A Generalized Disturbance Observer Theory and Application to Control of the Goldstar DD Robot Arm

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Abstract

There have been many approaches to solve the disturbance rejection problem in the control of LTI systems with state independent disturbances or possibly nonlinear state dependent disturbances. From the view point of each actuator, robot manipulators can be modeled as the second class of systems. With this model, M. Nakao et al.[1] introduced a decentralized control scheme based on interference estimation which is simple in its implementation and robust to the coupled dynamics and parameter variations. This paper systematically generalizes the control scheme to arbitrary finite dimensional LTI systems with disturbances. In doing so, we develope a disturbance observer theory for solving the disturbance rejection problem. We also present a discrete version of the theory with discussion of sampling and time-delay effects.

1. Introduction

The dynamic model of manipulators is non-linear and strongly coupled between each joint. The inertia matrix and the vector representing the Coriolis, centrifugal, and gravitational forces are varying as the position and the velocity of each joint vary. This makes the dynamic model even time-varying, which obviously brings about the difficulties in control. In the case of controlling direct drive manipulators, these difficulties even get worse because the nonlinearity and time-varying nature of the dynamic model directly affect the behavior of the actuators. There have been many approaches to handle these difficulties in manipulator control such as the computed torque methods[2], adaptive computed torque methods[3], and linearizing methods by nonlinear feedback[4]. Even though the methods are working well in some circumstances, the first methods may be sensitive to payload variation, and the second methods may require exciting signals to ensure robustness, and the third ones may also be vulnerable to parameter variation. In terms of implementation, the above methods require much centralized computation effort and correspondingly expensive underlying hardware.

In these aspects, the scheme proposed by M.Nakao et al.[1] seems to deserve much attention due to its decentralized and robust nature.

Following the idea of [1], this paper systematically developes a robust and decentralized control scheme based on a disturbance observer. In developing the theory, we consider general LTI systems as a reference model rather than a first order servo motor model in [1]. For the first step, we show that LTI systems with disturbances can be represented by a reference model with fictitious disturbances. The sufficient conditions for the fictitious disturbances to inherit the properties of the original disturbances are also derived. Based on this representation, we present a scheme for observing the fictitious disturbances which turns out to be uncausal(improper). Then, we design an approximated scheme to have a causal(proper) disturbance observer. We provide this estimated disturbance to the system input in order to neutralize the fictitious disturbance. We claim that the closed-loop system behave just like the reference model, which enables us to design outer loop control system based on the reference model. Finally, we apply the disturbance observer to the control of the GSIS direct drive arm. We present the simulation results.

2. Continuous Time Disturbance Observer

In this section we are dealing with a class of LTI systems described by the following block diagram.

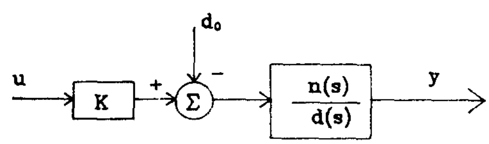


Figure 2.1 System Block Diagram

Here u, d_0, and y denote the system input, the disturbance to the system, and the system output, respectively. Moreover, n(s) and d(s) are the numerator and denominator of the system transfer function H(s), and K is the forward gain of the system.

For the first step, we show that the systems described by the Figure 2.1 can be transformed into the following nominal systems with the nominal forward gain K_n and nominal transfer function

$$H_m(s) = \frac{n_m(s)}{d_m(s)}$$

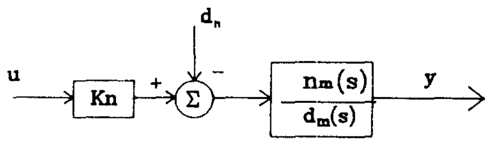


Figure 2.2 Nominal System Block Diagram

Lemma 2.1 : The systems represented by the Figure 2.1 can be transformed into the nominal system given by the Figure 2.2.

proof : From the Figure 2.1, we have the following :

$$d(s)y = n(s)Ku - n(s)d_o$$

Then, the following can be derived :

$$d_m(s)y + (d(s) - d_m(s))y = n_m(s)K_n + (n(s)K - n_m(s)K_n)u - n(s)d_o$$

Hence, it is clear that :

$$y = \frac{n_m(s)}{d_m(s)}\left[K_n u + \frac{d_m(s)-d(s)}{n_m(s)} y + \frac{n(s)K-n_m(s)K_m}{n_m(s)} u - \frac{n(s)}{n_m(s)} d_o \right]$$

By letting

$$d_n = \frac{n(s)}{n_m(s)} d_o - \frac{d_m(s)-d(s)}{n_m(s)} y - \frac{n(s)K-n_m(s)K_m}{n_m(s)} u$$

we have

$$y = \frac{n_m(s)}{d_m(s)}\left[K_n u - d_n \right]$$

which completes the proof.

It is desirable that the fictitious disturbance d_n inherit the properties of the original disturbance d_o. For this it is sufficient that

(S1) the original disturbance d_o is a smooth function,

(S2) the nominal transfer function $H_m(s)$ is minimum phase,

(S3) the system transfer function $H(s)$ is stable.

Now, we introduce a disturbance observer which is perfect but uncausal.

Lemma 2.2 : (Uncausal Disturbance Observer) Consider the following disturbance observer :

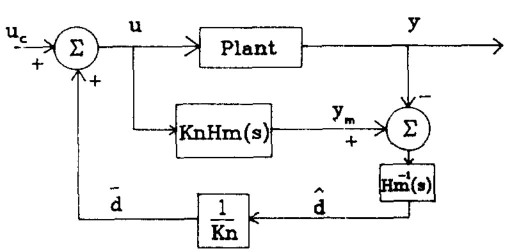


Figure 2.3 Uncasual Disturbance Observer

Then the closed-loop system becomes

$$y = K_n \frac{n_m(s)}{d_m(s)} u_c$$

which is the perfect nominal system model with no disturbance.

proof : From the Lemma 2.1, the plant can be represented by

$$y = H_m(s)\,[\,K_n u - d_n]$$

Then it can be derived that

$$e = y_m - y$$
$$= H_m(s)d_n$$

and

$$\bar{d} = \frac{1}{K_n} d_n$$

It follows that

$$y = H_m(s)\,[\,K_n(u_c + \bar{d}) - d_n\,]$$
$$= H_m(s)\left[K_n(u_c + \frac{1}{K_n} d_n) - d_n \right]$$
$$= H_m(s)K_n u_c$$

which completes the proof.

Note that if the conditions (S1) ~ (S3) are satisfied, the closed-loop system is internally stable. The stability follows from the observation that the bounded input u gives the bounded y and y_m, and the minimum phase $H_m(s)$ results in the stable $H_m(s)^{-1}$, i.e.,the bounded estimated disturbanc $\hat{d}$.Since the nominal transfer function $H_m(s)$ is causal(proper), its inverse $H_m(s)^{-1}$ is clearly uncausal(improper), which implies that the above disturbance observer is unimplementable.

For these reasons, we attempt to devise alternatives for uncausal disturbance observers which compensate the fictitious disturbarce d_n asymptotically with zero erros or with sufficiently small erros.

Theorem 2.3 : (Causal Disturbance Observer) We construct a causal disturbance observer with the following structure.

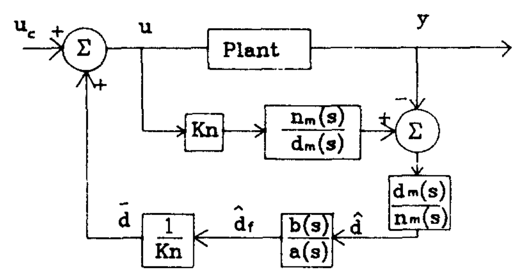


Figure 2.4 Casual Disturbance Observer

Here, the inserted filter is chosen such that

(T1). $\frac{d_m(s)b(s)}{n_m(s)a(s)}$ is causal,

(T2). $\lim_{s \to 0} \frac{b(s)}{a(s)} = 1$,

(T3). zeros of $a(s) \leq 0$.

Under these conditions and (S1) ~ (S3), we have the following results :

(1) the closed- loop system becomes stable in the sense of BIBS,

(2) the behavior of the closed-loop system becomes asymptotically

$$y = K_n \frac{n_m(s)}{d_m(s)} \left[u_c + \Delta d \right]$$

where $|\Delta d| \le \varepsilon$ for some sufficiently small $\varepsilon \ge 0$.

proof : Since the only difference between the uncausal and causal disturbance observer is the strongly stable filter b(s)/a(s),we can conclude the stability of the closed-loop system by using the same heuristics applied to the stability of the system with the uncausal disturbance observer.

With the same reasons in the proof of the Lemma 2.2, the fictitious signal $\hat{d}$ equals to d_n. From (T2) and (T3), it is clear that there exists a sufficiently small $\varepsilon \ge 0$ such that

$$|\hat{d}_f - d_n| \le \varepsilon$$

is satisfied asymptotically. It can be observed that

$$y = \frac{n_m(s)}{d_m(s)} \left[K_n(u_c + \frac{1}{K_n}\hat{d}_f) - d_n \right]$$
$$= \frac{n_m(s)}{d_m(s)} \left[K_n u_c + (\hat{d}_f - d_n) \right]$$

Taking $\Delta d = \hat{d}_f - d_n$ completes the proof.

If the characteristics of the original disturbance do is known a priori, the condition (S1)~(S3) ensures that d_n has the same characteristics.

In this case, we can apply the internal model principle to achieve asymptotic perfect disturbance compensation, i.e., $\varepsilon = 0$ in the above theorem. In doing so, we take the following steps to design the filter b(s)/a(s) giving perfect compensation. Here, we assume that $\Delta_d(s)$ denotes the characteristic equation for the original disturbance d_o.

(1). Take $\frac{1}{\Delta_o(s)}$ such that $H_m^{-1}(z)\frac{1}{\Delta_o(s)\Delta_d(s)}$ is causal(proper).

(2). Regarding $\frac{1}{\Delta_o(s)\Delta_d(s)}$ as a plant, design a pole-placement feedback compensator $g_c(s) = \frac{b_c(s)}{a_c(s)}$ such that the closed-loop system has a desired time constant.

(3). Take the filter

$$\frac{b(s)}{a(s)} = \frac{K_f \cdot b_c(s)}{a_c(s)\Delta_o(s)\Delta_d(s)+b_c(s)}$$

where K_f is chosen for giving

$$\lim_{s\to 0} \frac{b(s)}{a(s)} = 1 .$$

Then, from the internal model principle, we have

$$\lim_{t\to\infty} \hat{d}_f = \hat{d} = d_n$$

which, in turn, guarantees the asymptotic perfect disturbance compensation. However, it may be impractical to assume that the characteristics of d(s) be available.

Example 2.4 In this example, we consider a 2nd order plant with a sinusoidal disturbance and design the corresponding disturbance observer. The problem is given by

(1) system transfer function :

$$\frac{n(s)}{d(s)} = \frac{s+5}{s^2+3s+2}$$

(2) forward gain :

$$K = 0.4$$

(3) disturbance :

$$d_o = 0.3\sin(10\pi t)$$

The designed disturbance observer is reprented by

(1) nominal system :

$$K_n H_m(s) = K_n \frac{n_m(s)}{d_m(s)} = 10\cdot\frac{1}{s+10}$$

(2) filter :

$$\frac{a(s)}{b(s)} = \frac{1000}{s+1000}$$

The simulation is performed by the following block diagram :

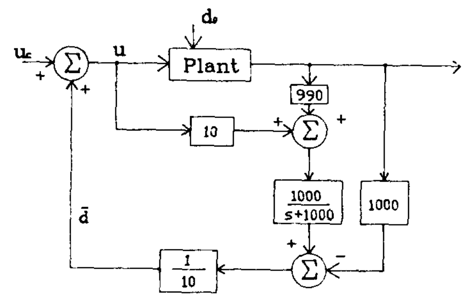


Figure 2.5 Simulation Block for Example 2.4

The simulation results are in the plot named Results of Example 2.4

3. Discrete Time Disturbance Observer

In the previous section we developed the disturbance observer theory in the continuous time domain and assumed that its implementation be based on an analog circuit. Since fast and inexpensive microprocessors become available, the controllers for industrial robots tend to be digitalized. Considering a microroprocessor based implementation, a discrete version of the disturbance observer theory is developed in this section.

There are two ways for designing a digital controller. First, we design a controller in the continuous time domain and then discretize it by using a transform method. Secondly, we discretize the plant and then use the discrete plant to design a digital controller in the discrete time domain. In this section, we take the second approach. In designing a digital disturbance observer, we take the same steps as the steps taken in the previous section.

We deal with a class of discrete LTI systems represented by

$$y_k = H(z)\cdot(u_k - d_k)$$
$$= \frac{n(z)}{d(z)}\cdot(u_k - d_k)$$

Here u_k and y_k denote the system input and output, and d_k denotes the system disturbance. The nominal system model is given by

$$y_k = K_n \cdot H_m(z)\cdot u_k$$
$$= K_n \cdot \frac{n_m(z)}{d_m(z)}\cdot u(k)$$

We choose a filter F(Z) such that
(D1) $F(z)\cdot H_m(z)^{-1}$ is causal,
(D2) $\lim_{z\to 1} F(z) = 1$,
(D3) $|$ poles of $F(z)$ $| \leq 1$.

As before, F(z) is used to construct a causal disturbance observer and may be chosen by applying the discrete internal model principle. With the nominal system model and the filter, we construct a causal disturbance observer as follows.

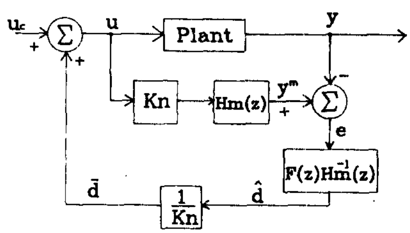


Figure 3.1 Digital Disturbance Observer

By applying this digital disturbance observer, we have the following results.
Theorem 3.1 (Digital Disturbance Observer)
We assume the following :
(C1) the system transfer function H(Z) is stable,
(C2) the nominal transfer function $H_m(Z)$ is minimum phase,
(C3) the plant disturbance is a smooth function
Under these conditions, we have the following results :
(1) the closed-loop system is BIBS stable,
(2) the behavior of the closed-loop system asymptotically converges to the behavior of the system represented by

$$y = K_n \cdot \frac{n_m(z)}{d_m(z)}\cdot\left[u_c + e_d \right]$$

where $| e_d | \leq \varepsilon$ for some sufficiently small $\varepsilon \geq 0$.

proof : Since the structure of the closed-loop system is same as in the continuous time case, we can use the same heuristics to prove BIBS stability of the colsed-loop system.

Using the Lemma 2.1, we have the following equivalent transform for the discrete plant :

$$y_k = \frac{n_m(z)}{d_m(z)}\cdot\left[K_n u_k - d_k^n \right]$$

From (C3), the fictitious disturbance d_k inherits the characteristics of the original disfurbance d_k. Then, the output error becomes

$$e_k = y_k^m - y_k$$
$$= H_m(z)\cdot d_k^n$$

From this, it follows that

$$H_m(z)^{-1}\cdot e_k = d_k^n$$

From (D1)~(D3), there exists a sufficiently small $\delta \geq 0$ such that

$$\lim_{k\to\infty} | \hat{d}_k - d_k^n | \leq \delta$$

Meanwhile, the response of the closed-loop system becomes

$$y_k = H_m(z)\cdot[\, K_n \cdot u_{ck} + (\, \bar{d}_k - d_k^n\,)\,]$$

Hence, taking $e_d = \hat{d} - d^n$ completes the proof.

In fact, the estimation $\bar{d}_k$ of the fictitious disturbance d_k^n can be applied to the system input at the time instant k+1. Thereby, there always exists contribution of this time-delay effect to the steady state error e_d. However, we claim that if the sampling time of the digital disturbance observer is sufficiently small, then the compensation error due to the time-delay be also sufficiently small.

Example 3.2 In this example, we consider the same plant as in Example 2.4 and design a digital disturbance observer. The digital disturbance observer is characterized by
(1) . nominal system model :

$$k_n \cdot H_m(z) = \frac{1-a}{z-a}$$

(2). filter :

$$F(z) = \frac{1-b}{z-b}$$

We take the sampling time as T = 1.6ms. The simulation is performed by the following block diagram :

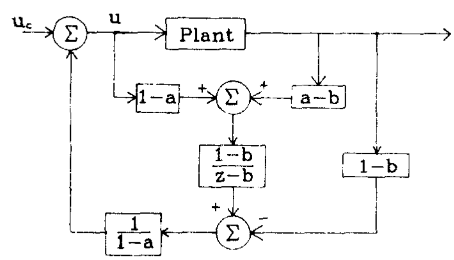


Figure 3.2 Simulation Block for Example 3.2

The simulation results are given in the plot named Results of Example 3.2

4. Application to the GSIS direct drive arm

The dynamic model of the GSIS direct drive arm can be derived as follows. Since the first two joints among the four joints are directly driven by the motors, we consider the dynamic model for those two joints.

$$T = H(\theta)\ddot{\theta} + V(\theta, \dot{\theta}) + G(\theta)$$

where T, $\theta \in R^{2\times1}$ denote the input torque and joint angle vector. The inertia matrix $H(\theta)\in R^{2\times2}$ is given by

$$H(\theta)=\begin{bmatrix} P_1 & -P_3\cos(\theta_1+\theta_2) \\ -P_3\cos(\theta_1+\theta_2) & P_2 \end{bmatrix}$$

and the vector $V(\theta, \dot{\theta}) \in R^{2\times1}$ representing the Coriolis and centrifugal forces is given by

$$V(\theta) = \begin{bmatrix} P_7\dot{\theta}_2^2 \sin(\theta_1+\theta_2) + P_4\dot{\theta}_1 \\ P_6\dot{\theta}_1^2 \sin(\theta_1+\theta_2) + P_5\dot{\theta}_2 \end{bmatrix}$$

and the vector $G(\theta) \in R^{2\times1}$ representing the gravitational force is given by

$$G(\theta) = 0$$

From this model, the first joint can be described by the following first order system with the state dependent noise :

$$\dot{\theta}_1 = \frac{1}{P_1 s}(T_1 - d_o)$$

$$= \frac{K}{P_1 s}\left(i_{a1} - \frac{d_o}{K}\right)$$

where $d_o = -P_3\ddot{\theta}_2\cos(\theta_1+\theta_2)$
$+ P_7\dot{\theta}_2^2\sin(\theta_1+\theta_2) + P_4\dot{\theta}_1$

With this model, we design the disturbance observer as follows :

$$K_n \cdot Hm(s) = K_n \cdot \frac{1}{J_n \cdot s + B_n}$$

$$F(s) = \frac{g}{s+g}$$

We apply this choice to the Figure 2.4. We also contruct a discrete disturbance observer for the first two joints and observe the speed response of the DD arm comparing with the response of the response of the reference model. The simulation results are given in the plot named Results for DD Arm Simulation. From the plot of the steady state error, it can be observed that there exists an osillation of 1.0×10^{-3}[rad/sec] order in the steady state. This may result in a few decades micron error of the end dffector, which is not good enough for the direct drive arm. In order to reduce the steady state error, we design a LQ speed controller regarding the reference model as the plant. Then, the simulation results of plot 5 and plot 6 show that the steady state error is reduced to the order of 1.0×10^{-5}[rad/sec] that corresponds to a few micron order error of the end effector.

5. Conclusion

In this paper, the idea of the interference estimation[1] is systematically generalized to set up a disturbance observer theory. It is shown that the disturbance observer results in the closed-loop system that behaves following a reference model with small errors. We then present a scheme for an asymptotic perfect disturbance observer. A discrete disturbance observer theory is also developed with the discussion of the time-delay effect. The disturbance observer is applied to the servo driver of the GSIS direct drive arm with the simulation results presented.

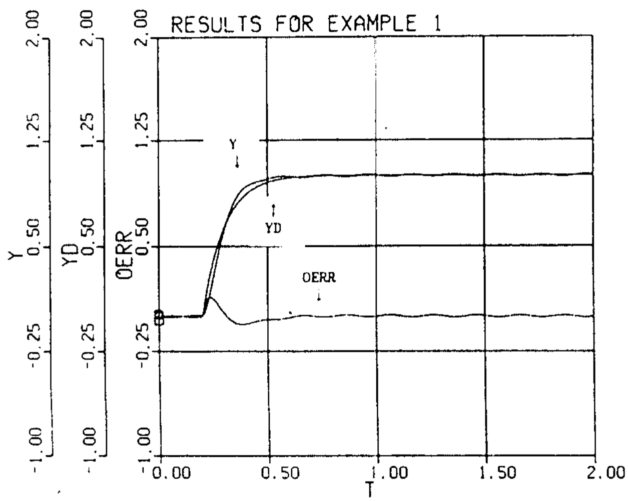
References

[1] M. Nakao et al., "A Robust Decentralized Joint Control Based on Interference Estimation," IEEE IECON '87 Conf. PP. 326 - 331.

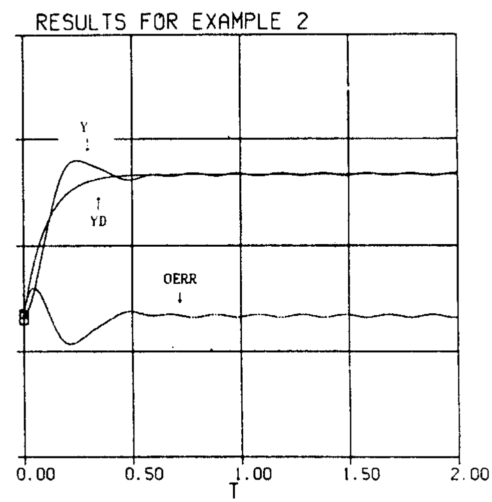
[2] J.J. Craig, Introduction to Robotics, Addison-Wesley, 1985.

[3] J.J. Craig, P. Hsu, and S. Sastry, "Adaptive Control of Mechanical Manipulators," IEEE Conf. on Robotics and Automation, San Francisco, Calif., 1986.

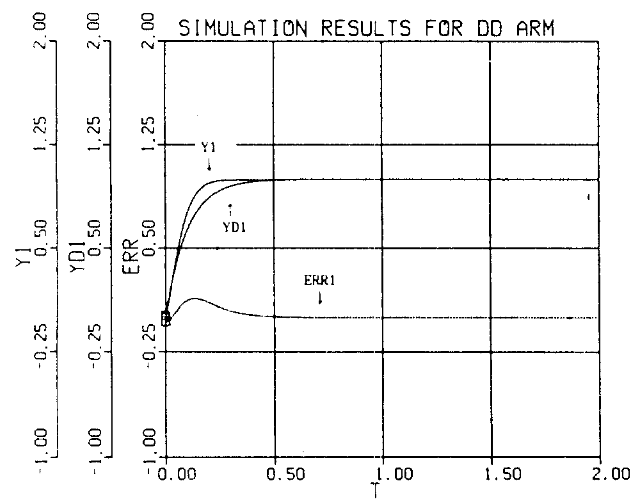
[4] E.G. Gilbert and I.J. Ha, "An Approach to Nonlinear Feedback Control with Applications to Robotics," IEEE Trans., Vol. SMC-14, No.6, 1984.



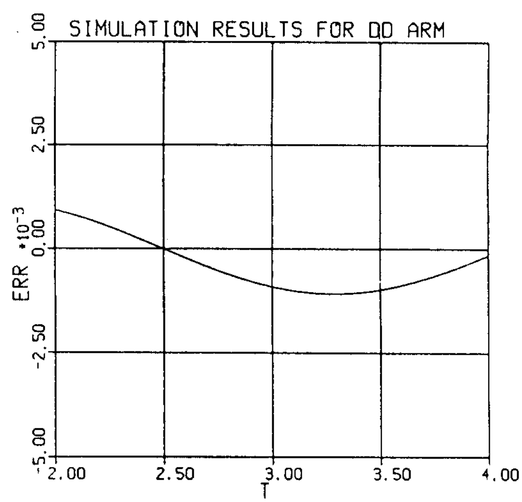
Plot 1 Results of Example 2.4



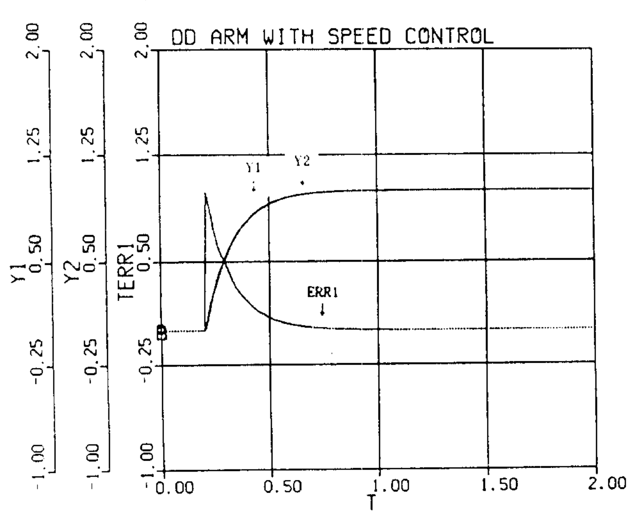
Plot 2 Results of Example 3.2



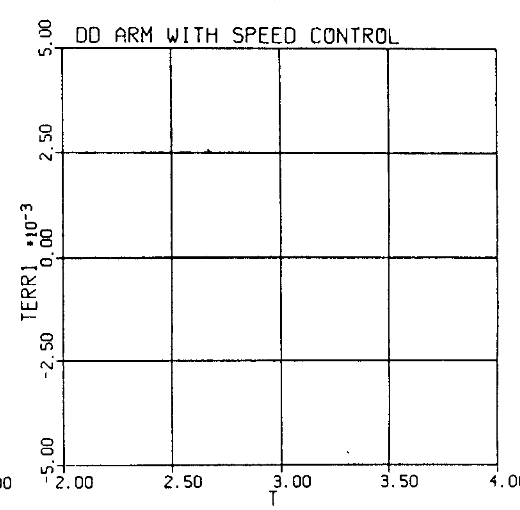
Plot 3 Results for DD Arm Simulation



Plot 4 Steady State Error



Plot 5 Disturbance Observer with a LQ controller



Plot 6 Steady State Error of Plot 5