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ROBUST SUBOPTIMAL REGULATOR DESIGN FOR LINEAR MULTIVARIABLE SYSTEM

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ABSTRACT

In this study, a design method to obtain a robust suboptimal regulator for linear multivariable system is presented. This new design method is based on the optimal regulator design method using eigen-structure assignment and it uses additional cost function which represent robustness of the closed loop system. When we design the regulator using pole assignment method for linear multivariable system we have extra degree-of-freedom after assigning desired eigenvalues of the closed loop system in determining the feedback gain. So we assign additional robust suboptimal eigenvectors so that we can obtain robust suboptimal regulator. In this study we also feedback the system output for more practical applications.

1. INTRODUCTION

It is well know that the LQR(Linear Quadratic Regulator) has a good robustness[1-3]. But not all LQR has a good robustness and some example give poor robustness[4]. So we must include some robustness factor in designing a LQR especially optimal LQR.

There are some method that try to include the robustness in designing optimal regulator [5-8]. But these methods use the assumption that the parameters of system are described by another variables and they differentiate the new variables. Or they assume that the sensitivity of the parameter can be described by some variables and then minimize this sensitivity function.

These methods have some weak points, first it is very strong assumption that we can describe the system parameter with some variables, for example, like aging effect of the components some variations can not be described by variables. Second, for the sensitivity function, it is also very difficult to formulate the sensitivity by known parameters. Third, though we assume that we made some formulation of sensitivity we must linearize the function at the operating point, so there will be linearization errors. Finally we introduce new variables, so the system order increase 2 times and there are computational difficulties.

In this study, we represent new design method of robust optimal regulator using eigen-structure assignment. This method use the desired system eigenvalues and eigenvectors which have robust and optimal property. This method does not require the formulation of sensitivity function or system parameter and is a more systematic and easy way to achieve desired regulator than the design method using $\{Q, R\}$ by try-and-error.

2. OPTIMAL REGULATOR DESIGN

In this section, we briefly describe the optimal regulator design method using eigen-structure. Regulator design by eigen-structure means we assign eigenvalues and eigenvectors to get the feedback gains.

Hence if we assign optimal eigenvector it becomes optimal regulator and if robust one then robust regulator.

The step of feedback gain determination by eigen-structure assignment for linear multivariable system is as follows[9].

i) determine the maximal rank matrix

$$N_i = \begin{bmatrix} N_{1i} \\ N_{2i} \end{bmatrix} \tag{2.1}$$

each N_i must satisfy eq(2.2)

$$(\lambda_i I - A, B)N_i = 0, \quad (i=1,\dots,n) \tag{2.2}$$

here

$$N_i \in C^{(n+m)*m}$$

n, m : *number of state, input*

A, B : *system matrix, input matrix*

λ_i : *desired eigenvalue* $(i=1,\dots,n)$

ii) determine parameter vector P_i

$$P_i = (P_{i1},\dots,P_{im})^T \tag{2.3}$$

here

$$V_0 = (N_{11}, \dots, N_{1n})$$

$$W_0 = (N_{21}, \dots, N_{2n})$$

$$P = \underset{i=1,n}{diag}(P_i)$$

calculate V , W

$$V = V_0 P, \quad W = W_0 P \tag{2.4}$$

iii) calculate feedback gain

$$F(P) = -W*V^{-1} \tag{2.5}$$

here P_i is arbitrary vector and if V is non-singular the F of eq(2.5) assign the desired eigenvalues. So first assign the desired eigenvalues and determine the eigenvector which minimize the object function determined by $\{Q, R\}$ weighting matrices.

3. ROBUST OPTIMAL REGULATOR DESIGN

There are many researches that assign the desired eigenvalues and ortho-normal eigenvectors[12-15] using the fact that when there is parameter perturbation the variations of eigenvalues are relatively small if the system eigenvectors are ortho-normal. In this section, we represent the robust optimal regulator design method using the previous optimal regulator design method[10] with the robust regulator design method [15]. By combining the performance index function of optimal and robust regulator we can continuously obtain the regulator from the optimal regulator to robust regulator. The target system can be described as below and we assume that the target system is completely controllable.

$$\dot{x}(t) = Ax(t) + Bu(t), \quad x(0) = X_0 \tag{3.1}$$

x : $n \times 1$ *state vector*

u : $m \times 1$ *input vector*

A, B : *system matrix(constant)*

We explain at 3.1 optimal regulator design and at 3.2 robust regulator design method. Finally, at 3.3 robust optimal regulator method.

3.1 Optimal Regulator Design

Assign the desired eigenvalues and determine feedback gain F which minimize the (3.2) performance index function[10].

$$J_1 = E\left\{ \int_{t=0}^{t=\infty} (x^T Q x + u^T R u)dt \right\} \tag{3.2}$$

here

Q : *symmetric positive semi-definite matrix*

R : *symmetric positive definite matrix*

X_0 : *random variable with* $E\{X_0 X_0^T\} = I$

$E\{\}$: *expectation function*

3.2 Robust Regulator Design

There are many design method which assign the ortho-normal eigenvector[12-15].

Cavin[12] use the fact that when the matrix made by the column eigenvectors is ortho-normal then it must be $V^T V = I$, so they define the performance index function to be minimized by.

$$J_2 = tr\ \{ (I - V^T V)^2 \} \tag{3.3}$$

here $tr\{\}$ means trace of a matrix.

Srinathkumar[13] used eq(3.4) as a performance index to be maximized.

$$J_2 = det(V) \tag{3.4}$$

here $det()$ means determinant of a matrix.

Kautsky[15] also used eq(3.5) as a performance index to be minimized.

$$J_2 = \kappa(V) \tag{3.5}$$

here $\kappa()$ means condition number of a matrix.

There are more performance index that represent the ortho-normality of a matrix and all these methods get the same result[15].

When the order of matrix $\{A\ , B\}$ is $(n \times n\ , n \times m)$, in the case $m=1$, the solution when exists, can be shown to be unique. In the case $1 < m < n$, various solution may exist, and, to determine a specific solution, additional condition must be supplied to eliminate the extra degree of freedom. In the case $m=n$, the pair $\{A\ , B\}$is always completely controllable, and any given closed-loop system matrix can always be achieved by feedback [15].

The relationship between the ortho-normality and transient response of $x(t)$ is as follows.

Property 1 :

Transient response of state $x(t)$ satisfy the (3.6) inequality.

$$\| x(t) \|_2 \leq \kappa(V) \cdot \max_j \{ e^{\lambda_j t} \} \cdot \| X_0 \|_2 \tag{3.6}$$

Proof : Theorem 5 of [15]

Property 2 :

When the feedback gain F assigns the stable eigenvalues $\{\lambda_j\}$ the perturbed system matrix $A + BF + \Delta$ is stable for all Δ that satisfy the (3.7) inequality.

$$\|\Delta\|_2 < \min_{s=i\omega} \sigma\ \{sI - (A+BF)\} \equiv \delta(F) \tag{3.7}$$

here $\sigma\{\}$ means singular value and the lower bound of $\delta(F)$ is

$$\delta(F) \geq \min_j \mathrm{Re}(-\lambda_j)/\kappa(V) \tag{3.8}$$

Proof : Theorem 6 of [15]

Property 1 means that the bound of transient response is smaller as $\kappa(V)$ is small. Property 2 means that as $\kappa(V)$ is smaller $\delta(F)$ becomes larger and that for the large perturbation the closed-loop system is stable.

3.3 Robust Optimal Regulator Design

Combining the performance index of 3.1 and 3.2 we can define new performance index as eq(3.9).

$$J = \alpha * J_1 + \beta * J_2 \tag{3.9}$$

here J_1 is performance index of optimality and J_2 is performance index of robustness.

This new robust optimal regulator can be also obtained by choosing a suitable pair of $\{Q , R\}$. But it is very difficult to select good $\{Q , R\}$ because there is no robustness index at $\{Q , R\}$. So the new method is more systematic and easy to get a desired regulator.

When $\alpha \neq 0$ and $B=0$ we get optimal regulator and if $\alpha=0$ and $\beta\neq 0$ then we get robust regulator if we choose suitable α , β we can get desired robust optimal regulator.

We showed 3 performance index of robustness at eq(3.3)-(3.5). Among them eq(3.3) has minimum value 0 so it is difficult to select suitable α , β combination and eq(3.4) is maximizing problem so we can not combine with minimizing problem. Finally eq(3.5) is a minimizing problem and has minimum value 1. So we can easily choose weighting factor α , β for our desired regulator.

3.4 Algorithm

Previous optimal regulator problem can be converted as follows [10].

$$J_1 = E\{X_0^T Q_1 X_0\} \tag{3.10}$$

and we can write again as

$$J_1 = tr\{Q_1\} \tag{3.11}$$

here Q_1 must satisfy eq(3.12)

$$(A+BF)^T Q_1 + Q_1(A+BF) + Q + F^T RF = 0 \tag{3.12}$$

Therefore the total performance index is

$$J = \alpha * tr(Q_1) + \beta * \kappa(V) \tag{3.13}$$

here Q_1 must satisfy eq(3.14)

$$(A+BF)^T Q_1 + Q_1(A+BF) + Q + F^T RF = 0 \tag{3.14}$$

when we try to get a solution parameter vector P we can use 2 type method. First, if we can get gradient of the performance index we can get the solution very fast using some gradient-based minimizing algorithm but it is sometimes difficult to get the gradient. Second, we can use the algorithm that does not need gradient. This method is rather slow but we do not have to calculate difficult gradient. Most famous method of the latter case is multivariable simplex method[17].

In this study, we do not have to calculate the parameter vector fast because we design the regulator in off-line. So we used commercial software package[18] which use multivariable simplex method.

4. OUTPUT FEEDBACK CASE

When we cat not access the all system states directly we must feedback outputs. In that case case we can use the intuitive concept that :

$$\min \| F^* - KC \| \tag{4.1}$$

where

F^* : *desired feedback gain*

K : *output feedback gain*

C : *output matrix*

Kosut[19] proposed 2 methods to solve eq(4.1). First one is so-called "minimum excitation method". We can get feedback gain as follows $F^o = KC$.

$$F^o = F^* PC^T (CPC^T)^{-1} C \tag{4.2}$$

where P satisfies eq(4.3).

$$(A+BF^*)P + P(A+BF^*)^T + I = 0 \tag{4.3}$$

F^* : *desired feedback gain*

Second one is so-called "minimum norm method:. We can get feedback gain as follows.

$$F^o = F^* C^T (CC^T)^{-1} C \tag{4.4}$$

These two methods are very simple but they do not guarante the stability of the closed-loop system. So we must be careful when we use one of these method. And we must have enough simulations before apply to a system.

The target feedback F^* can be robust one or optimal one or robust optimal one.

5. CONCLUSION

In this study, we proposed the optimal regulator design method for linear multivariable system using eigen-structure assignment. This method has the merit that is does not require the formulation of parameter or sensitivity function using another variables and it can obtain the continuous response from the optimal one to robust one. This method uses the full degree-of-freedom of eigen-structure assigning method. In output feedback, we introduced 2 type parameter-based methods.

For the further study, we are trying to find a algorithm which uses gradient to get the solution faster. And another output feedback regulator.

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