

Abstract

A NEW ALGORITHM FOR THE MAXIMAL CLOSURE PROBLEM

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A closure in a graph is defined as a subset of nodes such that if a node belongs to the subset, then all its successor nodes also belong to the subset. The maximal closure problem is the problem of finding a closure whose sum of node mass is the maximum among all closures. It has been shown that the maximal closure problem defined in a graph can be converted to the maximal flow problem (or the minimal cut problem) in a corresponding network. Thus, the maximal closure problem can be solved by any maximal flow algorithm (or any minimal cut algorithm).

We propose a new algorithm for the maximal closure problem. The new algorithm finds a maximal closure directly in a given graph while other algorithms first convert the graph to a corresponding network and then find a maximal closure from the network. Instead, our new algorithm employs the notion of the supply and the demand of a node in the given graph. The new algorithm finds the maximal closure through a series of iterations, each of which involves a last-labeled first-scanned search for flow augmenting paths from a different supply node. Although the new algorithm was originally designed to solve maximal closure problems, it is flexible in the sense that it can be

extended to the general maximal flow problem or specialized to the selection problem.

To examine the practical efficiency of the new algorithm, we implemented and tested it on selected classes of the maximal closure problem. We also implemented several other competing algorithms and compared their computing times. Based on our computational experience, we conclude that the new algorithm is superior or comparable to other best algorithms in computing times. In particular, our new algorithm was significantly faster in solving the problem of scheduling tasks under due dates.