FINITE-ELEMENT METHOD FOR THE IMPEDANCE ANALYSIS OF TRAVELING-WAVE MODULATORS

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ABSTRACT

A finite-element method is developed to calculate the impedance of arbitrarily shaped electrodes on traveling-wave modulators. This method employs the divergence theorem to obtain the total charge on an electrode from the node potential values. By using this method, the impedance of multi strip-line electrodes on anisotropic inhomogeneous dielectric media was analysed and the effect of non-zero electrode thickness was calculated.

I. INTRODUCTION

Various electro-optic modulators have been developed for guided-wave modulation on $LiNbO_3$ directional coupler, which is well known for its low loss propagation, wide bandwidth, and many areas of application. Among them, a modulator which utilises the advantages of both the traveling-wave modulator and the reversed $\Delta\beta$ modulator has been reported[1]. The configuration of this traveling-wave reversed $\Delta\beta$ (TWRD) modulator on z-cut $LiNbO_3$ directional coupler is shown in Figure 1.

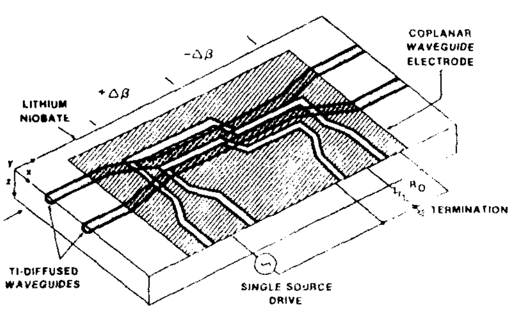

Figure 1. Traveling-Wave Reversed $\Delta\beta$ Modulator

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The electrodes consist of three coplanar transmission lines and are terminated by a 50 ohm resistance to form a traveling-wave modulator.

But in a traveling-wave modulator, the characteristic impedance of the electrode is desired not to deviate from the value of the shunt resistor which is usually 50 ohms [2].

The impedance of a transmission line can be obtained by a TEM-mode approximation[3] as

$$Z = \frac{1}{\sqrt{C_0 \cdot C \cdot V}} \quad \cdots (1)$$

where C_0 is the line capacitance of transmission line in free space and C, the line capacitance affected by dielectric materials surrounding transmission line, and V, the free space light velocity. So, we should first know the capacitance value to calculate the impedance of a modulator.

There are various methods for capacitance calculation of coplanar transmission lines. Examples are the conformal mapping method (CMM)[4] and the variational method in Fourier transformed domain (VMFD)[5]. Unfortunately, however, these methods cannot be applied to the three strip lines where the side strip line has a finite width. VMFD can only be applied to symmetric two strip lines and CMM can be applied to three parallel strip lines when the side strip width is assumed to be infinite.

In this paper, a new finite-element method (FEM) is developed to calculate the capacitance of these three coplanar strip lines whose cross-sectional geometry is shown in Figure 2.

II. FEM FOR THE CAPACITANCE CALCULATION.

From the Laplace equation, the Lagrangian functional can be obtained by using the Euler equation [6] as

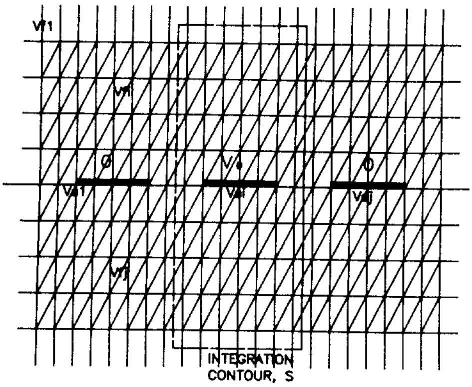

Figure 2. Cross-sectional geometry of the Finite-element discretization.

$$L=\int\int\left[\varepsilon x\left(\frac{dV}{dx}\right)^2+\varepsilon y\left(\frac{dV}{dy}\right)^2\right]dx\,dy \quad \cdots(2)$$

where εx, εy are the dielectric constants along the x, and y directions respectively. And then by the conventional finite-element technique[7], this functional can be expressed as discretized elements and formulated as a matrix summation such as

$$L=\sum_p\int\int\left[\varepsilon xp\left(\frac{dV}{dx}\right)^2+\varepsilon yp\left(\frac{dV}{dy}\right)^2\right]dxdy \cdots(3)$$

$$=\sum_p [Vp]\,[Ap]\,[Vp]^T$$

$$= [V]\,[A]\,[V]^T$$

where [V] is the nodal potential elements[V_1, V_2, V_3, ……]. A matrix [V] which minimizes the Lagrangian integral L in equation 3 is the solution to this variational problem .From this potential value,the capacitance can be evaluated using the divergence theorem as shown below

$$C=\frac{Q}{V}=\frac{\int\nabla\cdot D\,dv}{V_0-(0)}=\frac{\int\varepsilon E\cdot d\ell}{V_0} \quad \cdots(4)$$

$$=\frac{1}{V_0}\int\varepsilon\frac{dV}{dn}\cdot d\ell=\frac{1}{V_0}\sum\varepsilon\frac{\Delta Vi}{\Delta ni}\cdot\Delta\ell i$$

where V_0 is the magnitude of applied voltage, S is the integration contour and the subscript n means normal to contour S. The integration contour S is designed to encircle all the elecrtodes which have same charge polarity. For three parallel strip lines, it should encircle the center strip line or two side strip lines as shown in Figure 2. For two parallel strip lines, it should encircle only one strip line and, for four parallel strip lines, two other strip lines with the same charge polarity. Similarly, this method can be applied to arbitrarily shaped electrodes in anisotropic inhomogeneous dielectric media. And as this procedure does not have an iteration loop, it can reduce computing time compared with the other variational techniques.

Ⅲ. NUMERICAL CALCULATION

The results obtained by this method are compared with other methods. In Figure 3, the capacitance and impedance values obtained by several different methods are shown for symmetric two strip lines on z-cut $LiNbO_3$. And for electrode on z-cut $LiNbO_3$ substrate, the anisotropic dielectric constants are given as $\varepsilon x = 44.3\varepsilon_0$ and $\varepsilon y = 27.9\varepsilon_0$. So the effective dielctric constant is $\varepsilon f = (\sqrt{\varepsilon x\cdot\varepsilon y}+\varepsilon_0)/2$. And the thickness of the electrode is assumed to be zero. For this geometry , CMM and VMFD can give an accurate solution. In Figure 3, results from this FEM show good consistency with those from CMM and VMFD.

Now, the effect of the electrode thickness on the characteristic impedance is considered in the case of two parallel strip lines with electrode gap C= 5 μm and the results are shown in Figures 4.

In Figure 4-1, the free space capacitance increases by over 50 % when the electrode thickness is increased from 0 to 3 μm. This is a severe variation. But in Figure 4-2, the capacitance in dielectric media increases by less than 5 % at the same case. This little variation is due to the re-

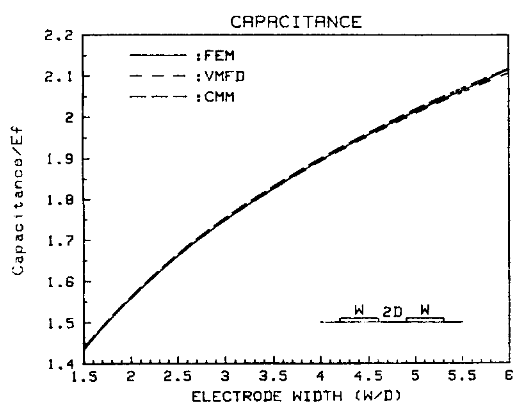

Figure 3. Capacitance of Two Parallel Strip Lines

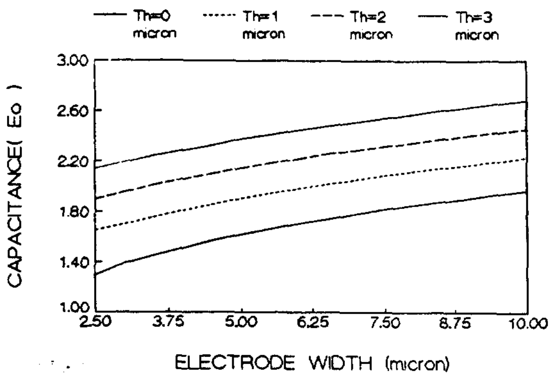

Fig.4-1. Free Space Capacitance.
Electrode Separation C = 5 micron.
Th : Electrode Thickness.

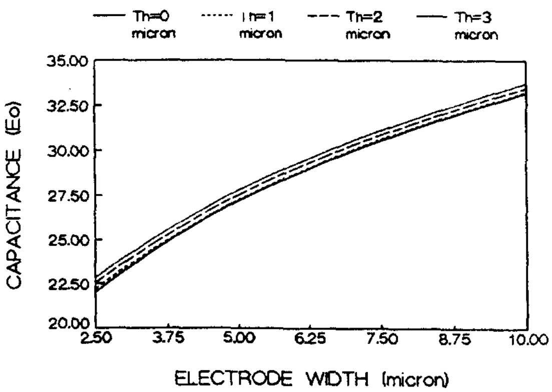

Fig.4-2. Capacitance in Dielectric Media
with E_{eff} = 35.7 E_o.

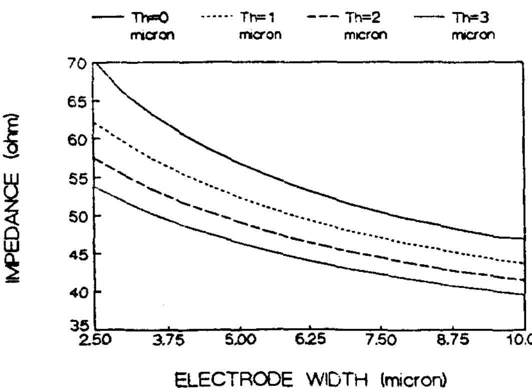

Fig.4-3. Impedance Variation.
C = 5 micron, E_{eff} = 35.7 E_o.

latively large dielectric constant of the substrate (ε_{eff}=35.7ε_0) compared to the free space dielectric constant ε_0 as can be seen in the equation shown below :

$$C = (\varepsilon_{eff} \cdot C_0 + \varepsilon_0 \cdot C_0') / 2 \quad \cdots(5)$$

where C_0 is zero-thickness capacitance and C_0' is non-zero-thickness capacitance in free space.

And the impedance variation is shown in Figure 4-3, and is inversely proportional to the root mean square value of free space capacitance as can be seen in Equation 1.

V. IMPEDANCE MATCHING OF THREE PARALLEL STRIP-LINE ELECTRODES

And now, the three parallel strip lines are to be considered by using this finite-element method.

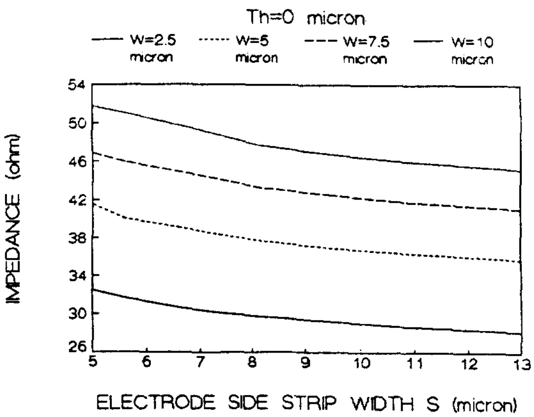

Fig.5-1. Triple Stripline Impedance.
Center Stripline Width = 20 micron.

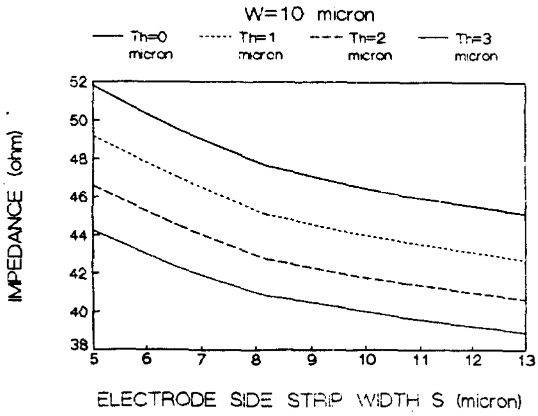

Fig.5-2. Triple Stripline Impedance.
C = 20 micron, W = 10 micron.

Parameterized plots of capacitance and impedance are shown in Figure 5-1. The center strip width C is 20 μm, gap width W is 2.5, 5, 7.5, 10 μm, and side strip width S is varied from 5 to 13 μm.

From these results, we can see that the impedance varies more significantly due to the electrode gap width W than due to the side strip width S. But the gap width, W must be over 10 μm to achieve 50 ohm impedance. It is a considerably large value because the applying voltage should increase in proportion to the gap width for sufficient electric field intensity in waveguides. But as S decreases, the gap width needed to achieve 50 ohm impedance, also decreases. So, to reduce the gap width, the side strip width should also be reduced. In Figure 5-2, the impedance variation at different electrode thickness is shown. As the electrode thickness increases, the impedance deviates farther from 50 ohm. If the electrode thickness is reduced severely, the sheet resistance of the electrode deteriorates the modulation characteristics. Therefore, the value of S should be reduced as small as possible while keeping the sheet resistance of the side strip itself not too small.

VI. CONCLUSION

A new finite-element method is developed to analyse the impedance of the traveling-wave reversed $\Delta\beta$ (TWRD) modulator. This method has the potential advantage of applicability to arbitrarily shaped electrodes in anisotropic inhomogeneous dielectric media. And as this method does not have any iteration loop, it can reduce the computing time. The validity of this method has also been shown. The design-parameters of this TWRD modulator are discussed by using this FEM technique.

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