < n$, we have

$$\dot{x} = J(\theta)\,\dot{\theta} \qquad (1)$$

where $\dot{x}$ and $\dot{\theta}$ indicates the Cartesian and joint velocity vectors defined in the task space R^n and the joint space R^m. J represents the m x n Jacobian matrix.

The Jacobian J defines the mapping from R^n to R^m. The unit sphere in R^n described by

$$\|\dot{\theta}\|^2 = 1 \qquad (2)$$

can be mapped into an ellipsoid in R^m through J.

$$\begin{aligned}\|\dot{\theta}\|^2 &= \dot{\theta}^T\dot{\theta} \\ &= \dot{x}^T (J^+)^T J^+ \dot{x} \\ &= \dot{x}^T (J J^T)^+ \dot{x} \\ &= [\ \dot{x}^T (JJ^T)^{-1} \dot{x} = 1\ ] \qquad (3)\end{aligned}$$

where the superscript "+" indicates the pseudo-inverse of the matrix, $J^+ = J^T (JJ^T)^{-1}$, and (3) represents an ellipsoid equation in R^m. This ellipsoid is called the ***manipulability ellipsoid*** and the the volume of this single arm mainpulability ellipsoid V_{sm} is given by

$$V_{sm} = D\ W_s \qquad (4)$$

$$D = \pi^{m/2} / \Gamma ((m/2) + 1) \qquad (5)$$

$$W_s = [\det (JJ^T)]^{1/2} \qquad (6)$$

where $\Gamma(\cdot)$ is the gamma function. W_s is defined as the manipulability measure, and represents the volume of the ellipsoid except for the constant coefficient which depends only on the dimension m. The ***single arm manipulability measure*** (SMM) is written formally as below.

$$SMM = [\det (JJ^T)]^{1/2} \qquad (7)$$

Let f denote the force (and moment) vector applied by the end effector, and let τ denote the joint driving torque(and force). Then we have,

$$\tau = J(\theta)^T f \qquad (8)$$

The unit sphere defined by

$$\|\tau\|^2 = 1 \qquad (9)$$

can be mapped into an ellipsoid in R^m through J.

$$\begin{aligned}\|\tau\|^2 &= \tau^T\tau \\ &= [\ f^T (JJ^T) f = 1\ ] \qquad (10)\end{aligned}$$

The ellipsoid defined in (10) is called the ***force ellipsoid*** , and volume of this single arm force ellipsoid V_{sf} is given by

$$V_{sf} = D / W_s \qquad (11)$$

Let the eigen values of JJ^T be denoted by σ_1^2, $\sigma_2^2, \ldots, \sigma_m^2$ with $\sigma_1^2 \geq \sigma_2^2 \geq \cdots \sigma_m^2$ and the corresponding eigen vectors be denoted by $u_1, u_2, \ldots, u_m$. Then the measure W_s can be expressed as

$$W_s = \sigma_1\ \sigma_2 \ldots \sigma_m \qquad (12)$$

The manipulabiliy ellipsoid expressed by (3) is an ellipsoid with the principal axes $\sigma_1 u_1, \sigma_2 u_2, \ldots \sigma_m u_m$. The force ellipsoid expressed by (10) share the same principal axes vector as the manipulability ellipsoid with the principal axes $(1/\sigma_1)u_1$, $(1/\sigma_2)u_2$, ..., $(1/\sigma_m)u_m$. The length of the each principal axis of the force ellipsoid is in inverse proportion to the principal axes of the manipulability ellipsoid, and hence ,the volume V_{sf} is in inverse proportion to that of the manipulability ellipsoid.

III. Dual Arm Manipulability Ellipsoid

Assume that the end effectors of the two robots are grasping an object as shown in Fig. 1, and the object is held rigidly so that no relative motion is possible between the object and the grippers.

For convenience, subscripts 1 and 2 are used to indicate the two robots. Let

- x_i = end position and orientation vector of the robots in Cartesian space. (mx1)
- θ_i = joint position vector of the robots in joint space. (nx1)
- J_i = Jacobian transformation between the robot joint spaces and the robot end position coordinates. (mxn)

We assume that neither of the two robots is in singular position, and the Jacobians always have full ranks.

Since the object is held rigidly by the grippers, we consider the object as being an integral part of the grippers, and divide the object conceptually at the reference point of the object. The reference point of the object is then viewed as the end position of the two robots (Figure 2) .

If we let $\dot{x}_1$ and $\dot{x}_2$ be the end velocities of the robots , we have

$$\dot{x}_1 = J_1(\theta_1)\,\dot{\theta}_1 \qquad (13)$$

$$\dot{x}_2 = J_2(\theta_2)\,\dot{\theta}_2 \qquad (14)$$

Since we assume that the object is held rigidly by the grippers and there is no relative motion between the object and the grippers, we get following constraints.

$$\dot{x}_1 = \dot{x}_2 \qquad (15)$$

$$\ddot{x}_1 = \ddot{x}_2 \qquad (16)$$

Using this constraints, we develope the expression for the dual arm manipulability as follows. Let

$$\dot{p} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}, \quad \dot{\theta} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

$$J = \begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix} \qquad (17)$$

Then

$$\begin{aligned}\dot{p} &= \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix}\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \\ &= J\ \dot{\theta} \qquad (18)\end{aligned}$$

The unit sphere in R^{2n} described by

$$\|\dot{\theta}\|^2 = 1$$

can be transformed into R^{2m} through J

$$\|\dot{\theta}\|^2 = \dot{\theta}^T\dot{\theta}$$
$$= \dot{p}^T (J^+)^T J^+ \dot{p}$$
$$= \dot{p}^T (J J^T)^+ \dot{p}$$
$$= \dot{p}^T (J J^T)^{-1} \dot{p} \quad (19)$$
$$= [\dot{x}_1^T \ \dot{x}_2^T] \begin{bmatrix} J_1J_1^T & 0 \\ 0 & J_2J_2^T \end{bmatrix}^{-1} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}$$
$$= [\dot{x}_1^T \ \dot{x}_2^T] \begin{bmatrix} (J_1J_1^T)^{-1} & 0 \\ 0 & (J_2J_2^T)^{-1} \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}$$
$$= 1 \quad (20)$$

Since the end velocity of the two arm must be equal as in (15), letting $\dot{x} = \dot{x}_1 = \dot{x}_2$ and by (20),

$$[\dot{x}^T \ \dot{x}^T] \begin{bmatrix} (J_1J_1^T)^{-1} & 0 \\ 0 & (J_2J_2^T)^{-1} \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{x} \end{bmatrix} = 1$$

$$\dot{x}^T [(J_1J_1^T)^{-1} + (J_2J_2^T)^{-1}] \dot{x} = 1 \quad (21)$$

The above equation (21) describes the ***dual arm manipulability ellipsoid*** in R^m space, and its volume V_{dm} is given by

$$V_{dm} = D \ W_d \quad (22)$$
$$W_d = [\det((J_1J_1^T)^{-1} + (J_2J_2^T)^{-1})]^{-1/2} \quad (23)$$

where D is given by (5). This dual arm manipulability ellipsoid describes the characteristics of the feasible motion of the object that can be accomplished by all unit norm vector $\dot{\theta} = [\dot{\theta}_1 \ \theta_2]$. ***Dual arm manipulability measure*** (DMM) is defined by the volume of this ellipsoid scaled by the coefficient D in the same manner as in the single arm case.

$$DMM = [\det ((J_1J_1^T)^{-1} + (J_2J_2^T)^{-1})]^{-1/2} \quad (24)$$

IV. Dual Arm Force Ellipsoid

In this section , the equation of the dual arm force ellipsoid is developed for two non-redundant arms, i.e. m = n. As explained in section III, we take the view that the object is divided at the reference point and the object reference point is regarded as the end position of the two robots. Let f_1 and f_2 denote the Cartesian force (and moment) vectors applied at the object reference point by the robot 1 and robot 2, and let τ_1 and τ_2 denote the joint driving forces (and torques) of the robot 1 and robot 2. Let f be the resultant force (and moment) applied at the object reference point. Then we have

$$\tau_1 = J_1(\theta_1)^T \ f_1 \quad (25)$$
$$\tau_2 = J_2(\theta_2)^T \ f_2 \quad (26)$$

Also,

$$f = f_1 + f_2$$
$$= J_1^{-T} \tau_1 + J_2^{-T} \tau_2$$
$$= [J_1^{-T} \ J_2^{-T}] \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \quad (27)$$

Let

$$K = [J_1^{-T} \ J_2^{-T}] \quad (28)$$
$$\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \quad (29)$$

Then, (27) becomes

$$f = K \ \tau \quad (30)$$

and the unit sphere in R^{2n} defined by

$$\|\tau\|^2 = 1$$

can be transformed through K into an ellipsoid in R^n .

$$\|\tau\|^2 = \tau^T\tau$$
$$= f^T (K^+)^T \ K^+ f$$
$$= f^T (K K^T)^+ f$$
$$= [f^T (K K^T)^{-1} f = 1] \quad (31)$$

Hence,

$$f^T \left[[J_1^{-T} \ J_2^{-T}] \begin{bmatrix} J_1^{-1} \\ J_2^{-1} \end{bmatrix} \right]^{-1} f = 1 \quad (32)$$

$$f^T [J_1^{-T}J_1^{-1} + J_2^{-T}J_2^{-1}]^{-1} f = 1 \quad (33)$$
$$f^T [(J_1J_1^T)^{-1} + (J_2J_2^T)^{-1}]^{-1} f = 1 \quad (34)$$

The above equation (34) describes the ***dual arm force ellipsoid*** in R^n space, and its volume V_{df} is given by

$$V_{df} = D \ / \ W_d \quad (35)$$

, and this ellipsoid describes the characteristics of the force that can be applied at the object reference point by the two robots, corresponding to all unit norm vector of $\tau = [\tau_1 \ \tau_2]$.

V. Some Properties of Dual Arm Manipulability Ellipsoid and Force Ellipsoid.

Inspection of the equations (21) and (34) shows that the relationship between the dual arm manipulability ellipsoid and the dual arm force ellipsoid is the same as that between the single arm manipulability and the single arm force ellipsoid. Let the eigen vectors of $[(J_1J_1^T)^{-1}+(J_2J_2^T)^{-1}]$ be denoted by $v_1, v_2, \ldots, v_m$ and its corresponding eigen values be denoted by $\phi_1^2, \phi_2^2, \ldots, \phi_m^2$. The equation (34) represents an ellipsoid with principal axes $\phi_1 v_1, \phi_2 v_2, \ldots, \phi_m v_m$, and equation (21) represents an ellipsoid with principal axes $(1/\phi_1)^{-1}v_1$, $(1/\phi_2)^{-1}v_2$, ..., $(1/\phi_m)^{-1}v_m$. Hence the lengths of the principal axes of the two ellipsoids is in inverse proportion which is the relationship between the two ellipsoids in the single arm case. The point is also made clear in equations (4),(11) and (22),(35). This result is on the contrary to that in [9] where the above inverse relationship is denied.

The relationship between the single arm and the dual arm manipulability measures are given next.

Lemma 1. Let the inside of the single arm manipulability ellipsoid of arm 1 and arm 2 be denoted by E_{sm1} and E_{sm2} such that

$$E_{sm1} = \{ \dot{x} \mid \dot{x}^T (J_1J_1^T)^{-1} \dot{x} \le 1 \}$$
$$E_{sm2} = \{ \dot{x} \mid \dot{x}^T (J_2J_2^T)^{-1} \dot{x} \le 1 \}$$

and let the inside of the single arm force ellipsoids be denoted by E_{sf1} and E_{sf2} , s.t.

$$E_{sf1} = \{ f \mid f^T (J_1 J_1^T)^{-1} f \le 1 \}$$
$$E_{sf2} = \{ f \mid f^T (J_2 J_2^T)^{-1} f \le 1 \}$$

Also let E_{dm} and E_{df} denote the inside of the dual arm manipulability and force ellipsoids , s.t.

$$E_{dm} = \{ \dot{x} \mid \dot{x}^T[(J_1 J_1^T)^{-1} + (J_2 J_2^T)^{-1}] \dot{x} \le 1 \}$$
$$E_{df} = \{ f \mid f^T[(J_1 J_1^T)^{-1} + (J_2 J_2^T)^{-1}]^{-1} f \le 1 \}$$

Then,

$$E_{dm} \subset E_{sm1} , \quad E_{dm} \subset E_{sm2} \quad (36)$$

and

$$E_{df} \supset E_{sf1} , \quad E_{df} \supset E_{sf2} \quad (37)$$

(pf)

Let α be any vector that lies on the single arm manipulability ellipsoid of arm 1 such that

$$\alpha^T (J_1 J_1^T)^{-1} \alpha = 1 \quad (38)$$

Let β be any vector that lies on the dual arm manipulability ellipsoid,and let g be the function representing this ellipsoid such that

$$g(\beta) = \beta^T [(J_1 J_1^T)^{-1} + (J_2 J_2^T)^{-1}] \beta = 1$$

Then, since $J_1 J_1^T$ and $J_2 J_2^T$ are positive definite,

$$g(\alpha) = \alpha^T [(J_1 J_1^T)^{-1} + (J_2 J_2^T)^{-1}] \alpha \ge \alpha^T (J_1 J_1^T)^{-1} \alpha = 1 \quad (39)$$

by (38). Equation (39) holds for any vector α on the single arm manipulability ellipsoid of arm 1 and thus this ellipsoid includes the dual arm manipulability ellipsoid. This relation is the same for the manipulability ellipsoid of the arm 2 and (36) is proved.

Let γ be any vector on the single arm force ellipsoid of the arm 1 such that

$$\gamma^T (J_1 J_1^T) \gamma = 1 \quad (40)$$

Let δ be any vector on the dual arm force ellipsoid,and let h be the function representing this ellipsoid such that

$$h(\delta) = \delta^T [(J_1 J_1^T)^{-1} + (J_2 J_2^T)^{-1}]^{-1} \delta = 1$$

Then,

$$h(\gamma) = \gamma^T [(J_1 J_1^T)^{-1} + (J_2 J_2^T)^{-1}]^{-1} \gamma \le \gamma^T (J_1 J_1^T) \gamma = 1 \quad (41)$$

by (40). Equation (41) holds for any vector γ on the single arm force ellipsoid of arm 1 and thus this ellipsoid is included inside the dual arm force ellipsoid. This relation is the same for the force ellipsoid of the arm 2 and (37) is proved. ■

In other words, the dual arm manipulability ellipsoid is contained inside the single arm manipulability ellipsoids, and conversely , the single arm force ellipsoids are contained inside the dual arm force ellipsoid. As a consequence of this lemma, we get the following lemma 2.

Let the single arm manipulability measure of arm 1 and arm 2 be denoted by SMM1 and SMM2 respectively, and let the volume of the single arm force ellipsoid of the two arms be denoted by V_{sf1} and V_{sf2}, s.t.

$$SMM1 = [\det (J_1 J_1^T)]^{1/2} \quad (42)$$
$$SMM2 = [\det (J_2 J_2^T)]^{1/2} \quad (43)$$
$$V_{sf1} = D [\det (J_1 J_1^T)]^{1/2} \quad (44)$$
$$V_{sf2} = D [\det (J_2 J_2^T)]^{1/2} \quad (45)$$

Lemma 2 . The dual arm manipulability measure is less than the single arm manipulability measure of each arm, i.e.

$$DMM \le \min [SMM1 , SMM2] \quad (46)$$

and the volume of the dual arm force ellipsoid is greater than either volume of the single arm force ellipsoids, i.e.

$$V_{df} \ge \max [V_{sf1} , V_{sf2}] \quad (47)$$ ■

The lemma 2 indicates that the the manipulability measure defined in (24) can not be greater than either of the two individual arm's manipulability measure. This implies that if one or both of the two arms is in a singular position, then the single arm manipulability measure becomes zero, and the dual arm manipulability measure must be zero, and hence the dual arm is in a singular configuration. This result of course agrees with the human intuition.

The definition of the dual arm manipulability measure in this paper differs from the works in [8], and [9] in the following respects. All the definitions are based on the volume of the dual arm manipulability ellipsoid. However, the definitions in this paper and [9] are drawn from the exact equation of the manipulability ellipsoid obtained by transforming the unit sphere in $\dot{\theta} = [\dot{\theta}_1 \ \dot{\theta}_2]$ space, while the definition in [8] is drawn from the ellipsoid that approximately represents the intersection of the two arm's individual manipulability ellipsoid. Hence, the manipulability ellipsoid in this paper and in [9] are based on the subspace S_1 of R^{2n} , where $S_1 = \{(\dot{\theta}_1, \dot{\theta}_2) \mid \|\dot{\theta}_1\|^2 + \|\dot{\theta}_2\|^2 = 1\}$ while that in [8] is based on the subspace S_2 of R^{2n} where $S_2 = \{(\dot{\theta}_1, \dot{\theta}_2) \mid \|\dot{\theta}_1\|^2 = 1 \text{ and } \|\dot{\theta}_2\|^2 = 1 \}$. The second difference is that the manipulability in this work and in [8] is defined in the task space R^m which is natural since the motion and the force vectors of the object belong to R^m . However, The manipulability ellipsoid in [9] is defined in R^{2m} which includes redundant dimensions.

VI. Examples

To illustrate the properties of the definitions presented, we take a dual two link revolute arm as an examples. In figure 3, Two revolute arms forming a closed kinematic chain is shown. All link lengths are equal to one meter for convenience, and the base coordinate coincides with the base of the arm 1. The bases of the two arms are seperated by two meters in x direction, and located at (0,0) and (2,0). The arms are assumed to be holding a imaginary point mass object.

The manipulability measure of the two robots, SMM1 and SMM2 are shown in the figure 4a , 4b, and the dual arm manipulability measure (DMM) is shown in figure 4c. The manipulability measures reduces to zero at the edge of the common workspace, and the DMM is symmetric over the workspace as the two arms are identical and the link lengths are all equal.

The manipulability ellipsoids for the two single arms and the dual arm are presented in figure 5a, 5b and 5c. DMM is less than SMM1 and SMM2 as shown in lemma 2 and vanishes to zero at the edge of the common workspace.

The force ellipsoids for the two single arms and the dual arm are shown in figure 6a, 6b and 6c. The volume of the dual arm force ellipsoid is greater than that of the single arm's force ellipsoid as shown in lemma 2, and approaches to infinity at the edge of the workspace.

VII. Conclusions

The concept of the manipulability measure of the robotic mechanism is extended to the dual arm forming a closed kinematic chain. New definitions of manipulability and force ellipsoids are drawn by transforming the unit sphere in dual arm joint velocity space and dual arm joint torque space to Cartesian velocity space and Cartesian force space respectively. The manipulability measure is defined by the volume of the dual arm manipulability ellipsoid as in the single arm case. It is shown that the definitions presented in this paper for manipulability ellipsoid and the force ellipsoid share the same inversely proportional properties as that existing in the single arm case.

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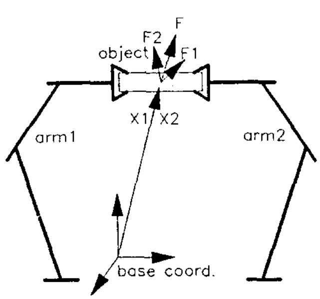


Figure 1. Two robot arms forming a closed kinematic chain

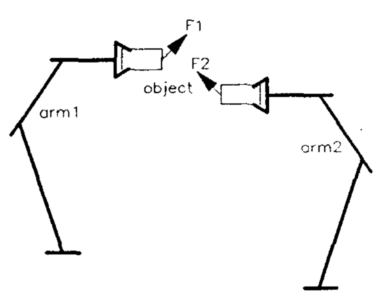


Figure 2. Two arms conceptually seperated at the object reference point

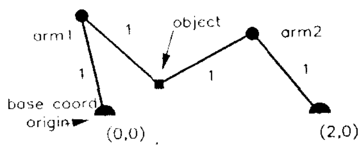


Figure 3. Dual two link revolute arm

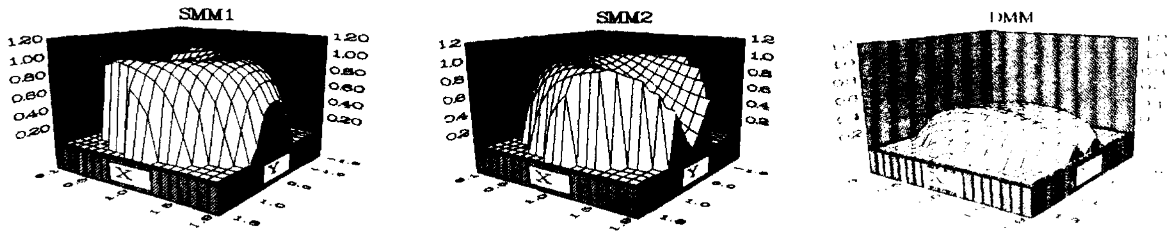
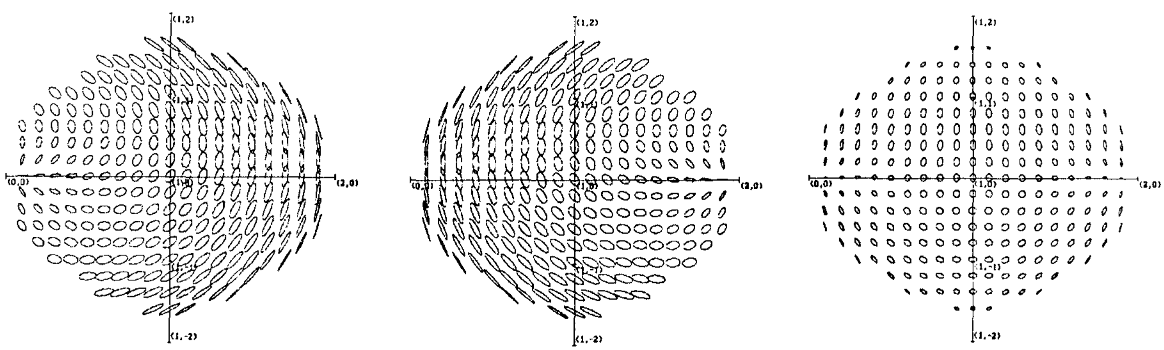


Figure 4. a) SMM 1, b) SMM 2 , c) DMM

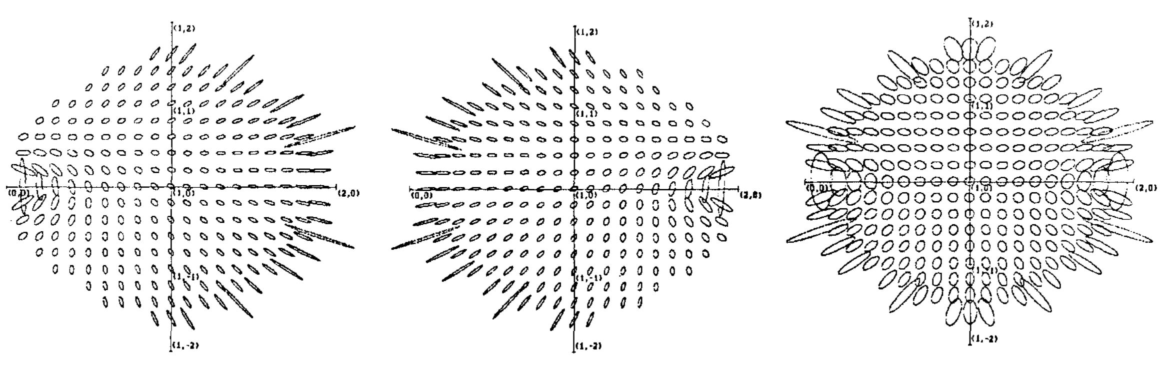


a) Manipulability ellipsoid for arm 1

b) Manipulability ellipsoid for arm 2

c) Dual arm manipulability ellipsoid

Figure 5.



a) Force ellipsoid for arm 1

b) Force ellipsoid for arm 2

c) Dual arm force ellipsoid

Figure 6.