THE APPLICATION OF THE ORIENTATION DENSITY FUNCTION TO THE MECHANICS OF FIBROUS ASSEMBLY

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ABSTRACT

This paper shows the possibility of the application of the orientation density function of fibers to the mechanics of fibrous assembly. As an example, the orientation density function of a single yarn was theoretically derived in consideration of the idealized helical yarn. And the theoretical derivation of the tensile modulus of the fibrous assembly was performed in view of the fiber orientation. Application of this orientation density function to the obtained tensile modulus and to the contraction factor of the yarn was also performed so that the theoretical equations of the tensile modulus and the contraction factor of the yarn were obtained. Close agreement was shown between the theoretical and the existing equations. Consequently it was confirmed that the application of the orientation density function to the mechanics of the fibrous assembly is sufficiently possible.

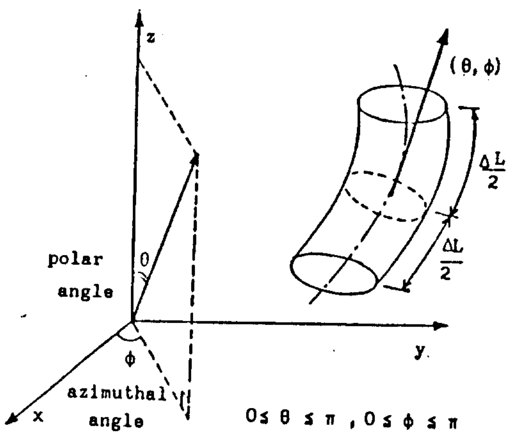

Fig.1. The two angles of orientation of a fiber segment.

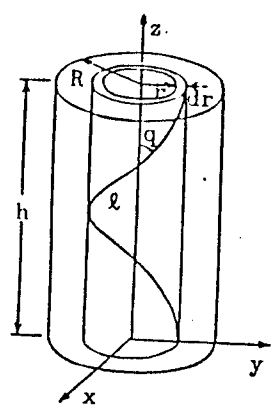

Fig.2. Idealized helical yarn geometry.

$$\Omega_y(\theta,\phi)\sin\theta = 2\sin\theta/\pi\tan^2\alpha\cos^3\theta \qquad (13)$$

$$(0 \le \theta \le \alpha,\ 0 \le \phi \le \pi),$$

where α is the surface angle of twist and $\Omega_y(\theta,\phi)\sin\theta$ is the orientation density function of fibers in the yarn.

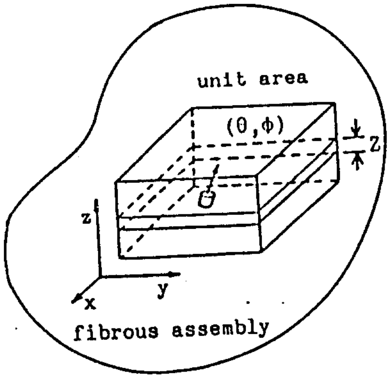

Fig.3. The volumetric element of a fibrous assembly.

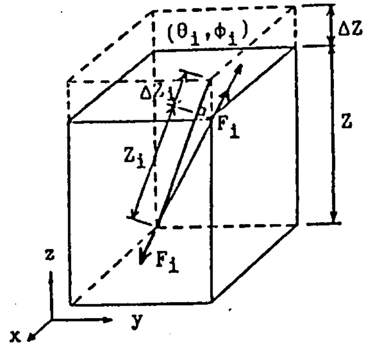

Fig.4. Tensile deformation of a fiber segment of orientation (θ_i, ϕ_i).

$$E_a = E_f\langle\cos^3\theta\rangle / \langle\sec\theta\rangle \quad . \tag{30}$$

where E_a is the specific tensile modulus of fibrous assembly, E_f is the specific modulus of the fiber and the notation $\langle\ \rangle$ represents the mean value.

$$E_y = E_f\langle\cos^3\theta\rangle/\langle\sec\theta\rangle = E_f 3\cos^3\alpha/\ (1+\cos\alpha+\cos^2\alpha) \quad . \tag{33}$$

where E_y is the specific tensile modulus of the yarn .

$$\begin{aligned} C_y &= \langle\sec\theta\rangle = \int_0^{\pi} d\phi \int_0^{\alpha} d\theta\ 2\sin\theta\sec\theta\ /\ \pi\tan^2\alpha\cos^3\theta \\ &= 2(1+\sec\alpha+\sec^2\alpha)\ /\ 3(1+\sec\alpha) \quad . \end{aligned} \tag{35}$$

where C_y is the contraction factor of the yarn .

Table 1. Comparison of the calculated results from eq.(33) and eq.(35) with those from the wellknown equations, $E_y=E_f\cos^2\alpha$ and $C_y=(1+\sec\alpha)/2$.

twist angle α, (degree)	coefficient of tensile modulus of the yarn		contraction factor of the yarn	
	$\cos^2\alpha$	$\frac{3\cos^3\alpha}{1+\cos\alpha+\cos^2\alpha}$	$\frac{1+\sec\alpha}{2}$	$\frac{2(1+\sec\alpha+\sec^2\alpha)}{3(1+\sec\alpha)}$
0	1.0	1.0	1.0	1.0
10	.970	.970	1.0077	1.0077
20	.883	.882	1.0321	1.0324
30	.750	.745	1.0774	1.0792
40	.587	.573	1.1527	1.1594
50	.413	.388	1.2779	1.2980

CONCLUSIONS

The orientation density function of a single yarn and the tensile modulus of the fibrous assembly were theoretically calculated. The applications of the orientation density function of a single yarn to the obtained equation of the tensile modulus and the contraction factor of the yarn were performed. Consequently, the theoretical tensile modulus and contraction factor of the yarn were derived and they were compared with the existing equations. Close agreement was shown between them. Therefore, it was confirmed that the application of the orientation density function to the mechanics of the fibrous assembly is sufficiently possible.

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