홀로그램 렌즈어레이를 이용한 적응 2차 비선형 연상기억
Adaptive quadratic associative memory using holographic lenslet arrays

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Abstract

Optical implementation of adaptive quadratic associative memory for two-dimensional patterns is described by using holographic lenslet arrays and spatial light modulators. Basic experimental results demonstrating its feasibility are reported.

Recently, optical realization of associative memory based on neural network models is studied by many researchers.[1-12] A neural network stores information distributively in its weight interconnections, and processes its information in parallel. Another important feature of the neural network is learning capability by changing its interconnection weights between neurons. In this Letter, optical implementation of two-dimensional (2-D) quadratic associative memory (QAM) that needs parallel N^6 weighted interconnections is described. We show that fully adaptive interconnections for the 2-D QAM are realizable by using two 2-D holographic lenslet arrays and two spatial light modulators (SLM's). Of course, this adaptive interconnection method can be applied to other neural net models based on the connectionism such as the Hopfield model[13] and bidirectional associative memory[14]. For example, the adaptive interconnects for Hopfield model may be implemented by using one holographic lenslet array and one spatial light modulator. Thus two extensions of our previous work[12] are proposed; they are learning capability and storage of 2-D images in QAM. We also show basic experimental results.

Extension of the 1-D QAM[15] to the 2-D QAM is straightforward. Introducing an operator $\mathcal{T}$ that transforms 1-D vectors to 2-D matrices may be a solution[8]. We consider here only autoassociative memory, which can be readily extended to a heteroassociative memory. A set of M binary matrices V_{ij}^s (s=1, 2, ..., M and i, j=1, 2, ..., N) are stored in a sixth rank tensor,

$$W_{ijklmn} = \sum_{s=1}^{M}(2V_{ij}^s - 1)(2V_{kl}^s - 1)(2V_{mn}^s - 1) \qquad (1)$$

where the unipolar binary {1, 0} is assumed. The tensor W_{ijklmn} is used for the retrieval of stored information from erroneous or partial input matrices. The tensor-matrix product

$$\hat{V}_{ij}^t \equiv \sum_{k,l}^{N}\sum_{m,n}^{N} W_{ijklmn} V_{kl}^{t'} V_{mn}^{t'} \qquad (2)$$

with thresholding operation on $\hat{V}_{ij}^t$ yields an estimate of stored matrix that is most like the input $V_{ij}^{t'}$. The thresholded estimate matrix is fed back to the input, and it converges to the correct stored image.

In the optical implementation it is convenient to use unipolar W_{ijklmn}^{*} by adding a constant to every W_{ijklmn}. This is compensated by input-dependent thresholding operation.[8,12] Eq. (2) may be cast into two equations as follows:

$$T_{ijkl}^{t*} \equiv \sum_{m,n}^{N} W_{ijklmn}^{*} V_{mn}^{t'} \qquad (3a)$$

$$\hat{V}_{ij}^{t*} = \sum_{k,l}^{N} T_{ijkl}^{t*} V_{kl}^{t'} \qquad (3b)$$

where the terms marked by * means they have unipolar values. Then, V_{ij}^{t} is obtained by an appropriate input-dependent thresholding operation on $\hat{V}_{ij}^{t*}$. In this case the thresholding level $\theta_i^{t'}$ is $(\sum_j V_j^{t'})^2$. If both T_{ijkl}^{t*} and $\hat{V}_{ij}^{t*}$ are to be thresholded like Ref. 12, $\theta_i^{t'}$ is $\sum_j V_j^{t'}$.

Consider the implementation of Eqs. (3a) and (3b) with optics. We explain how each of the two tensor-matrix multiplication can be realized by using

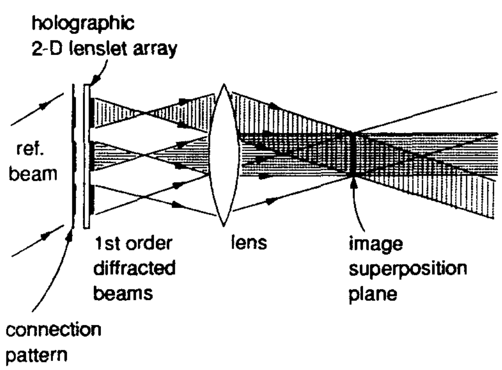

Fig. 1. Image superposition operation of holographic lenslet array. The image of patterns in front of each holographic lenslet array element is superposed in the image plane.

both a holographic lenslet array and an SLM. Each holographic lens plays the role of a lens for the first order diffracted beam when the reference beam is illuminated. Thus it is possible to superpose the images of patterns positioned in front of the holographic lenses with the help of a lens as shown in Fig. 1. Each holographic lens is made by exposing in a small area of the hologram a parallel reference beam with an object beam that is expanding from a focus. The array of such lenses are obtained by shifting the holographic plate and then exposing the two beams, repeatedly.

Since Eq. (3a) is a weighted sum, $\sum_{mn} W_{ijklmn}^{*} V_{mn}^{t'}$, the holographic lenslet array is used to obtain T_{ijkl}^{t*}. First, we encode the tensor weight W_{ijklmn}^{*} into 2-D matrix pattern and position the pattern in front of the holographic lenslet array. Then, by illuminating a light beam through the input $N \times N$ SLM that represents an input matrix $V_{mn}^{t'}$, Eq. (3a) is realized in the superposed image plane as shown in Fig. 1. A coding rule of the sixth rank tensor into the 2-D SLM is shown in Fig. 2a, in which each value of W_{ijklmn}^{*} is normalized and encoded by the degree of light transmission through the pixel of the SLM. Thus an $N^3 \times N^3$ SLM is required for W_{ijklmn}^{*}. Similarly, Eq. (3b) is realized after T_{ijkl}^{t*} is obtained. The total system is shown in Fig. 3. Part A and B of Fig. 3 is the implementation of Eqs. (3a) and (3b), respectively. Part B is similar to Part A except that the weight pattern T_{ijkl}^{t*} is the result of Part A.

Note that the weight pattern W_{ijklmn}^{*} is not recorded in the hologram array. It is only positioned in front of the holographic lenslet array. Thus an adaptive system can be realized by changing W_{ijklmn}^{*} value of the $N^3 \times N^3$ SLM.

To show the feasibility of our system, a simple experiment is executed. Two binary images "L" and "T"

$$L \equiv \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix}, \qquad T \equiv \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix} \tag{4}$$

are stored in 3×3 neurons. The $3^3 \times 3^3$ weight pattern W^*_{ijklmn} are calculated and the film mask for W^*_{ijklmn} is fabricated as explained in Fig. 2a. The fabricated film mask for W^*_{ijklmn} is shown in Fig. 2b. The element size of 3×3 holographic lenslet array we made for the demonstration of our idea is 4mm×4mm, which may be made far smaller. Inserting erroneous input images in the implemented system of Fig. 3, we obtain the correct output of the system. Experimental results of Part A, T^{t*}_{ijkl}, and Part B, $\hat{V}^{t*}_{ij}$ are shown in Fig. 4. The lumped 3×3 image blocks shown in the center photographs of Fig. 4 are detected by CCD camera of Part A, and displayed by an $N^2 \times N^2$ SLM of part B. We use a liquid crystal television (LCTV) for an $N^2 \times N^2$ SLM. The 3×3 image blocks displayed by an LCTV is positioned in front of the 3×3 holographic lenslet array elements in Part B, with each image block aligned with a corresponding lenslet array element. Only the pixels of the input $N \times N$ SLM that are switched on contribute corresponding block images to forming the output superposed image

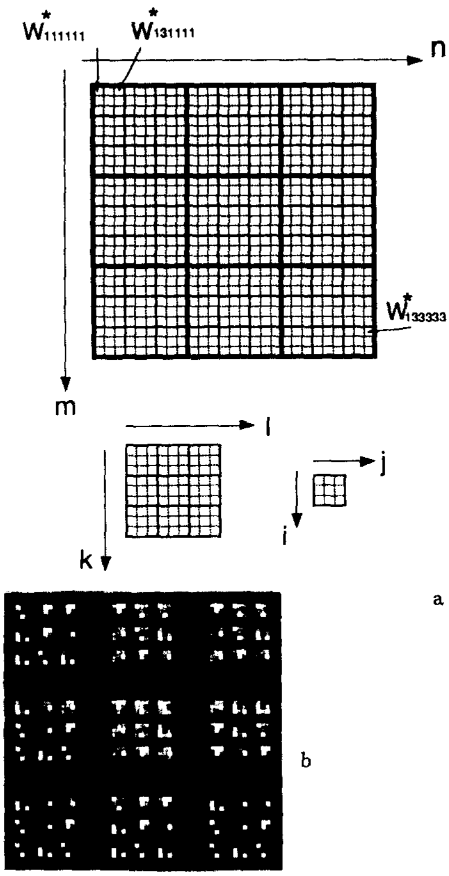

Fig. 2. (a) An example of the coding rule of the sixth rank tensor W^*_{ijklmn} into 2-D SLM when N=3. The value of each W^*_{ijklmn} is represented by the degree of light transmission through the pixel of 2-D SLM.

(b) The fabricated film for W_{ijklmn} in the experiment.

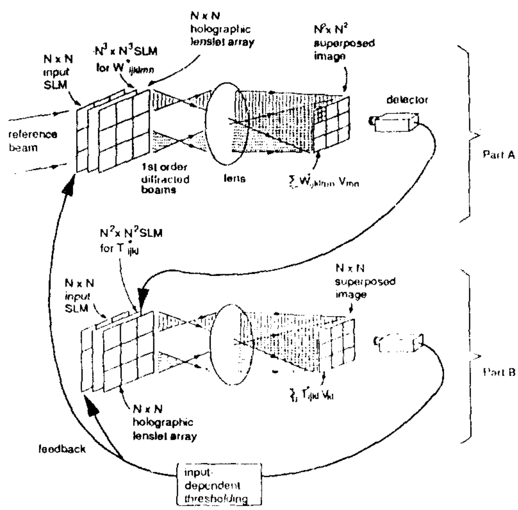

Fig. 3. Total system setup.

$\hat{V}_{ij}^{t*}$. Fig. 4 shows that exact L and T are obtained in the image plane of Part B. We did not perform the feedback operation in this experiment. Thus only an iteration was made. In many cases one iteration is enough to obtain the correct output. Thresholding and feedback using a CCD camera in Part B will improve the memory performance.

The storage capacity of this system M_q for $N \times N$ neurons is proportional to N^4 (about $0.03N^4$).[15] The required maximum pixel number of 2-D SLM is $N^3 \times N^3$. If we want N^4 rate memory capacity with linear associative memory such as the Hopfield model memory, we require $N^2 \times N^2$ neurons and $N^4 \times N^4$ SLM for the weight pattern.

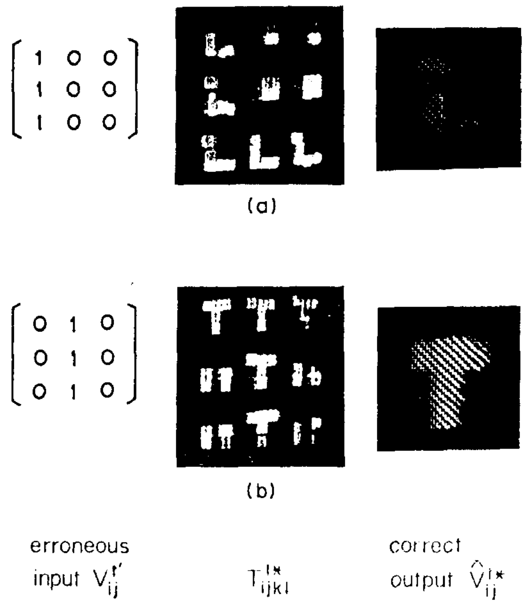

Fig. 4. Experimental results: Photographs of the output T_{ijkl}^{t*} in Part A and the output $\hat{V}_{ij}^{t*}$ in Part B of Fig. 3. (a) The case when the input matrix is an erroneous L. (b) The case when input matrix is an erroneous T.

Pixel size of a few microns in the connection pattern can be imaged with our holographic lenses. Thus 10×10 neurons may be easily implemented by a holgraphic lenslet array whose total size is about 5mm×5mm. In this case M_q is about 300, which is a storage capacity applicable to practical memory usages. To obtain a fully adaptive system with 10×10 neurons, a programmable SLM with $10^3 \times 10^3$ pixels is required. However, there are a variety of applications in which full adaption is not needed. Probably, partial learning using an SLM with a small number of pixels is worthy of being studied until the advent of an SLM with a large number of pixels.

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