ANALYSIS OF WAVE VELOCITY FOR TEMPERATURE PROPERGATION IN A MECHANICAL FACE SEAL

(기계평면시일에서 온도전파를 위한 파속도의 이론적해석)

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1. Introduction

A mechanical face seal is most commonly used to seal liquids and gases at various speeds, pressures and temperatures. The primary seal ring is in sliding contact with the seal seat and as a result heat in the vicinity of the interface is generated.

Local temperatures at points along the circumferential direction will fluctuate as asperities on the surfaces pass. This kind of fluctuation of temperature has been investigated to take place [1,2]. This may lead to the hot spots phenomenon between the contacting asperities. Sibley and Allen [3] showed photographic evidence of systemically moving hot spots in the contact zone. The appearance of such a temperature disturbance has been attributed to a kind of thermoelastic instabilities [4,5] between two surfaces: This involves a feedback loop which comprises localized elevation of frictional heating, resultant localized thermal bulding, localized pressure increase as the result of the bulging and futher elevation of frictional heating as the result of the pressure increase. The heating of hot spots will be continued until the expanded material due to the frictional heating is worn off. Therefore to predict the speed of temperature propagation into the body is essential to the analysis of heat transfer on the edge of the seal.

In order to predict the wave velocity in the body, classical equation of heat flow has been applied to a seal-like configuration with one face having a sinusoidally varying temperature distribution. This analytical method may be very useful to explain the thermoelastic instability phenomenon in the sliding contact.

2. Analysis

A thermally conductive plate slides on a thermal insulator. The conductor moves against a stationed insulator at velocity U along the x axis. The problem of a semi-infinite blade geometry is shown in Fig. 1.

We assume that the heat generated by viscous friction between the parallel plates is transferred into the solid. The face geometry with a sinusoidal waviness will cause the non-uniform heating. This may be led to the thermoelastic deformation in the interface. A problem on the conduction of heat of non-steady state and moving temperature disturbance will be considered.

To simplify the equation of heat flow, we assume the width z of the blade to be small. It is assumed that the thermal diffusivity, α_m within the metal does not vary with temperature. The governing differential equation can then be written

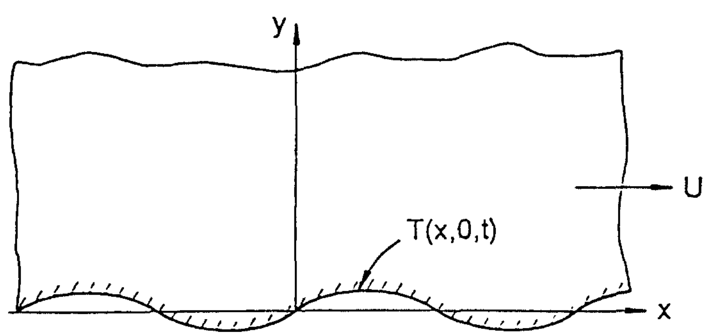

Fig. 1 Semi-infinite blade with a sine variation in the surface temperature

$$\frac{\partial^2 T}{\partial y^2} = \frac{1}{\alpha_m} \frac{\partial T}{\partial t} \tag{1}$$

where T is the temperature distribution in the body and t is time. Eq.(1) may be solved using the following boundary conditions. The temperature variations relative to the fixed surface are assumed as follow:

$$T(x,0,t) = |T_i| e^{\beta t} \sin[\chi\{x - (c + U)t\}] \tag{2a}$$

$$T(x,y,t) = 0 \quad \text{as } y \to \infty \tag{2b}$$

where $|T_i|$ is constant amplitude of temperature, β represents the exponent of growth of temperture wave, c denotes the traversal velocity of temperature wave, and χ is the wave number defined as

$$\chi = 2\pi/\lambda \tag{3}$$

where λ is a wavelength.

The corresponding S problem is defined by

$$\frac{\partial^2 S}{\partial y^2} = \frac{1}{\alpha_m} \frac{\partial S}{\partial t} \tag{4}$$

with the boundary conditions

$$S(x,0,t) = |T_i| e^{\beta t} \cos[\chi\{x - (c + U)t\}] \tag{5a}$$

$$S(x,y,t) = 0 \quad \text{as } y \to \infty \tag{5b}$$

If we introduce the complex combination $\tilde{T} = S + i \cdot T$, it is constructed by multiplying Eq.(1) to Eq.(2b) by i and adding them to Eq.(4) to Eq.(5b), respectively. The modified equation for $\tilde{T}$ is then given by

$$\frac{\partial^2 \tilde{T}}{\partial y^2} = \frac{1}{\alpha_m} \frac{\partial \tilde{T}}{\partial t} \tag{6}$$

with the boundary conditions

$$\tilde{T}(x,0,t) = |T_i| e^{i\{\varkappa x - [\varkappa(c + U) + i\beta]t\}} \tag{7a}$$

$$\tilde{T}(x,y,t) = 0 \quad \text{as } y \to \infty \tag{7b}$$

The solution form of the modified equation (6) may be written as

$$\tilde{T} = Y(y) e^{i\{\varkappa x - [\varkappa(c + U) + i\beta]t\}} \tag{8}$$

where the function Y(y) is determined so that the heat transfer equation (6) and its boundary conditions (7a,b) must be satisfied. Substituting Eq.(8) into Eq.(6) gives

$$Y'' + i \frac{[\varkappa(c + U) + i\beta]}{\alpha_m} Y = 0 \tag{9}$$

Substituting Eq.(8) into the boundary conditions (7a,b) yields

$$Y(0) = |T_i| \tag{10a}$$

$$Y(y) = 0 \qquad y \to \infty \tag{10b}$$

Therefore the ordinary diferential equation (9) can be solved as

$$Y = B_1 e^{i\sqrt{i\xi}\, y} + B_2 e^{-i\sqrt{i\xi}\, y} \tag{11}$$

where

$$\xi = \frac{\varkappa(c + U) + i\beta}{\alpha_m} \tag{12}$$

Consider the complex relationship for Eq.(11) and substitute it into Eq.(8). Then

$$\tilde{T} = B_1 e^{-\sqrt{\frac{\xi}{2}}\, y \;+\; i\left\{ \varkappa x - [\varkappa(c + U) + i\beta]t + \sqrt{\frac{\xi}{2}}\, y\right\}}$$

$$+ B_2 e^{\sqrt{\frac{\xi}{2}}\, y \;+\; i\left\{\varkappa x - [\varkappa(c + U) + i\beta]t - \sqrt{\frac{\xi}{2}}\, y\right\}} \tag{13}$$

Using the first boundary condition (7a), the unknown coefficient B_1 of Eq.(13) is obtained as, $B_1 = |T_i|$. Since The temperature disturbance should be finite as y becomes infinite, B_2 is zero. Thus, the solution of Eq.(6) becomes

$$\tilde{T} = |T_i| e^{-(a+b)y + \beta t} \left\{\cos[\varkappa(x - (c + U)t) + (a - b)y] + i \sin[\varkappa(x - (c + U)t) + (a - b)y]\right\} \tag{14}$$

where

$$a = \frac{\beta}{4\, b\, \alpha_m} \tag{15a}$$

$$b = \left[\frac{-\varkappa(c + U) + [\varkappa^2(c + U)^2 + \beta^2]^{1/2}}{4\alpha_m} \right]^{1/2} \tag{15b}$$

The negative case of Eq.(15b) will be discarded because the temperature should be bounded as y goes to infinite. Since $\tilde{T} = S + i \cdot T$, the solution of the temperature perturbation in the body may be found

$$T = | T_i | e^{-(a+b)y+\beta t} \sin\{\varkappa[x -(c + U)t] + (a - b)y\} \qquad (16)$$

The temperature fluctuations due to the frictional heating on the edge of the body are propagate into the body with the wave velocity c given by [6]

$$c = \{2\alpha_m[\varkappa(c + U)]\}^{1/2} \qquad (17)$$

We may consider limiting case; non-moving plate, i.e., U = 0. The temperature wave can propagate into the solid even though the body does not move. Thus we have to discard the negative case. The wave equation of Eq.(17) may be rearranged as

$$c = \frac{2\pi\alpha_m}{\lambda}\left[1 + \left(1 + \frac{\lambda U}{\pi\alpha_m}\right)^{1/2}\right] \qquad (18)$$

This equation indicates an importance of the wavelength to the wave velocity into the body.

3. Conclusion

Fig.2 shows the distributions of wave velocity c with the sliding velocity U of the conductor. Curves are plotted for various values of the wavelength. As the sliding speed increases, the distribution of the wave velocity increases with approximately half of a parabolic shape. At low value of the wavelength, the wave velocity is much higher than the long wavelength.

Equation (18) serves to provide the estimate of the wave velocity into the body as a function of material property, wavelength and speed of the blade. The wavelength of temperature disturbance appears to be an important factor to predict the wave velocity when the heat transfers to the body. The wave velocity expressions (18) may be essential to understand the thermoelastic instability phenomenon in frictionally heated contact.

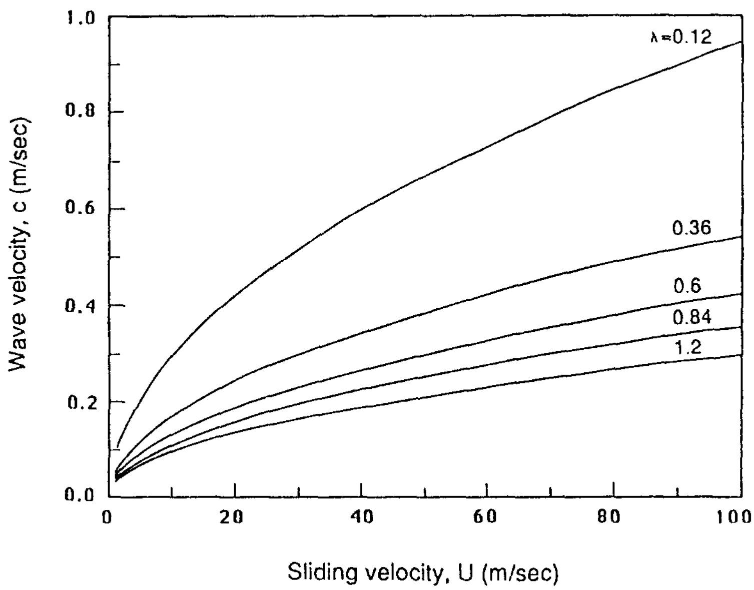

Fig. 2 Relationship between the wave velocity and the sliding speed of body with various values of the wavelength.

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