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# DESIGN OF ADAPTIVE CONTROLLER OF DC SERVO MOTOR

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Abstract - Design procedure of adaptive controller with variable load condition is present and applied to velocity control of small, permanent magnet DC servo motor. The state feedback control scheme is adopted and Recursive Least Squares algorithm is used for parameter estimation. In order to reduce the time consuming in the procedure of adaptation gain tuning of state feedback controller, approximate curve fitting technique is applied to the relations between load condition and poles of the system, load condition and feedback gains. With this method, fast adaptation can be accomplished. It is shown that this procedure can be applied not only to variable load condition but also to variation of other system constnts, for example variation of resistance and inductance etc.. Simulatiom results is present for both cases - variable inertia load, variable motor resistance to verify performance improvements. This design procedure produces an adaptive controller which is feasible for implementation with microprocessor by reducing calculation time.

#### I Introduction

The direct current motor has been widely used in many areas - industrial robot, medical manipulator and electrical wheelchair etc. In many of these applications, parameter variations occur due to the change of operating conditions and that of load conditions, for example do motor on electrical wheelchair may experience the variations of load inertia of 600 % or more. [1] These parameter variations result in performance degradation or instability of the control algorithm.

Recognizing the variation of parameters, controller is able to satisfy the prescribed performance characteristics by suitable adaptation.

In this work, adaptive controller is designed and applied to do servo motor. State feedback control and optimal regulator theory are used for determination of controller gains and Recursive Least Squares is used for parameter identification. In real application of adaptive controller, the calculation time - time consumed in estimation and adaptation - is very important problem. The optimal regulator theory, is ricatti equation is too time-consumed to be

applied directly in adaptation. Thus in order to reduce caculation time an approximate method is introduced for fast adaptation. The relations between load and system poles, load and feedback gains are obtained by approximate curve fitting technique. Through parameter estimation, coefficients of characteristic equation is estimated and poles of the system can be calculated. And with the relations which are fitted approximately, load can be expected and proper feedback gains can be obtained.

# II Dynamic Modelling of System

The fundamental equations of the motor are written by examining the electrical and mechanical characteristics of the DC servo motor.

The fundamental electrical equation is

e=Ri+Ldi/dt+Kew --(1)

and mechanical equation is

Kti=Jdw/dt+Bw+TL--(2)

In these equations,

R: Armature resistance

L'armature inductance

K: Back E.M.F constant, Torque constant

J: Inertia

B: Viscous damping

TL: Load torque

If the state variables are chosen as

x1=1 x2=w

then equations (1),(2) can be represented as matrix form as follows

x = A x + B u --(3)

where

$$x = (xi x2)^{T}$$

$$A = \begin{bmatrix} -R/L - K/L \\ K/J - B/J \end{bmatrix}$$

B = 
$$\begin{bmatrix} 1/L & 0 \\ 0 & -1/J \end{bmatrix}$$
  
u =  $\begin{bmatrix} ea & TL \end{bmatrix}$ 

The purpose of this paper is to investigate the effect of inertia load variation to the system, say transient response, so from now on TL load is neglected in the equation.

For the purpose of this paper, this continuous system is desired to be converted to discrete representation. If armature voltage is generated through the ZOH(zero order hold), it can be assumed piece-wise constant between the sampling instants. This allows the converted discrete model is as follows.

$$x(k+1)=\widehat{A}x(k)+\widehat{B}u(k)$$
 --(4)  
 $y(k)=\widehat{C}x(k)$ 

where

$$\widehat{A} = \int_{\mathbb{R}} [(sI - A)^T]$$

$$\widehat{B} = \int_{\mathbb{R}} A ds \cdot B$$

$$\widehat{C} = [0 1]$$

The constants of the motor used in this paper are shown in table 1. And each equations above is obtained.

$$A = \begin{bmatrix} -340.3 & -56.0 \\ 303.1 & 0.1354 \end{bmatrix} B = \begin{bmatrix} 523.56 \\ 0 \end{bmatrix}$$

$$\hat{A} = \begin{bmatrix} 0.1103 & -.1256 \\ 0 & 0.6852 & 0.8727 \end{bmatrix} \hat{B} = \begin{bmatrix} 1.1739 \\ 1.1837 \end{bmatrix}$$

Table 1. Motor constants

R	0.65 Ω
L	1.91 mH
J	3.53E-4 Kg·m³
В	4.78E-5 Nm/rad-5"
K	0.107 Nm/A

III The structure of the controller

## (i) State feedback control

The state feedback control scheme is chosen and the structure is shown in figure 1. In order to track the reference command without offset, the integral action should be contained in the controller[4]

To develop this state feedback scheme, the state variable representation of the system is first augmented with an integral of error state such that state feedback on the augmented system will incorperate integral action. As integral state represents the sum of the past values, it can be described as follows,

Therefore the system equation (4) above is augmented

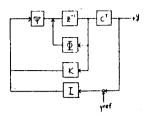


Figure 1.

$$\begin{bmatrix} \mathbf{x}(\mathbf{k+1}) \\ \mathbf{q}(\mathbf{k+1}) \end{bmatrix} = \begin{bmatrix} \mathbf{h} & \mathbf{Q}_{\mathbf{x}}(\mathbf{k}) \\ -\mathbf{C} & \mathbf{I}_{\mathbf{q}}(\mathbf{k}) \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ \mathbf{Q} \end{bmatrix} \mathbf{u}(\mathbf{k}) + \begin{bmatrix} \mathbf{0} \\ \mathbf{1} \end{bmatrix} \mathbf{yref}$$

$$\mathbf{If} \ \mathbf{z}(\mathbf{k+1}) = \begin{bmatrix} \mathbf{x}(\mathbf{k+1}) & \mathbf{q}(\mathbf{k+1}) \end{bmatrix}^{\mathsf{T}} \mathbf{then}$$

 $z(k+1) = \overline{\phi} z(k) + \psi u(k) + \phi \text{ yref } --(6)$ 

#### (2) State feedback gain

There are several methods used in obtaining the gains of state feedback controller, for example pole-placement method and optimal linear quadratic theory by which the gains of state feedback controller is obtained from the optimization of a quadratic control performance criterion, is investigate in this paper.

In equation (6), if the control is selected

and a quadratic performance criterion of the form.

$$J=Z(N)HZ(N)+\sum_{i=0}^{N-1}[Z(i)QZ(i)+U(i)RU(i)]$$

is chosen, the solution of the optimal control problem is given by discrete Ricatti equation,

$$F(k)=[R+\psi^{T}H(k+1)\psi^{-}]^{'}\psi^{T}H(k+1)\Phi$$
 $H(k)=\Phi^{T}H(k+1)\Phi^{-}+Q$ 
 $-F(k)[R+\psi^{T}H(k+1)\psi^{-}]F(k)$  --(7)

The recursive Ricatti equation is solved backwardly and provided the augmented system (6) is completely controllable, this solution will converge to a unique, steady state solution which will stabilize the resulting closed loop system and minimize the quadratic criterion.[4]

In figure 2, solutions of Ricatti equation are shown for three states of system. The selection of H,Q and R matrix, which appear in euqation (7), is performed by trial and error method. The Q matrix is a weighting factor on the state trajectory while R is a weighting factor for the control effort used in obtaining that trajectory. In this paper, Q is selected as the identity matrix and R as 0.5.

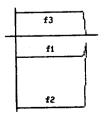
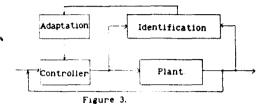


Figure 2.

# IV Adaptive controller

Figure 3 shows the scheme of self-tuning adaptive controller. It is composed of two parts-the one for parameter estimation and the other for adaptation with respect to the results of parameter identification.



## (1) Parameter estimation

If the load inertia is changed in operation of DC servo motor, the system parameter are also varied and if its change of the control system is not recognized and the feedback gains is not changed properly, the performance of the system is degraded and somtimes even the stability of the system can not be guaranteed. In order to avoid these problem, first of all, the controller should identify the system parameter that is changed with variable load conditions.

There are several parameter estimation algorithm [5] which are applicable to this case and one of which, Recursive Least Squares algorithm is used in this work.

The RLS algorithm is given by [5]

$$\hat{\mathbb{G}}_{(K+1)} \hat{\mathbb{G}}_{(K+1)} \Psi_{(K+1)} = \frac{1}{1 + \hat{\Psi}_{(K+1)} \hat{\mathbf{h}}_{(K+1)} \hat{\mathbf{h}}_{(K+1)} \mathbf{h}_{(K+1)}} \frac{1}{\mathbf{h}_{(K+1)} \hat{\mathbf{h}}_{(K+1)} \hat{\mathbf{h}_{(K+1)}} \hat{\mathbf{h}_{(K+1)} \hat{\mathbf{h}}_{(K$$

$$F(x) = [ [1 - Y(x)] \Psi^{T}(x+1) ] P(x)$$

$$\hat{F}(x) = 0$$

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 $\hat{\theta}$ : Estimated parameters vector  $\psi$ : Data vector

Now in order to apply RLS to DC motor, the trasfer function is obtained

$$G(s) = \frac{b_1}{S^4 + a_1S + a_2}$$

and it can be also expressed in second order Z-transform as follows

$$G(z) = \frac{\hat{b}_1 \mathcal{Z} + \hat{b}_0}{\mathcal{Z}^2 + \hat{a}_1 \mathcal{Z} + \hat{a}_0} - --(10)$$

and it is converted to

$$W(k)=-\hat{a}1 \quad W(k-1)=\hat{a}0 \quad W(k-2)+\hat{b}1 \quad U(k-1)$$
  
+ $\hat{b}0 \quad U(k-2) = --(11)$ 

The equation (ii) is an ARMA model of the system to be identified. With known data, velocity w and control input u, the coefficients a b can be estimated.

In algorithm (9),

 $\widehat{\theta}(\mathbf{k}) = [-\widehat{\mathbf{a}}\mathbf{1}(\mathbf{k}) - \widehat{\mathbf{a}}\mathbf{0}(\mathbf{k}) \widehat{\mathbf{b}}\mathbf{1}(\mathbf{k}) \widehat{\mathbf{b}}\mathbf{0}(\mathbf{k})]$ 

 $\Psi(k)=[\Psi(k-1) \ \Psi(k-2) \ U(k-1) \ U(k-2)]$ 

The least squares algorithm is selected primarily because of its simplicity when compared to other algorithms. However the least squares technique will not perform well unless persistent exitation of the system exists. The application for which this work was developed is DC servo motor with state feedback and so the commands of the system is continuously changing. Consequently it is thoght that least squares approach is sufficient.

The simmulation results of parameter identification is shown in figure 4.

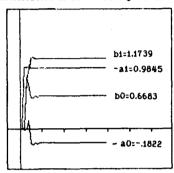


Figure 4.

#### (2) Adaptation

Parameter estimation makes it possible to adjust the gains of state feedback controller with respect to the changing operating conditions. But in real time application of adaptive control, the calculation time consumming in identification and gain-tuning is very serious problem. So the optimal state feedback control theory mentioned in section III can not be applied directly in real time adaptation.

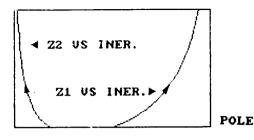
Thus Approximate Curve Fitting method is introduced for fast gain-tuning of the state feedback controller in this work. Through the results of simulation, the change of feedback gains and movement of system poles due to the variation of inertia load can be obtained and shown in figure 6. First, three feedback gains Fi,F2 and F3 are changing linearly with respect to the changing inertia load from no load to 300% of no load. They can be approximately fitted by first order lines.

f1=134.8 
$$\hat{J}$$
=0.4669  
f2=-1014.4  $\hat{J}$ =0.4617 --(12)  
f3=188.4  $\hat{J}$ +0.2413

Second, movements of two poles of the system can be also approximately fitted by second order curve.

$$\hat{J}=0.0223$$
  $\hat{Z}=0.0333$   $\hat{Z}+0.0128=---(13)$ 

As shown, calculation of system poles through parameter estimation makes it possible to estimate inertia load and to find proper feedback gains.



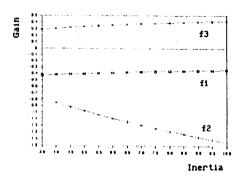


Figure 5.

#### (3) Overall control algorithm

With these approximate curves, fast gain-tuning is possible and whole control algorithm is

<1> Through identification, the characteristic equation of the system can be obtained.

<2> With this characteristic eq. above, two pole Zi,Z2 can be calculated.

<3> The changed inertia load can be found by the approximate curve.--- eq.(13)

<4> The proper gains F1,F2,F3 can be found by the approximate lines.--- eq.(12)

# V Simulation results

Motor performance with above procedure is shown in figure 6. The velocity reference is 50 rad/sec. Velocity and input data are stored in memory in period I, and estimation of coefficients, and adaptation is performed in period II. From the beginning of period III; control input is determined with adjusted feedback gains. As shown in the figure, approximate method above shows good adaptation and performance improvement.

Figure 7 is a result of applying this method to variation of motor resistance. Resistance is assumed to be increased 300% of normal terminal resistance. Because pole of system carries the information of operating conditions - variation of resistance, inductance as well as inertia load , the above technique can be applied to directly. Figure 8 shows pole movement remains stable region when inertia and resistance varied up to 300%.

## VI Conclusion

With approximate adaptation above, good performance of self-tuning adaptive controller is obtained. And this procedure can be extended to the variation of other constans like resistance, inductance and damping factor.

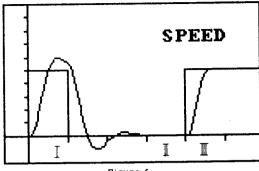


Figure 6.

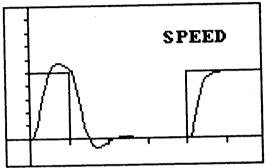


Figure 7.

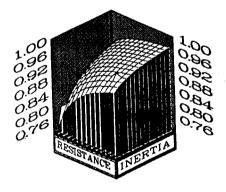


Figure 8.

## VII Reference

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