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단안시에 의한 무늬 그래디언트로 부터 면 방향 복구

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Recovering Surface Orientation from Texture Gradient by Monocular View

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ABSTRACT

Texture provides an important source of information about the three dimensionary information of visible surface, particulary for stationary monocular views.

To recover three dimensionary information, the distorting effects of projection must be distinguished from properties of the texture on which the distortion acts.

In this paper, we show an approximated maximum likelihood estimation method by which we find surface orientation of the visible surface(hemisphere) in gaussian sphere using local analysis of the texture. In addition,assuming that an orthographic projection and a circle is an image formation system and a texel(texture element) respectively, we derive the surface orientation from the distribution of variation by means of orthographic projection of a tangent direction which exists regulary in the arc length of a circle. we present the orientation parameters of textured surface with slant and tilt, and also the surface normal of the resulted surface orientation as needle map.

This algorithm was applied to geographic contour and synthetic textures.

1. Introduction

One central problem of image understanding is the recovery of three dimensional scene information from a single two dimensional image. A single two dimensional image is an ambiguous representation of the three dimensional world, many different scenes could have produced the same image, yet the human visual system is extremely successful at recovering a qualitatively correct characteristic from this type of representation. Workers in the field of computational vision have devised a number of distinct schemes that attempt to emulate this human capability;these schemes are collectively known as "shape from intrinsic properties"(e.g.,shape from shading,shape from texture, or shape from contour)[3, 4, 7].

In this paper we consider the computation of shape from texture.

Witkin[9,10] proposed an effective technique for recovering surface orientation from images of natural textured surfaces must rest on texture descriptions that can actually be computed from such images, and must avoid highly restrictive assumptions about texture geometry. More specifically, he related the texture of a surface to the distribution of directions of reflectance boundaries on that surface, and posed the problem of how a uniform distribution of such direction is transformed by projection. Once this transformation is determined, the problem of recovering the surface orientation of a small planar patch of texture based on observing the distributon of direction in an image can be posed as a maximum likelihood problem.In this paper we describe more efficient algorithm, that is,approximated maximum likelihood estimation of local surface orientation,for solving the shape from texture problem. This method is more efficient for recovering surface orientation which is useful for large value(tangent distribution:n>100). In such cases, the prior distribution of surface orientation parameters can be ignored and original surface orientation can be simply recovered from the distribution of tangent direction.

2. Geometric Model

We assume orthographic projection and use the geometric model used by Witkin[9]. Following his notation we assume an object plane S in space with an orthogonal coordinate system(x', y'). there is also an image plane I with coordinate system(x,y). The orientation of S with respect to I can be denoted by two angles s and t; the slant is the angle between I and S (which we will always take to be acute.i.e., $s\in[0, \pi/2]$) and the tilt is the angle between the projection of the normal of S onto I and the x axis in I ($t\in[-\pi/2, \pi/2]$). Figure 1 shows the relationships between I,S,s, and t.

The projective distortion on an orthographically projected planar surface is simple a one dimensional scalling and compression in the direction of steepest decent away from the viewer(tilt) whose magnitude is the cosine of the angle between the surface and the image plane(slant).The slant and tilt representation of surface orientation[13] is related to gradient space representation[16]. Figure 2 shows the slant and tilt in gradient space.

We want to find the angles of their orthographic projection onto I with the x axis. First let t=0. Then the projection of S's normal is parallel to the axis,the y and y'axes are both parallel to the line of intersection between

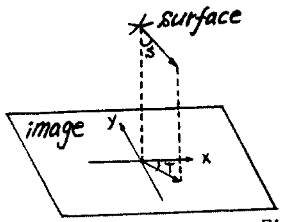


Fig.1 Geometrical model

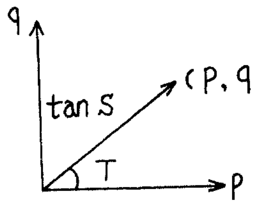


Fig.2 Representation of and tilt in gradient space

I and S and the angle between the x and x'axes is s.Since in this case there will only be projective shortening in the x direction by a factor cos(s) it follows immediately that the relation between an angle B in S and an angle A in I is given by

$$\tan(a) = \tan(B)/\cos(s) \qquad (2.1)$$

To introduce tilt, rotate the object plane around the z axis (which is perpendicular to the image plane), keeping the slant constant. The projected normal(and the projected x'axis) then rotates away from the x axis over an angle t. Since the projected line still makes an angle A with the projected normal it now follows that its angle a' with the x axis is given by a'=A+t, or using (2.1), we obtain

$$a' = \arctan(\tan(B)/\cos(s)) + t \qquad (2.2)$$

as the relation between a line at angle B with the x'axis in S and a line at angle A with the x axis in I. Figure 1 shows the relationships between A, a',B, s, and t. Rewritten, we obtain the following as our geometric model of the imaging process:

$$\tan(a'-t) = \tan(B)/\cos(s) \qquad (2.3)$$

where a'is the projected tangent angle, B is the angle between the unprojected tangent and the tilt direction's projection onto S, and (s,t) is the oientation of the curve in space. This expression relates a', which can be measured in the image,to (s,t), which we wish to recover, yielding what we sought.

3. Measuring Texture Distortion due to Surface Orientation

3.1 Definition of the problem

The tangent to a curve at a given point is defined as the first derivative of position on the curve with respect to arc length. The tangent is a unit vector and may be visualized as an arrow that just grazes the curve at the specified point[9]. The distortion is demonstrated by the projection of a circle to an ellipse: arc length on a circle is uniformly distributed over tangent direction,but the distribution for an ellipse assumes maxima and minima in the directions of the major and minor axes respectively. For the projection of a circle,the direction of the minimum coincides with the tilt direction, t, and the relative height of the peak varies with the slant, s. The particular tangent distribution obtained in the image depends on the distribution prior to projection, as well as the orientation of the textured surface.

3.2 Maximum likelihood estimation of surface orientation[9,10]

For any hypothesized surface orientation, the geometric relation translates each value of a' into a corresponding value of B, and so translates the observed distribution of a' into a corresponding distribution of B; a possible distribution of B may be obtained for each value of (s,t). Given an expected distribution for (B, s, t), the likelihood of an observed distribution at any hypothesized surface orientation can be evaluated. If B, s, and t are treated as randomly variables,and a j.p.d.f.is assumed for those variables,then (s, t) may be estimated statisticaly.

We will express the uniform strategy by assuming that tangent direction and surface orientation are isotropic and independent.Isotropy reasonably supposes that all surface orientations are equally likely to occur in nature and that tangents to surface curves are equally likely in all direction. The statement that all surface orientations are equally likely requires clarification: the orientation of a surface can be given by the unit normals, a needle of unit length normal to the surface. The set of normals (called the Gaussian sphere[18] (Fig.3)) which contains the points of the needles. When we say that all surface orientations are equally likely, we mean the needle is as likely to land at any one point on the sphere as any other[9].

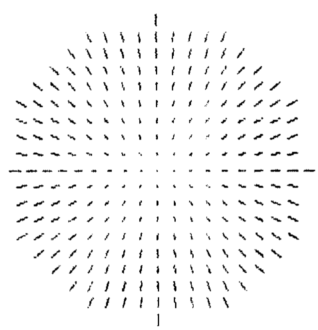


Fig.3 The surface normal orientation exisiting on gaussian sphere

Most succinctly,it is assumed that the quantities (B,s,t) are randomly distributed and their joint probability density function D(B, s, t) is given by

$$D(B, s, t) = \frac{1}{\pi^2}\sin(s) \qquad (3.2\text{-}1)$$

We assume that the ranges of the angles are : $0 \le s < \pi/2$, $0 \le s < \pi$, $0 \le B < \pi$. Similarly, the density of the (s, t) is given by

$$D(s, t) = \frac{1}{\pi}\sin(s). \qquad (3.2\text{-}2)$$

The independence assumption requires that the image tangents $\{a^i : 1 < i < n\}$ are statically independent. That is, it is assumed that the tangent directions at different points on the image curve are independent.
From (3.2-1) and the transformation (2.2) we find that the joint density D(A, s, t) is given by

$$D(A, s, t) = \frac{1}{\pi^2}\frac{\cos(s)\sin(s)}{\cos^2(A-t)+\cos^2(s)\sin^2(A-t)} \quad (3.2\text{-}3)$$

For the conditional density D(A/s, t) we find

$$D(A/s, t) = \frac{1}{\pi}\frac{\cos(s)}{\cos^2(A-t)+\cos^2(s)\sin^2(A-t)} \quad (3.2\text{-}4)$$

This density function tells us, under the assumption of isotropy and independence for (B,s,t) ,how the image tangent direction is distributed as a function of surface orientation. This distribution is graphed at several at several values of s and t in Fig 4.

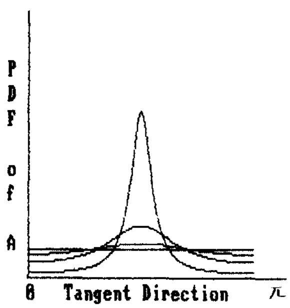
We shall denote by A the sample $\{A_1, A_2, A_3 \cdots A_n\}$ of the projected direction and by D(A/s, t) its conditional density:

$$D(A/s, t) = \prod_{i=1}^{n} D(A_i/s, t) \quad (3.2\text{-}5)$$

By Bayes' formula we obtain

$$D(s, t/A) = \frac{D(A/s,t)\,D(s,t)}{\int_0^{\pi}\int_0^{\pi/2} D(A/s,t)\,D(s,t)\,ds\,dt} \quad (3.2\text{-}6)$$

where integration is performed over the ranges of s and t. Dividing by the integral simply normalizes the function to integral 1.
The value of (s,t) for which this function assumes a maximum likelihood estimated for surface orientation and the integral of the function over a region gives the probability that the surface orientation lies inside that region.



(a) slant(0, π/2),tilt(0)

Fig.4 Curves in the function D(A_i|s,t),plotted against A_i ,with t=0,at several values of s

4. Obtaining the Estimators using approximated Maximum Likelihood Estimation method.

4.1 Approximated likelihood function

From (3.2-8) it follows that for a likelihood function L(s,t/ A) for the sample A = $\{A_1 \cdots A_n\}$, we can use the expression

$$L(s, t/A) = \frac{\sin(s)\cos^n(s)}{\prod_{i=1}^{n}[1-\sin^2(s)\sin^2(A_i-t)]} \quad (4.1\text{-}1)$$

< Algorithm >
This algorithm is only valid for large samples of directions(tangent sample;n >100). We shall begin by showing that for large n the termD(s,t) contributes little to D(s, t /A).

[1]. IF s is large
sin(s) is approximately 1
Thus, for large s, D(s, t) contributes little to the expression of D(s, t/ A).
[2]. IF s is small
sin(s) is approximately s
cos(s) is approximately $1 -(s^2/2)$, ignoring high order terms
cos(s) is approximately $1-(n*s^2/2)$.

Now since sin(s) sin(Ai-t) is bounded above by 1, $[1 - \sin^2(s)\sin^2(Ai - t)]$ is approximately $[1+\sin^2(s)\sin^2(Ai-t)]$ again choosing to ignore the high order term,

$$\prod[1-\sin^2(s)\cdot\sin^2(Ai-t)]\cong[1+\sin^2(s)\cdot\sin^2(Ai-t)] \quad (4.1\text{-}2)$$

We can expand the product, and keeping only the second order terms we arrive at the expression

$$\prod_{i=1}[1+\sin^2(s)\sin^2(Ai-t)] \cong 1+s^2\sum_{i=1}\sin^2(Ai-t) \quad (4.1\text{-}3)$$

Maximizing D(s,t/A) for small s and large n can this be approximated by maximizing the expression

$$D(s,t/A) = s(1-\tfrac{n}{2}s^2)(1+ s^2\sum_{i=1}^{n}\sin^2(Ai-t) \quad (4.1\text{-}4)$$

Hence we are interested in maximizing (4.1-4) with respect to s and t.

< THEOREM >
Since the sum sin(Ai-t) does not involve s directly, we can replace the sum by its upper bound give

$$\sin^2(Ai-t) < \frac{n}{2\cos^2(s)}$$

< PROOF >
The proof involves differentiating LOG L with respect to s and t, and setting the sulting expression to zero. This gives

$$\frac{\cos(s)}{\sin(s)} - n\frac{\sin(s)}{\cos(s)} + 2\sum_{i=1}^{n}\frac{\sin(s)\cos(s)\sin^2(t-A_i)}{1-\sin^2(s)\sin^2(t-A_i)} \quad (4.1\text{-}5)$$

We may simplify the first equation of (4.1-5) by dividing it by sin(s)cos(s).Thus we obtain the system to be solved in the form

$$\frac{1}{\sin(s)} - n\frac{1}{\cos(s)} + 2\sum_{i=1}^{n}\frac{\sin^2(A_i-t)}{1-\sin^2(s)\sin^2(t-A_i)} \quad (4.1\text{-}6)$$

To persue our strategy of solving (4.1-6) we now derive an inequality involving s and t.
Making the substitution

$$M = \frac{1}{\cos^2(s)} \quad (4.1\text{-}7)$$

in the first equation of (4.1-6) we bring it into the form

$$\frac{1}{n}\sum_{i=1}^{n}\frac{\sin^2(A_i-t)}{\sin^2(A_i-t)+M\cos^2(A_i-t)}+\frac{1}{2n}\frac{1}{M-1} = \frac{1}{2} \quad (4.1\text{-}8)$$

Denoting the left side of (4.1-8) by F(M), we observe that $F(M)\to\infty$ as $M\to 1$ from right and $F(M)\to 0$ as $M\to\infty$. Since F is decreasing we conclude that (4.1-8) has a unique solution on $(1, \infty)$ for every sample A = $\{A_1 \cdots A_n\}$. Assuming that M is a solution of (4.1-7) we observe that we make the left side of (4.1-8) smaller if we:
(a) multifly sin(Ai-t) by M in the denominator of (4.1-8), (b)leave out the expression $\frac{1}{2n}\cdot\frac{1}{M-1}$. Thus we arrive at the inequality

$$\sin^2(A_i-t) < \frac{n}{2\cos^2(s)}$$

Therefore, after replace the term $\cos^{-2}(s)$ by $1+s^2$

, we are left with maximizing

$S(1-n*s^2/2)(1+n*s^2/2)$ with respect to s.

The maximum is obtained at $s=\frac{1.18\pi}{n}$, as n increase, s will approach zero, which would be the maximum if the first term s were left out.

Therefore we can ignore D(s,t) and simply treat s and t as parameters to be estimated. Thus we can simply maximize

$$D(A/s,t) = \frac{\cos(s)}{\cos^2(A-s)+\cos^2(s)\sin^2(A-s)} \quad (4.1\text{-}9).$$

The maximum of D(/s, t)

$$= \frac{1}{\pi}\frac{1}{\cos(s)} \qquad \text{at } A = t \pm \pi/2 \qquad (4.1\text{-}10)$$

The minimum of D(/s,t)

$$= \frac{\cos(s)}{\pi} \qquad \text{at } A = t \qquad (4.1\text{-}11).$$

4.2. Approximated Estimator of Surface Orientation.

If we subdivide interval $[-\pi/2,\pi/2]$ in w subintervals i.......n of the same length π/w and denote by H1....Hw the number of Ai's falling in these intervals, respectively, we observe that

$$\frac{H_i}{n} \approx D(A_i/s,t) \quad i=1.......w \quad (4.2\text{-}1)$$

where a is the center of the interval i. Denoting by Hmax and Hmin value of $H:i=1...w£ and considering (4.1-9)-(4.2-1)

$$\frac{1}{\cos(s)} = \frac{\pi H_{max}}{n}$$
$$t = A^* \pm \pi/2 \qquad (4.2\text{-}2)$$

where A* is the center of the interval i for H = Hmin.

Concluding we suggest the following formulas for the estimated S and T

$$S = \cos^{-1}(n/w \cdot H_{max})$$
$$T = A^* \pm \pi/2 \qquad (4.2\text{-}3)$$

,where the sampling interval n is large value.

5. Computer simulation and results

We have presented a simple algorithm for the estimation of the slant and tilt from a random sample of the tangent directions. This algorithm was applied to geographic contour and synthetic textures. The computed results are presented in Fig.5, Fig.6 respectively. The errors between actual surface orientation and computed one are presented in table 1 . Table 2 represents an example for (45,30). In order to achieve a unique maximum-minimum pair in this algorithm, the subinterval size must be on the order of 10 ($w=\pi/10$). It was revealed that the error for the original orientation was taken place mainly by slant. This fact coincides with the results of research by a series of psychologists[2]. In addition it may be said that the research should be made under relaxed supposition since the supposition of being isotropy is strong one for natural image.

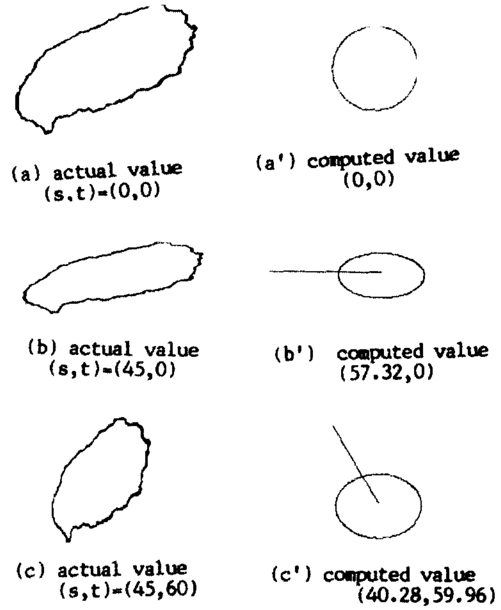


Fig.5 an example applied in geographic contour

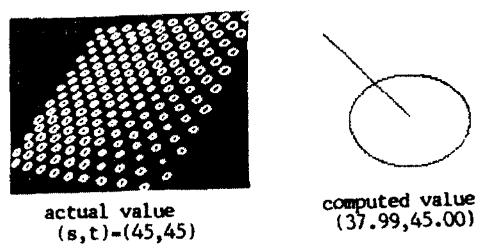


Fig.6 an example applied in synthetic textures

Table 1. (a) The computed surface orientation

Original Value		Computed Value		Original Value		Computed Value	
Slant	Tilt	Slant	Tilt	Slant	Tilt	Slant	Tilt
0	0	0	0	0	45	0	45.02
45	0	57.32	0	0	90	0	90
60	0	57.55	0	0	130	0	129.94
0	30	0	30.01	30	0	31.49	0
0	60	0	59.98	30	30	33.79	30.00
45	45	37.99	45.00	30	60	38.50	59.97
30	45	38.33	45.01	30	90	32.50	89.99
45	60	40.28	59.96	45	30	57.47	29.99
30	130	23.40	129.92	45	90	57.48	89.98
90	30	83.20	29.96	45	130	55.45	129.00
90	45	81.26	44.97	60	30	68.28	29.99
90	60	73.37	59.93	60	45	54.28	45.00
90	90	79.34	89.95	60	90	57.43	89.97
90	130	72.62	129.93	60	130	66.19	129.91
90	0	77.53	0	60	60	75.34	59.95

(b) The computed error of surface orientation

	Tilt					
slant	0	30	45	60	90	130
0	0	0	0	0	0	0
	0	-0.01	0.02	-0.02	0	-0.06
30	1.49	3.79	8.33	8.50	2.50	-7.40
	0	0	0	-0.03	-0.01	0.08
45	12.32	12.47	-7.01	-3.28	12.48	10.45
	0	-0.01	0	0.04	-0.02	-1.00
60	-2.55	8.28	-5.72	15.34	-2.57	6.19
	0	-0.01	0	-0.05	-0.03	-0.09
90	-12.47	-7.20	-8.74	-14.63	-10.66	-17.38
	0	0.04	0.03	0.07	0.05	0.07

Table 2. a portion of distributed tangent angle at(45,30)

Tilt = 45 , Slant = 30			
a		b	
0	-44.98	89.9	44.97
1.8	-43.42	91.7	47.05
3.6	-41.86	93.5	49.12
5.4	-40.30	95.3	51.20
7.2	-38.74	97.1	53.26
8.9	-37.17	98.9	55.33
10.8	-35.60	100.8	57.39
12.6	-34.02	102.6	59.43
14.4	-32.44	104.4	61.47
16.2	-30.86	106.2	63.50
18	-29.26	108	65.52
19.8	-27.66	109.8	67.53
21.6	26.05	111.6	69.52
23.4	-24.44	113.4	71.50
25.2	-22.81	115.1	73.47
27	-21.17	117	75.42
28.7	-19.52	118.8	77.36
30.6	-17.86	120.4	79.28
32.4	-16.19	122.3	81.18
34.2	-14.50	124.1	83.07
36	-12.80	125.9	84.94
37.8	-11.09	127.7	86.79
39.6	-9.36	129.5	88.63

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