Rayleigh Fading AWGN 채널에 대한 Dual-k 길쌈부호의 평균자승오차

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MSE of Dual-k Convolutional Codes for an AWGN Channel with Rayleigh Fading

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ABSTRACT

We are concerned with transmitting numerical source data of $\{0, 1, 2, . . . , 2^k-1\}$ through a channel coding system. The rate 1/v dual-k convolutional code with the orthogonal MFSK modulation and the Viterbi decoding is employed for the implementation of the channel coding system. The mean square error of the dual-k convolutional code is evaluated for the numerical source transmitted over an additive white Gaussian noise channel with Rayleigh fading.

1. INTRODUCTION

Suppose that we have a source of numerical data to be transmitted through a channel coding system. A mean square error (MSE) can be a good criterion for judging the performance of the channel coding system.

This paper evaluates the MSE performance of dual-k convolutional codes employed for transmitting numerical data of source S_k, where S_k is an equiprobable discrete memoryless source with alphabets of the form,

$$S_k = \{ 0, 1, 2, . . . , 2^k-1 \} , \quad (1)$$

and k is a positive integer.

A channel over which the data are transmitted is assumed to be a M $(=2^k)$-ary additive white Gaussian noise (AWGN) channel with Rayleigh fading. A few communication systems such as spread spectrum and multiple access communications have such M-ary channels. Dual-k convolutional codes with orthogonal MFSK modulation and the Viterbi decoding are simple enough for a practical implementation of a channel coding system for the M-ary channels[1].

The block diagram of a communication system to be investigated here is sketched in figure 1.

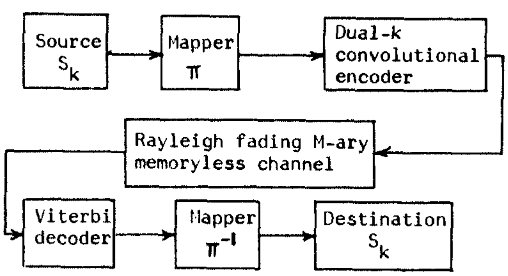

Figure 1: The block diagram of the communication system

Let G be the k-dimensional binary vector space. The upper mapper encodes a numerical element of S_k into a k-tuple binary vector of G under the one-to-one and onto mapping rule , i.e.

$$\pi : S_k \longrightarrow G \quad (2)$$

A k-tuple binary vector enters the rate 1/v dual-k convolutional encoder. The encoder outputs v

M-ary symbols, where $M = 2^k$. They are orthogonally MFSK-modulated and then transmitted over an AWGN channel with Rayleigh fading. Through the Viterbi decoder and the lower mapper in cascade, we have a numerical output of S_k. Let s and t be the input and the final output value of S_k respectively. Then the mean square error e_π^2 can be given by

$$e_\pi^2 = \underset{s\ t}{E\ E}\ (s - t)^2$$
$$= 2^{-k} \sum_s \sum_t (s - t)^2 p(t|s). \quad (3)$$

2. Rayleigh fading $M(=2^k)$-ary memoryless channel

Let $\{x_i(t) = \sin(w_i t + \theta),\ i=1, 2, .., M\}$ be a M-ary orthogonal signal set, where θ is uniformly distributed over $[0, 2\pi]$. A discrete M-ary memoryless channel can be completely characterized by conditional probability between input and output alphabets as shown in figure 2. Let p_e be the

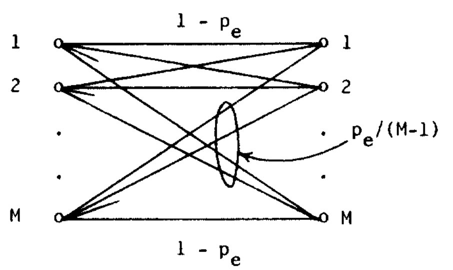

Figure 2: M-ary discrete memoryless channel

probability that $x_j(t)$ is detected when $x_i(t)$ is sent, where $i \neq j$ and $i, j \in \{1,2, . . . , M\}$. Then we have

$$p(j|i) = \begin{cases} 1 - p_e & , i = j \\ p_e/(M-1) & , i \neq j \end{cases} \quad (4)$$

where k denotes $x_k(t)$ and k = i and j. Suppose that $x_i(t)$ sent over a channel with Rayleigh fading. Then the received signal can be written by

$$y(t) = A \sin(w_i t + \theta) + n(t)\ ,\quad 0 \leq t \leq T \quad (5)$$

where n(t) is a zero mean white Gaussian noise of double-sided power spectral density $N_o/2$, and A is a Rayleigh random variable with the probability density of

$$P_A(a) = \frac{a}{\sigma^2} e^{-a^2/(2\sigma)} \quad , \quad a > 0 \quad (6)$$

The channel symbol error probability p_e of the AWGN channel with the Rayleigh fading is given by [2]

$$p_e = \sum_{i=1}^{M-1} \binom{M-1}{i} (-1)^{i+1} (1 + i(1 + \frac{E}{N_o}))^{-1} \quad (7)$$

where $E = \sigma^2 T$.

3. Dual-k convolutional codes

The finite field representation of a rate 1/v dual-k convolutional code is sketched in figure 3 [1].

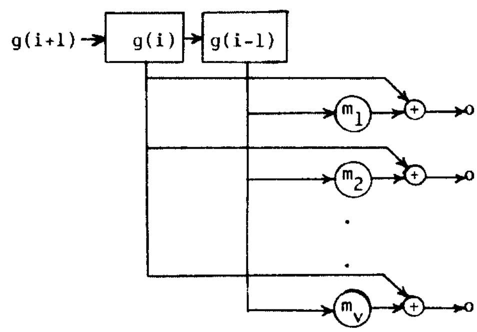

Figure 3: Finite field representation of a rate 1/v dual-k convolutional code

In figure 3, g(i) is an element of a Galois field $GF(2^k)$ at time i, and $m_j \in GF(2^k)$, where j = 1, 2, .., v. Notice that the k-dimensional binary vector space, G can be represented by the $GF(2^k)$.

Odenwalder[1] has derived an explicit form of an upper-bounded transfer function of the rate 1/v dual-k convolutional code.

$$T(D,N,L) = \frac{(2^k-1)D^{2v}L^2N}{1-NL(vD^{v-1} + (2^k-1-v)D^v)} \quad (8)$$

The transfer function represents all the paths of the trellis diagram of the dual-k convolutional code with the reference path of all zeros. In (8), the powers of D, N, and L denote the channel symbol errors, input symbol errors and lengths, respectively, of the unmerged segments in the trellis diagram. Therefore, by differentiating T(D,N,L) with respect to N and letting N = 1 and L = 1, we have the upper-bounded symbol error probability [3]

$$\frac{\partial}{\partial N} T(D,N,L)\bigg|_{N=1,L=1} = ((2^k-1)D^{2v})(1-(vD^{v-1}+(2^k-1-v)D^v)^2)^{-1} \quad (9)$$

where D is the Chernoff parameter given by [2]

$$D = 0.5(p_e(1-p_e)/(M-1))^{-1/2} + p_e(M-2)/(M-1) \quad (10)$$

and p_e is defined by (7).

4. MSE calculation

Let p_o be the symbol error probability defined on S_k through the communication system of figure 1. Since (9) is an upper-bounded symbol error probability, we have

$$p_o \leq ((2^k-1)D^{2v})(1-(vD^{v-1}+(2^k-1-v)D^v)^2)^{-1} \quad (11)$$

We also have

$$p_o = \sum_{\substack{t \in S_k \\ t \neq s}} p(t|s),$$

due to the orthogonal MFSK signalling,

$$= (M-1)\, p(t|s) \quad (12)$$

where $s \neq t$ and any $s \in S_k$ and $t \in S_k$. From(3), we have

$$e_\pi^2 \leq 2^{-k} \sum_s \sum_t (s-t)^2 (M-1)^{-1} \frac{\partial}{\partial N} T(D,N,L)$$

$$= 2^{1-k}\Big(\sum_s s^2 - (M-1)^{-1} \frac{\partial}{\partial N} T(D,N,L) \sum_s \sum_t s\cdot t\Big)$$

$$= 2^{1-k}((2^{2k}-1)/6 - (2^{k-1}(2^k-1))^2 D^{2v}(1 - vD^{v-1} + (2^k-1-v)D^v)^{-2}) \quad (13)$$

For uncoded case, we have $p_o = p_e$. Therefore we have

$$e_o^2 = 2^{1-k}((2^{2k}-1)/6 - 2^{2(k-1)} p_e) \quad (14)$$

where p_e is defined by (7).

5. Conclusion

We have derived the MSE of the rate 1/v dual-k convolutional code used for transmitting the numerical data of $\{0, 1, 2, .., 2^k-1\}$ over the AWGN channel with the Rayleigh fading. Using the expression of the MSE, we can have a value of the code rate parameter v which provides a desired distortion in MSE. Since the orthogonal MFSK modulation is employed, the MSE is independent of any choice of the mapping rule π .

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