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Equivalent network analysis of optical strip waveguides

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ABSTRACT

Taking the normal mode transmission direction perpendicular to the substrate, an alternative equivalent network formulation is developed for the analysis of optical strip waveguides. All kinds of mode couplings between the normal modes are included in this formulation. Compared to the previous equivalent network formulations, the calculations are simplified especially when the thickness of the uniform slab is thin enough such that no well guided modes are available along the uniform slab waveguide.

In integrated optics, strip waveguides[1] comprising the strip loaded waveguide[2] and the rib waveguide[3] are easy to be fabricated and their characteristics are stable under fabrication errors that are critical to rectangular dielectric waveguides. Several approximate methods[1,2] and numerical methods[4,5] have been reported for their analysis. Recently, equivalent network approaches[6-13] are published to analyze such structures more simply and accurately. These approaches regard the whole structure of a strip waveguide as a junction of three asymmetric slab waveguide regions along the normal mode transmission direction chosen parallel to the substrate surface. The mode matching technique is used at the junction planes which separate the slab waveguide regions. When there are no well guided modes in the uniform slab waveguide below the loading strip, the continuous mode contributions are important and the calculations are complicated.

In this paper, choosing the transmission direction normal to the substrate surface, an alternative equivalent network formulation is developed for the analysis of optical strip waveguides. Our choice of transmission direction simplifies the calculation since the simplest form of the continuous modes, i.e. plane waves, is available in the upper and the lower regions to the central region. The central region continuous modes may be easily included in the analysis since the symmetric slab waveguide admits simple form of mode functions and has no substrate radiation modes. This formulation has been well used lately to analyze the rectangular dielectric waveguides and their arrays.[14] As is apparent, our approach holds its usefulness even when there are no discrete modes in the uniform slab waveguide and when the height of the loading strip becomes large.

The typical structure of the optical strip waveguides is shown in Fig. 1 where n_a, n_c, n_f, and n_s are the refractive indices corresponding to the permittivities, ϵ_a, ϵ_c, ϵ_f, and ϵ_s, respectively. The electric permittivity, magnetic permeability, and wavenumber in free space are denoted as ϵ_o, μ_o, and k_o, respectively. We assume implicit t (time) dependence of the fields to be $\exp(j\omega t)$, where ω is the angular frequency.

Representing ξ as a propagation constant for a guided mode of the strip waveguide in Fig. 1, we assume that all the fields have $\exp(-j\xi z)$ dependence. There are two types of guided modes in this waveguide, $E^x_{\mu\nu}$ and $E^y_{\mu\nu}$.[1] The subscripts μ and ν denote the number of the electric (or magnetic) field extremum in x and y directions, respectively. The superscripts x and y denote the principal direction of the electric field.

To provide an appropriate boundary condition for the fields inside the upper region, however, the tangential fields at the upper junction plane can not be given arbitrarily but should be compatible with each other.[15] Also, the tangential fields at the lower junction plane should be compatible with each other to provide an appropriate boundary condition for the fields in the lower region.

In order to concentrate on the upper and lower region, we use the Equivalence principle[15] and replace the central region with a perfect magnetic conductor. Both the upper and the lower regions have uniform unbounded cross sections transverse to the y direction and the permittivity may be written as $\epsilon(y)$. For these regions, we use the network formulation presented in Chapter 2 of Ref. 16 to find the relations between the tangential fields at the junction planes.

Network representations for these regions are shown in Fig. 2. Perfect magnetic conductor is defined to have zero tangential magnetic fields at its surfaces and hence the transmission lines are terminated open at the surfaces of the magnetic conductor. To make the values of the fields outside the magnetic conductor unchanged, we introduce electric surface currents on and under the magnetic conductor with infinitesimal gaps from the magnetic conductor. Although there should exist magnetic currents also, their effects are cancelled by the opposite images inside the magnetic conductor.

As a result, we obtain the following relations between the tangential fields at $y = l$ and $y = 0$:

$$E_x(x,l) = \frac{-1}{2\pi}\int_{-\infty}^{\infty} d\alpha \frac{\exp(j\alpha x)}{\alpha^2+\xi^2}[\alpha\xi\{Z^E_\uparrow(\alpha) - Z^H_\uparrow(\alpha)\}\mathcal{H}_x(\alpha,l) + \{\alpha^2 Z^E_\uparrow(\alpha) + \xi^2 Z^H_\uparrow(\alpha)\}\mathcal{H}_z(\alpha,l)], \quad (1a)$$

$$E_z(x,l) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\alpha \frac{\exp(j\alpha x)}{\alpha^2+\xi^2}[\{\xi^2 Z^E_\uparrow(\alpha) + \alpha^2 Z^H_\uparrow(\alpha)\}\mathcal{H}_x(\alpha,l) + \alpha\xi\{Z^E_\uparrow(\alpha) - Z^H_\uparrow(\alpha)\}\mathcal{H}_z(\alpha,l)], \quad (1b)$$

$$E_x(x,0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\alpha \frac{\exp(j\alpha x)}{\alpha^2+\xi^2}[\alpha\xi\{Z^E_\downarrow(\alpha) - Z^H_\downarrow(\alpha)\}\mathcal{H}_x(\alpha,0) + \{\alpha^2 Z^E_\downarrow(\alpha) + \xi^2 Z^H_\downarrow(\alpha)\}\mathcal{H}_z(\alpha,0)], \quad (1c)$$

$$E_z(x,0) = \frac{-1}{2\pi}\int_{-\infty}^{\infty} d\alpha \frac{\exp(j\alpha x)}{\alpha^2+\xi^2}[\{\xi^2 Z^E_\downarrow(\alpha) + \alpha^2 Z^H_\downarrow(\alpha)\}\mathcal{H}_x(\alpha,0) + \alpha\xi\{Z^E_\downarrow(\alpha) - Z^H_\downarrow(\alpha)\}\mathcal{H}_z(\alpha,0)]. \quad (1d)$$

where the impedance seen looking up from the current source at $y = l^+$ is denoted as $Z_\uparrow$ and the impedance seen looking down from the current source at $y = 0^-$ is denoted as $Z_\downarrow$. Note that $\mathcal{H}_x(\alpha, y)$ and $\mathcal{H}_z(\alpha, y)$ are the Fourier transform of the tangential magnetic fields $H_x(x, y)$ and $H_z(x, y)$ for the x axis, respectively.

In the central region where the permittivity may be denoted as $\epsilon(x)$, the tangential fields are expanded for $x \neq \pm W/2$ in terms of the slab waveguide normal modes that can be separated into obliquely propagating TE (H, $E_x \equiv 0$) and TM (E, $H_x \equiv 0$) modes with respect to the x direction[17] as follows:[7,14]

$$E_x(x,y) = \sum_n V_n''(y)\phi_n''(x),$$
$$H_x(x,y) = \sum_n I_n'(y)\phi_n'(x),$$
$$E_z(x,y) = \sum_n V_n'(y)\phi_n'(x) - \sum_n \frac{j\xi}{\beta_n''^2} V_n''(y)\frac{d\phi_n''(x)}{dx},$$
$$H_z(x,y) = -\sum_n \frac{j\xi}{\beta_n'^2} I_n'(y)\frac{d\phi_n'(x)}{dx} - \sum_n \epsilon_r(x) I_n''(y)\phi_n''(x), \quad (2)$$

where $\epsilon_r(x) = \epsilon(x)/\epsilon_o$ is the relative electric permittivity in the central region and

$$V'(y) = \frac{j\omega\mu_o}{\beta'^2}\frac{dI'(y)}{dy}, \quad I''(y) = \frac{j\omega\epsilon_o}{\beta''^2}\frac{dV''(y)}{dy}. \quad (3)$$

The summations in Eqs. (2) may include continuous modes when they are discretized in a proper way.[6,7,11] The contents of the following set { ϕ', I', V', β' } and { ϕ'', I'', V'', β'' } represent the TE and TM mode mode function, modal current, modal voltage, and propagation constant, respectively.

From Eqs. (3), we may find the following relations:

$$V'(0) = \frac{j\varsigma'\omega\mu_o}{\beta'^2\sin(\varsigma' l)}\{I'(l) - \cos(\varsigma' l)I'(0)\},$$
$$V'(l) = \frac{j\varsigma'\omega\mu_o}{\beta'^2\sin(\varsigma' l)}\{\cos(\varsigma' l)I'(l) - I'(0)\},$$
$$I''(0) = \frac{j\varsigma''\omega\epsilon_o}{\beta''^2\sin(\varsigma'' l)}\{V''(l) - \cos(\varsigma'' l)V''(0)\},$$
$$I''(l) = \frac{j\varsigma''\omega\epsilon_o}{\beta''^2\sin(\varsigma'' l)}\{\cos(\varsigma'' l)V''(l) - V''(0)\}. \quad (4)$$

We insert Eqs. (2) into Eqs. (1) and use the following orthogonal properties of the real mode functions for $m \neq n$:[7,14]

$$\int_{-\infty}^{\infty} dx\ \phi_m'(x)\phi_n'(x) = 0, \quad \int_{-\infty}^{\infty} dx\ \phi_m''(x)\epsilon_r(x)\phi_n''(x) = 0. \quad (5)$$

Then, we obtain the following linear coupled equations for arbitrary m-th TE mode modal current and n-th TM mode modal voltage at $y = l$ and $y = 0$:

$$V_m'(0) = \sum_p P_{mp}(\xi)I_p'(0) + \sum_q Q_{mq}(\xi)I_q''(0) + \sum_q R_{mq}(\xi)V_q''(0),$$
$$V_n''(0) = \sum_p U_{np}(\xi)I_p'(0) + \sum_q V_{nq}(\xi)I_q''(0),$$
$$V_m'(l) = \sum_p \tilde{P}_{mp}(\xi)I_p'(l) + \sum_q \tilde{Q}_{mq}(\xi)I_q''(l) + \sum_q \tilde{R}_{mq}(\xi)V_q''(l),$$
$$V_n''(l) = \sum_p \tilde{U}_{np}(\xi)I_p'(l) + \sum_q \tilde{V}_{nq}(\xi)I_q''(l), \quad (6)$$

where P's, Q's, R's, U's and V's characterize the mode couplings between the slab waveguide modes at $y = 0$ and $\tilde{P}$'s, $\tilde{Q}$'s, $\tilde{R}$'s, $\tilde{U}$'s and $\tilde{V}$'s characterize the mode couplings between the slab waveguide modes at $y = l$.

These coupling coefficients can be evaluated from the numerical integration in Fourier domain except R's and $\tilde{R}$'s that have closed form expressions. Each integrand for the numerical integration in Fourier domain is proportional to the product of the Fourier transform of one mode function with the complex conjugate of the Fourier transform of the other coupled mode function. The TM mode functions are often multiplied by $\epsilon_r(x)$. Since the spatial contents of the slab waveguide mode functions are greater than the effective thickness[18] of the fundamental TE_0 mode, the Fourier transform of the mode functions and, hence, the integrands are appreciable within a bounded range in Fourier domain.

As a result, we have the same number of equations (4) and (6) for the number of the unknown values of the modal currents and voltages at $y = l$ and $y = 0$, e.g., eight equations for eight unknowns when one TE and one TM modes are used in the central region. Scanning ξ/k_o from n_s to $\max(n_f, n_c)$, we seek all the values of ξ that satisfy the nontrivial condition for these modal currents and voltages.

Dispersion characteristics for the E^x_{m1} modes of three different rib waveguide structures ($n_f = n_c$) are calculated and the results are shown in Figs. 3 and 4. Neglecting the slab waveguide TE mode contributions in the central region, only Eqs. (1a) and (1c) are used in the analysis with $\mathcal{H}_x(\alpha, y) = 0$. Accordingly, of all the coupling coefficients, only V's and $\tilde{V}$'s are calculated in the analysis. It is noteworthy that we have made no restrictions to the polarizations of the upper and lower region normal modes.

Slab waveguide radiation modes are discretized by introducing perfectly conducting boundaries at the right and the left to the loading strip at intervals of two free space wavelengths except for near cutoff cases.[6,7] For near cutoff cases, four free space wavelengths are found to be sufficient. One may use the method established by Dagli and Fonstad[11]. The number of the TM slab waveguide normal modes used is limited to M. In Fig. 3, good convergence is found for M $\geq$ 5 and M = 10 is chosen in the analysis. In Fig. 4, M = 15 is used except for the high frequency range ($h \geq 10$) of the second and higher order rib waveguide modes where M is increased up to 30 to account for the diffraction effects occurring from the slab waveguide end opened normal to the uniform slab. At $h = 15$, for example, total 20 guided TM modes plus 10 discretized TM radiation modes are used for the E^x_{21} and E^x_{41} modes.

In Fig. 3, when $l = h$, our results agree with those of the mode-matching methods done by Yasuura et al.[5,9] and those of the scalar wave analysis done by Koshiba and Suzuki[9] and, when $l = 2h$, our results are in close agreements with those of Koshiba and Suzuki.[9] rather than Yasuura et al.[5,9] In Fig. 4, our results agree well with those of the vectorial wave analysis done by Koshiba and Suzuki [10] and those of Yasuura et al.[5]

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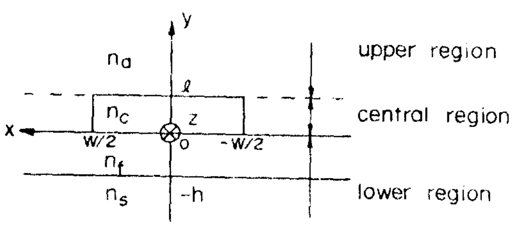

Fig. 1 Optical strip waveguide.

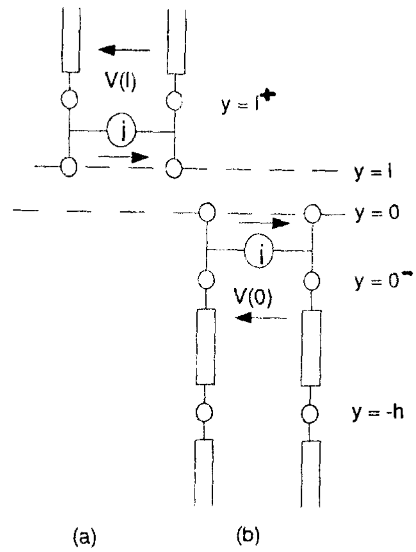

Fig. 2

Network representations for the (a) upper region and (b) lower region when the central region is replaced by a perfect magnetic conductor.

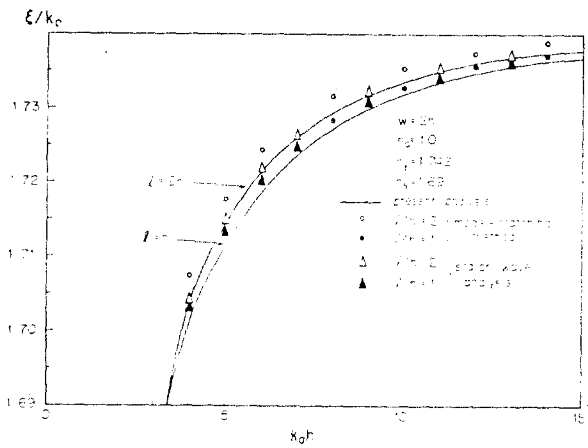

Fig. 3

Dispersion characteristics for the E^x_{11} mode of rib waveguides with $W = 2h$ for $l = h$ and $l = 2h$.

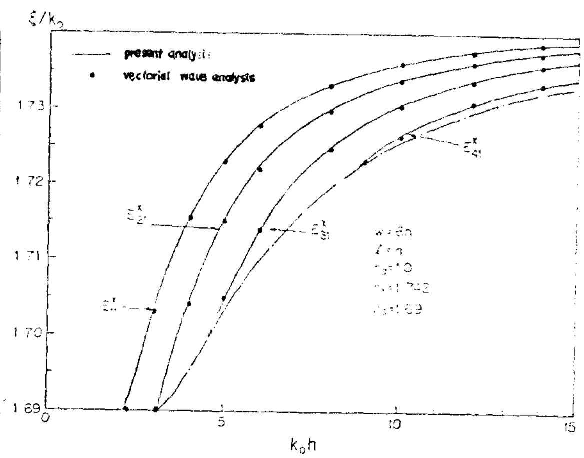

Fig. 4

Dispersion characteristics for the E^x_{m1} modes of a rib waveguide with $W = 6h$ and $l = h$.

— · — · — the fundamental mode of the uniform slab waveguide.