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콘크리트 포장구조의 평면응력 해석을 위한 컴퓨터모델

(A Computer Model for the Planar Effects of Concrete Pavements with Skewed Joints.)

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Abstract

The planar effects on the concrete pavements is mainly due to the concrete shrinkage, subgrade friction, and thermal expansion or contraction. A complete understanding of analytical behavior of concrete pavement requires the development of computer model, stiffness matrix and equivalent nodal load matrices due to the effects mentioned above. A computer program, INPLANE II, has been written to evaluate the planar effects on concrete pavements. The planar effects determine to what degree the joint open and also help in determining factors which affect the joint stiffnesses and structural behavior of concrete pavements.

1. Introduction

Despite the development of structural analysis programs for the concrete highway pavement over Westergaard's equations, the planar effects of concrete shrinkage, subgrade friction, and thermal expansion or contraction are largely ignored. Although they are generally not as high as the stresses induced by gravity loads, they may become significant depending on the initial condition of joint opening. The planar effects will also determine to what degree the joint will open, primarily due to concrete shrinkage strain. This will have an effect on the joint load transfer mechanism, or stiffness.

2. Parallelogram In-Plane Stress Element.

The element used in this analysis is the four node, parallelogram, plane-stress, membrane element, which becomes a rectangular element when the skew angle equals 90 o.

This parallelogram element has two degrees of freedom (DOF) at each node for a total of eight DOF per each element as shown in Figure 1.

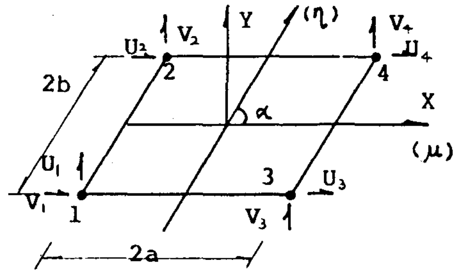

Fig.1 Degree of Freedom for Parallelogram In-plane Element.

Displacement function in the -1 to 1 normalized coordinate system is assumed to be:

$$u = c1 + c2R + c3S + c4RS$$
$$v = c5 + c6R + c7S + c8RS$$

Shape functions which relate nodal displacements(q) to generic element displacement(U) are given as follows.

$$f1 = 1/4\ (\ 1 - R\ )\ (\ 1 - S\ )$$
$$f2 = 1/4\ (\ 1 - R\ )\ (\ 1 + S\ )$$
$$f3 = 1/4\ (\ 1 + R\ )\ (\ 1 - S\ )$$
$$f4 = 1/4\ (\ 1 + R\ )\ (\ 1 + S\ )$$

Having the necessary operators for stresses and strains, we can evaluate the element stiffness matrix.

$$K = \int_v \mathbb{B}T\ \mathbb{E}\ \mathbb{B}\ dV$$
$$= t\ ab\ \sin(\alpha) \int_{-1}^{1} \int_{-1}^{1} \mathbb{B}T\ \mathbb{E}\ \mathbb{B}\ dRdS$$

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3. Boundary Condition

The pavement is modeled by using a three slab system with two intermediate joints. To provide stability to the system, it is necessary to restrain some degrees of freedom. Since the nodes along the slab's centerline will have the smallest displacements perpendicular to those centerlines, (equal to zero for a symmetric loading), rollers were placed along the lateral (Y-direction) centerlines of the slabs, and pins at the centers as shown in Figure 2.

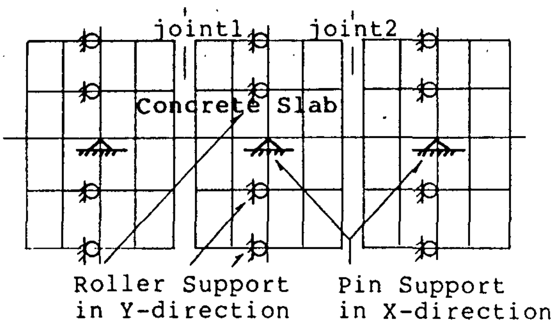


Fig. 2 Boundary Conditions of Finite Element Model for 3 Slab

These boundary conditions will prevent the nodes along the lateral centerlines from displacing perpendicular to those centerlines and provide structural stability.

Dividing the slabs along their axes of symmetry, or centerlines, also helps in determining which direction the friction forces will act. Fig. 3 shows which directions the friction forces act in response to the concrete shrinkage and negative thermal gradient effects. The directions of the friction forces at a node depend on which quadrant that node is in. The directions of the friction forces shown in Figure 3 are reversed for thermal expansion.

4. Equivalent Nodal Loads

4.1 Concrete Shrinkage

The equivalent nodal load, F_{shr}, due to uniform concrete shrinkage strain ε_{shr} is calculated by performing the following integrations.

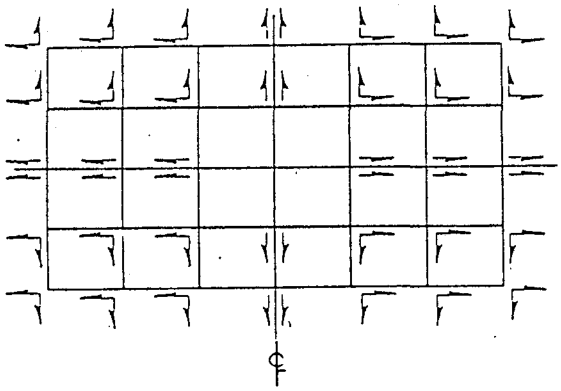


Fig. 3 Direction of Nodal Friction Forces.

Setting an initial strain $\{\varepsilon_o\}$ equal to ε_{shr},

$$[F_{shr}] = -t\, ab\, \sin(\alpha)\cdot \varepsilon_{shr} \cdot \int_R\int_S [B^T]\cdot[C] \begin{bmatrix}1\\1\end{bmatrix} dR\, dS.$$

where [B] is the 8x3 matrix containing the derivatives of the shape functions.

Using Gauss Quadrature method, the integration is performed by inserting the correct sampling points into the function to be integrated, multiplying the results by the weighing factors, and then summing the results.

4.2 Frictional Forces

Since shrinkage and thermal effects involve no actual applied loads on a highway pavement system, stresses induced in the pavement will be those due to closing of the pavement joints and subgrade friction. However, a careful investigation should be made to see if there is a void beneath the element, then the forces due to friction are set to zero.

The equivalent nodal load due to friction is given:

$$[F_{fr}] = \phi\cdot K_o \int_{area} [f]^T [f_b]\cdot[q_b]\, dA.$$

$$\int_A dA = \int_x\int_Y dx\, dY = |J|\cdot\int_R\int_S dR\, dS$$

where [f] is a shape function of parallelogram plane stress element.

[q] is the vector of nodal displacement

K0 = subgrade stiffness.

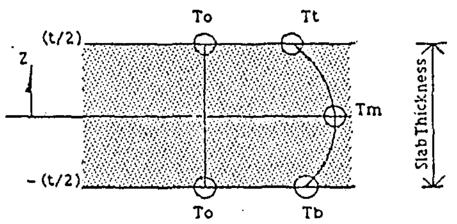
{fb} = 1x12 array of shape functions of plate bending element.
{qb} = 12x1 array of nodal displacements due to plate bending only.
= Skew angle

Therefore,

$$[F_{fr}] = \phi\, ab\, \sin(\alpha)\cdot K_o \cdot \int_R \int_S [f]^T \cdot [f_b]\; dR\, dS \cdot [q_b]$$

4.3 Temperature Loading

The thermal gradient in the slab is assumed to be a second degree polynomial defined by three measured temperatures across the slab thickness, one at the surface (Tt) , one at the center (Tm), and the third at the bottom of the slab (Tb). These three temperature readings and a reference temperature (T0) are then fitted to the polynomial equation F(z) = A + Bz + Cz2. T0 is the assumed temperature at which there are no thermal strains in the slab as shown in the Figure 4.



$$F(Z) = A + BZ + CZ^2$$

where $A = Tm$
$B = 1/t\,(Tb - Tt)$
$C = 2/t^2(Tt + Tb - 2Tm)$

FIG 4. ASSUMED DISTRIBUTION OF THERMAL GRADIENT ACROSS THE THICKNESS.

The equivalent nodal loading due to thermal effects can be computed by equating an initial strain ($\varepsilon 0$) equal to εtemp,

$$[\,Ft\,] = \alpha \cdot \int [B]T\; [E] \begin{bmatrix} F(Z) \\ F(Z) \\ 0 \end{bmatrix} dx\,dy\,dz$$

The thermal gradient function F(Z) may be integrated over the slab thickness and the temperat -ure readings substituted for the function constants to obtain

$$T = \int_{-t/2}^{t/2} F(z)\, dz$$
$$= t\,[\,T_m - T_o + (T_t + T_b - 2T_m)/6\,]$$

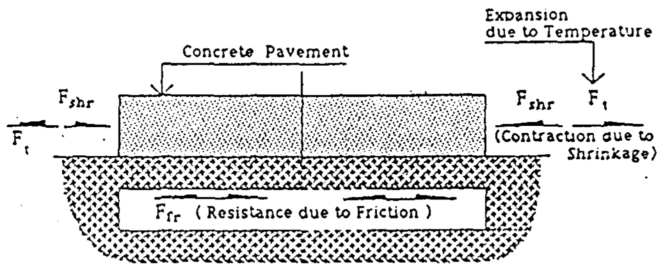
and for the normalized coordinate system,

$$[F_t] = \alpha \cdot ab\, \sin(A) \cdot \iint [B^T][E] \begin{bmatrix} T \\ T \\ 0 \end{bmatrix} dR\, dS$$

After computing equivalent nodal forces due to thermal gradients, compare the sum of shrinkage and thermal induced stresses with friction forces.

If an absolute magnitude of Fshr , Fshr , is less than Ffr , check to see if the sum of Ftemp and Fshr is greater than Ffr . If so, total applied load will be the sum of [Fshr], [Ftemp] and [Ffr]. Otherwise the total applied load will be zero as shown in Figure 5.

After the nodal displacements due to thermal effects are determined, they are added to those obtained for the concrete shrinkage.



If $|F_{shr}| > |F_{fr}|$,
Then Total Active Load (PLOAD) Due to Thermal Gradients : PLOAD = F_t

If $|F_{shr}| < |F_{fr}|$,
Then Let TV = $F_{shr} + F_t$
If $|TV| > |F_{fr}|$,
Then PLOAD = $F_t + F_{shr} + F_{fr}$
Else, PLOAD = 0.0

Fig. 5 Shrinkage, Friction and Thermal Forces

5. Element Stresses

The main routine then determines the nodal stresses. This is done by extracting the local displacements for each element from the array of structural displacements. The most accurate estimation of the element strains are those at the Gauss points. The coordinates of the Gauss points are inserted in to the [B] function matrix and the multiplication

$\{ \varepsilon t \} = [B] \{ q \}$

is carried out.

The initial strains due to shrinkage and temperature are subtracted out and then stresses at the Gauss points are obtained.

A least squares fit is then performed on the four sets of stresses to obtain the linear interpolation equations for stress. The nodal stresses for each element are determined, an for those nodes that are common to more than one element, the average stresses are calculated.

6. Conclusions

Boundary conditions for the planar effects of concrete pavement modeling have been studied with the development of the stiffness matrix of parallelogram in-plane stress element and the equivalent nodal load matrices due to friction on the subgrade, concrete shrinkage and thermal expansion or contraction. The derivations are quite good even for a test problem with a skewed angle up to 30 degree. To obtain better accuracy of the results, higher order function and larger meshes are also of interest.

A preliminary analysis shows that planar effects on concrete pavement are important. Although they are generally not as high as the stresses induced by gravity loads, they may become significant depending on the initial condition of joint opening. The planar effects will also determine to what degree the joint will open, primarily due to concrete shrinkage strain. This

will have an effect on the joint load transfer mechanism, or stiffness.

7. References

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