THE COMPLETENESS OF A PROPOSITIONAL FRAGMENT OF LESNIEWSKI'S ONTOLOGY AND ITS RELEVANCE TO LOGICAL GRAMMAR

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§1 Introduction In Ishimoto [2] and subsequently in Kobayashi and Ishimoto [5] it was proved that a propositional fragment of Lesniewski's ontology is embedded in first-order predicate logic with equality *via* a translation. (See also Shimizu and Kagiwada [12].) Now, in this paper a much simpler proof will be given of this embedding theorem along with some philosophical as well as linguistic observations. The embedding theorem will also be employed for proving the elimination theorem and other ones for the proposed fragment which are usually proved syntactically.

§2 Tableau method As is the case with our previous work, we first introduce following Schütte [9], [10], [11] the notion of positive and negative parts of a fromula, which has the effect of simplifying the subsequent discussion though not indispensable.

Definition 2.1 The positive and negative parts of a formula A are defined recursively as follows:

2.11 A is a positive part of A,

2.12 If $B \vee C$ is a positive part of A, then both B and C are positive parts of A,

2.13 If $\sim B$ is a positive part of A, then B is a negative part of A, 2.14 If $\sim B$ is a negative part of A, then B is a positive part of A.

For making the notion clearer, we are presenting some examples:

$$F[A+] = A,$$
$$F[A+] = \sim\sim A,$$
$$F[A+] = \sim\sim(A \vee \sim B),$$
$$G[A-] = \sim A,$$
$$G[A-] = \sim\sim\sim A \vee B,$$
$$G[A-] = \sim\sim(\sim A \vee B) \vee C,$$

where $F[A+](G[A-])$ means that A occurs in $F[A+](G[A-])$ as a positive (negative) part thereof. The expressions like $F[A+, B+]$, $G[A+, B-, C-]$ and the like are understood analogously subject to the condition that A, B and C be not overlapping.

As proposed by Ishimoto [2] the propositional fragment of Lesniewski's ontology called L1 is defined in its Hilbert-type version to be the smallest class of formulas containing all the instances of tautology as well as the formulas of the following froms:

2.21 $\vdash \in ab \supset \in aa,$

2.22 $\vdash \in ab \wedge \in bc. \supset \in ac,$

2.23 $\vdash \in ab \wedge \in bc. \supset \in ba,$

being closed under detachment.

Lesniewski's (elementary) ontology is, on the other hand, defined to be the smallest class of formulas containing all the formulas of the form:

$$\vdash \in ab \equiv .(\exists x) \in xa \wedge (x)(y)(\in xa \wedge \in ya. \supset \in xy) \wedge (x)(\in xa \supset \in xb),$$

or $\vdash \in ab \equiv .(\exists x)(\in xa \wedge \in xb) \wedge (x)(y)(\in xa \wedge \in ya. \supset \in xy)$

as well as all theses of first-order predicate loric (without equality) being closed under quantificational rules.

The tableau method proposed of L_1 by Kobayashi and Ishimoto 〔5〕 is defined by the following reduction rules:

$$\vee_{-}\ \frac{G\,[A \vee B_{-}]}{G\,[A \vee B_{-}] \vee \sim A \;\mid\; G\,[A \vee B_{-}] \vee \sim B}\ ,$$

$$\in_{1}\ \frac{G\,[\in ab_{-}]}{G\,[\in ab_{-}] \vee \sim \in aa}\ ,$$

$$\in_{2}\ \frac{G\,[\in ab_{-}\ ,\ \in bc_{-}]}{G\,[\in ab_{-}, \in bc_{-}] \vee \sim \in ac}\ ,$$

$$\in_{3}\ \frac{G\,[\in ab_{-}\ ,\ \in bc_{-}]}{G\,[\in ab_{-}, \in bc_{-}] \vee \sim \in ba}\ ,$$

of which the first one is known to be sufficient for developing classical propositional logic.

The (well-formed) formulas of L_1 are defined in the well-known way in terms of a (countably) infinite list of name variables a, b, c, ... and the like as well as of two logical symbols, namely, $\vee$ (disjunction) and $\sim$(negation) along with the Lesniewski's $\in$ (epsilon) and some technical symbols. (Other logical symbols are defined, if necessary, in terms of them.) The formulas thus defined will be denoted by such meta-logical variables as A, B, C,... (Outermost parentheses will be suppressed almost in every case.) It is remarked in this connection that all the symbols thus introduced will be employed only meta-logically. (For Lesniewski's ontology consult, among others, Slupecki 〔13〕, Lejewski 〔6〕 and Luschei 〔7〕.)

Now, on the basis of the reduction rules as above introduced, we wish to prove some these of L_1 by the tableau method. (The detailed definition of tableau is not given here. For the formal definition refer to Smullyan 〔14〕.)

2.31 The proof of $\in ab \supset \in aa$ (2.21):

$$\in_1 \frac{\sim \in ab \vee \in aa}{(\sim \in ab \vee \in aa) \vee \sim \in aa.}$$

2.32 The proof of $\in ab \wedge \in bc. \supset \in ac$ (2.22):

$$\in_2 \frac{\sim\sim(\sim \in ab \vee \sim \in bc) \vee \in ac}{(\sim\sim(\sim \in ab \vee \sim \in bc) \vee \in ac) \vee \sim \in ac.}$$

2.33 The proof of $\in ab \wedge \in bc. \supset \in ba$ (2.23):

$$\frac{\sim\sim (\sim \in ab \vee \sim \in bc) \vee \in ba}{(\sim\sim (\sim \in ab \vee \sim \in bc) \vee \in ba) \vee \sim \in ba.}$$

2.34 The proof of $(\sim \in aa \vee \sim \in ab) \supset \sim \in ab$:

$$\vee_{-} \frac{\sim (\sim \in aa \vee \sim \in ab) \vee \sim \in ab}{\in_1 \frac{(1)}{(2)} \qquad\qquad (3)} \quad ,$$

where

(1) = $(\sim (\sim \in aa \vee \sim \in ab) \vee \sim \in ab) \vee \sim\sim \in aa$,

(2) = $((\sim (\sim \in aa \vee \sim \in ab) \vee \sim \in ab) \vee \sim\sim \in aa) \vee \sim \in aa$,

(3) = $(\sim (\sim \in aa \vee \sim \in ab) \vee \sim \in ab) \vee \sim\sim \in ab$,

As exemplified above, a tableau, every branch of which ends with a formula of the form $F[A_+, A_-]$, is said to be closed. A closed tableau constitutes a proof of the given formula.

§3 Fundamental theorem For presenting the fundamental theorem (Theorem 3.2), the notion of Hintikka formula is in order, which runs as follows:

Difinition 3.1 A Hintikka formula A is defined only as follows:

3.11 A is not of the form $F[B_+, B_-]$,

3.12 If A contains $B \vee C$ as a negative part, then it contains B or C as a negative part thereof, 3.13 If A contains $\in ab$ as a negative part, then it contains $\in aa$ as a negative part thereof,

3.14 If A contains $\in ab$ and $\in bc$ as negative parts, then it contains $\in ac$ as a negative part thereof,

3.15 If A contains $\in ab$ and $\in bc$ as negative parts, then it contains $\in ba$ as a negative part thereof.

Now, we state the theorem:

Theorem 3.2 _Given a formula of L_1, reducing it by way of reduction rules, every branch of the tableau ends in a finite number of steps with a formula of the form $F(A_+, A_-)$ or with a Hintikka formula, whereby a branch is not extended if the application of a reduction rule gives rise to a formula which is already in occurrence as its negative part in the formula to be reduced. And, in case a branch ends with a Hintikka formula, every constituent formula of the branch constitutes a positive part of the Hintikka formula._

The proof is not given formally, and we shall rest content with the presentation of an example, which, we hope, will be sufficient for taking care of the general case.

Let the given formula be:

$$\sim(\sim\in aa \vee \in ba) \vee \sim \in ac ,$$

with the different name variables a, b and c, which could possibly be combined in the course of the reduction by way of $\in_1$, $\in_2$, and $\in_3$ only in the following ways giving rise to an atomic formula involving the indicated variables in this order, namely,

(a,a), (a,b), (a,c), (b,a), (b,b), (b,c),
(c,a), (c,b), (c,c).

Now, by reduction the given formula we obtain the following tableau:

$$\sim(\sim\in aa \vee \in ba) \vee \sim\in ac$$

$\in_1$ (1)	$\in_2$ (3)
(2) (aa)	(4) (bc)
	$\in_3$
	(5) (ab)
	$\in_1$
	(6) (aa)
	$\in_1$
	(7) (bb)

where

$(1) = (\sim(\sim\in aa \vee \in ba) \vee \sim \in ac) \vee \sim\sim\in aa,$

$(2) = ((\sim(\sim\in aa \vee \in ba) \vee \sim\in ac) \vee \sim\sim\in aa) \vee \sim\in aa,$

$(3) = (\sim(\sim\in aa \vee \in ba) \vee \sim\in ac) \vee \sim\in ba,$

$(4) = ((\sim(\sim\in aa \vee \in ba) \vee \sim\in ac) \vee \sim\in ba) \vee \sim\in bc,$

$(5) = (((\sim(\sim\in aa \vee \in ba) \vee \sim\in ac) \vee \sim\in ba) \vee \sim\in bc) \vee \sim\in ab,$

$(6) = ((((\sim(\sim\in aa \vee \in ba) \vee \sim\in ac) \vee \sim\in ba) \vee \sim\in bc) \vee \sim\in ab) \vee \sim\in aa,$

$(7) = (((((\sim(\sim\in aa \vee \in ba) \vee \sim\in ac) \vee \sim\in ba) \vee \sim\in bc) \vee \sim\in ab) \vee \sim\in aa) \vee \sim\in bb.$

Each reduction by $\in_1$, $\in_2$, or $\in_3$ uses, every time, one possible combination of (orderd) pairs of name variables a, b and c as indicated on the left hand of each constituent formula of the tableau, and any combination once used is never employed again, because this would be against the restriction imposed upon the extension of a formula. And, in the case of $\vee_-$ any prin ipal formula of $\vee_-$ used as such is never employed again in the same branch. If $\vee_-$ were applied to the formula once used as its principal formula, this application would be against the requirement of the Theorem. Since there are only finite number of subformulas of the given formula and the combination of name variables, in this case, nine in total, the extension of a branch come to the end in a finite number of steps even if we do not come across a formula of the form $F[A_+, A_-]$.

Once we come across a formula, which is not of the form $F[A_+, A_-]$ and to which no reduction rule is applicable any more without violating the requirement of the Theorem, the formula already constitutes a

Hintikka formula. In fact, if the formula contained a formula of the form $A \vee B$ as a negative part without having neither A nor B as its negative parts $\vee_-$ would remain to be applicable to the formula extending the branch against the assumption. The other properties of the Hintikka formula are dealt with similarly.

The second statement of the Theorem is proved by induction on the length of the branch which ends with a Hintikka formula.

This completes the proof of Theorem 3.2.

A tableau constructed subject to the requirement of the Theorem is called a normal tableau. The tableaux shown in § 2 are all normal. For reference we wish to present hereunder a tableau which is not normal:

$$\in_2 \frac{\in_1 \dfrac{\sim \in ab}{\sim \in ab \vee \sim \in aa}}{(\sim \in ab \vee \sim \in aa) \vee \sim \in ab},$$

where the application of $\in_2$ gives rise to $\in ab$ as a negative part, which already occurs in the second line to be reduced.

As will be demonstrated in what follows, if a formula is provable at all by the tableau method, it is proved by a normal tableau.

§ 4 <u>Translation and Soundness</u> In preparation for the soundness and completeness of L_1, we wish to define a translation denoted by T:

Definition 4.1 The translation T, which transforms every formula of L_1 into that of first-order predicate logic with equality, is defined as follows:

4.11 $T \in ab = F_b \,\iota x F_a x$,

4.12 $T\, A \vee B = Ta \vee TB$,

4.13 $T \sim A = \sim Ta$,

where F_a, F_b,... are the monadic predicate variables of first-order logic corresponding to the name variables a, b, ... of L_1. $F_b \,\iota x F_a x$ is the Russellian-type definite description, namely,

$\exists x(F_a x \wedge F_b x) \wedge \forall x \forall y(F_a x \wedge F_a y. \supset x = y)$.

As easily seen the translation T is defined by induction on the length of the formula of L_1. It is also not difficult to see that the T-transforms of the formulas of L_1 do not exhaust the formulas of first-order logic.

Lemma 4.2 <u>The formulas of the following forms are provable in forst-order predicate logic with equality</u>:

$$T\,F[A_+] \equiv T\,F[A_+] \vee A \;,$$

$$T\,G[A_-] \equiv T\,G[A_-] \vee \sim A \;.$$

For proving the Lemma it suffices to demonstrate the following two implications :

$$\vdash T\,A \supset T\,F[A_+] \;,$$

$$\vdash T\sim A \supset T\,G[A_-] \;,$$

which are proved simultaniously by induction on the number of the procedures applied for specifying A as a positive (negative) part of $F[A_+]$ $(G[A_-])$.

The basis does not present any difficulties since $TA \supset TA$ is a tautology.

For taking care of induction steps, suppose $F[A_+] = F_1[A \vee B_+]$. We, then, have $\vdash T\,A \vee B \supset T\,F_1[A \vee B_+]$ by induction hypothesis. This, in conjunction with $\vdash T\,A \supset .\; T\,A \vee T\,B$ $(= T\,A \vee B)$, gives rise to $\vdash T\,A \supset T\,F_1[A \vee B_+]$ as required. The case that $F[A_+] = F_1[B \vee A_+]$ is taken care of analogously.

We, next, suppose that $F[A_+] = G_1[\sim A_-]$, from which obtains by induction hypothesis $\vdash T \sim\sim A \supset T\,G_1[\sim A_-]$. Since $\vdash T \sim\sim A \equiv T\,A$, we easily obtain $\vdash T\,A \supset T\,G_1[\sim A_-]$ as required.

Finally, assume $G[A_-] = F_1[\sim A_+]$. By induction hypothesis $\vdash T \sim A \supset T\,F_1[\sim A_+]$, which is nothing but the looked for $\vdash T\sim A \supset T\,G[A_-]$.

We are now in a position to prove the soundness of L_1 with respect to the proposed translation T.

Lemma 4.3 (Soundness theorem) <u>If A is a theses of L_1 in its tableau method (not necessarily proved by a n rmol tableau), then T A is provable in first-order predicate logic with equality.</u>

The proof is carried out by induction on the length of the tableau on the basis of the preceding lemma. (The proof proceeds upwards beginning with the end of the branch.)

The basis is forthcoming from the following equivalences in first-order logic.

$\vdash$ T F [A_+, A_-]

$\equiv$ T F [A_+, A_-] $\vee$ A Lemma 4.2,

$\equiv$ T (F [A_+, A_-] $\vee$ A) $\vee \sim$ A Lemma 4.2,

$\equiv$ T (F [A_+, A_-] $\vee$ T A) $\vee \sim$ T A Definition 4.12, 4.13

We are proceeding to induction steps, which are taken care of by the following equivalences:

For $\vee_-$:

$\vdash$ T G [A $\vee$ B_-] $\vee \sim$ A and $\vdash$ T G [A $\vee$ B_-] $\vee \sim$ B induction hypothesis,

$\Leftrightarrow$ $\vdash$ T (G[A $\vee$ B_-] $\vee \sim$ A) $\vee \sim$ (A $\vee$ B)

and $\vdash$ T (G [A $\vee$ B_-] $\vee \sim$ B) $\vee \sim$ (A $\vee$ B) Lemma 4.2,

$\Leftrightarrow$ $\vdash$ T G [A $\vee$ B_-] $\vee \sim$ (A $\vee$ B) propositional logic,

$\Leftrightarrow$ $\vdash$ T G [A $\vee$ B_-] Lemma 4.2.

For $\in_1$:

$\vdash$ T G [$\in ab_-$] $\vee \sim \in$ aa induction hypothesis,

$\Leftrightarrow$ $\vdash$ T (G[$\in ab_-$] $\vee \sim \in$ aa) $\vee \sim \in$ ab Lemma 4.2,

$\Leftrightarrow$ $\vdash$ T (G[$\in ab_-$] $\vee \sim$ T $\in$ aa) $\vee \sim$ T $\in$ ab Definition 4.12, 4.13,

$\Leftrightarrow$ $\vdash$ T G[$\in ab_-$] $\vee \sim$ $F_a \iota xF_a x$ $\vee \sim$ $F_b \iota xF_a x$ Definition 4.11,

$\Leftrightarrow$ $\vdash$ T G[$\in ab_-$] $\vee \sim$ $F_b \iota xF_a x$ predicate logic,

$\Leftrightarrow \vdash T\ G[\in ab\_] \vee \sim T \in ab$ Definition 4.11,

$\Leftrightarrow \vdash T\ G[\in ab\_] \vee \sim ab \in$ Definition 4.12, 4.13,

$\Leftrightarrow \vdash T\ G[\in ab\_]$ Lemma 4.2,

where in the equivalence of the fourth and fifth lines use is made of the following theses of first-order predicate logic with equality.

$$\vdash F_b \iota x F_a x \equiv .. F_a \iota x F_a x \wedge F_b \iota x F_a x.$$

The cases corresponding to $\in_2$ and $\in_3$ are similarly taken care of, and we rest content with presenting the following equivalences in first-order logic, which are employed in the proof.

$$\vdash F_b \iota x F_a x \wedge F_c \iota x F_b x. \equiv . F_b \iota x F_a x \wedge F_c \iota x F_b x \wedge F_c \iota x F_a x,$$

$$\vdash F_b \iota x F_a x \wedge F_c \iota x F_b x. \equiv . F_b \iota x F_a x \wedge F_c \iota x F_b x \wedge F_a \iota x F_b x.$$

§5. Completeness With a view to proving the completeness of L_1 in its tableau method version with respect to the translation T, namely, the converse of the soundness theorem just proved, we first prove in advance the following lemma.

Lemma 5.1 *Every Hintikka formula contains at least one atomic formula as its positive or negative part.*

This is easily proved by *reductio ad absurdum* by supposing to the contrary.

Assume, if possible, a formula of the form $A \vee B$ were the shortest positive part of the given Hintikka formula. By Definition 2.12, then, A and B would be both positive parts of the formula against the hypothesis. Again, if possible, suppose $A \vee B$ were the shortest negative part of the formula. By Definition 3.12, then, A or B would be a negative part of the formula against the hypothesis. If $\sim A$ were the shortest positive part of the formula, A would be a negative part thereof (2.13) against the hypothesis. If $\sim A$ were a negative part of the given formula, A would be a positive part thereof (2.14) against the hypothesis.

We next introduce the notion of chains thus:

Definition 5.2 Given a Hintikka formula, a chain is a finite series of name variables $a_1, a_2, \ldots, a_n$ $(1 \leq n)$ such that:

5.21 $a_1, a_2, \ldots, a_n$ are connected by the relation $R(a_i, a_j)$ to be defined as the occurrence of $\in a_i\, a_j$ and $\in a_j\, a_i$ $(1 \leq i, j \leq n)$ as negative parts in the Hintikka formula,

5.22 The series $a_1, a_2, \ldots, a_n$ is maximal.

As easily seen, the relation as above defined is reflexive, symmetric and transitive. This obtains from the properties of the Hintikka formula.

For the purpose of illustration a number of Hintikka formulas will be presented along with the chains associated thereto:

5.31 $((\sim\in ab \vee \sim\in bc) \vee (\sim\in ac \vee \sim\in ba)) \vee (\sim\in aa \vee \sim\in bb)$,

where the series consisting of a and b constitute a chain if c is different from a and b.

The following one, however, does not contain any chain:

5.32 $(\sim(\sim\in ab \vee \in aa) \vee \sim\sim\in ab) \vee \in ba$,

where no name variables are connected by a chain.

5.33 $((\sim\in ab \vee (\sim\in aa \vee \sim\in ca)) \vee \sim\in cc) \vee \sim\in ac$,

where the series consisting of a and c is the only chain with b different from a and c.

5.34 In the next example a formula is reduced by reduction rules with one branch ending with a Hintikka formula. (All the name variables involved are assumed to be different from each other.):

$$\vee_{-}\ \frac{(\sim aa \sim\in bb) \vee ((\sim\in ab \vee \sim\in dc) \vee \in cb)}{\in_1 \dfrac{(1)}{(2)} \qquad\qquad \in_3 \dfrac{(3)}{(4)}}$$

$$\in_1 \frac{(4)}{(5)}$$

$$\in_1 \frac{(5)}{(6)}$$

where

$(1) = \sim(\sim\in aa \vee \in bb) \vee \sim\in ab \vee \sim\in dc \vee \in cb \vee \sim\sim\in aa$,

$(2) = \sim(\sim\in aa \vee \in bb) \vee \sim\in ab \vee \sim\in dc \vee \in cb \vee \sim\sim\in aa \vee \sim\in aa$,

$(3) = \sim(\sim\in aa \vee \in bb) \vee \sim\in ab \vee \sim\in dc \vee \in cb \vee \sim\in bb$,

$(4) = \sim(\sim\in aa \vee \in bb) \vee \sim\in ab \vee \sim\in dc \vee \in cb \vee \sim\in bb \vee \sim\in ba$,

$(5) = \sim(\sim\in aa \vee \in bb) \vee \sim\in ab \vee \sim\in dc \vee \in cb \vee \sim\in bb \vee \sim\in ba \vee \sim\in aa$,

$(6) = \sim(\sim\in aa \vee \in bb) \vee \sim\in ab \vee \sim\in dc \vee \in cb \vee \sim\in bb \vee \sim\in ba$
$\vee \sim\in aa \vee \sim\in dd$.

(Here and in what follows the association of disjuncts will not be indicated since it is easily recoverable.)

The end formula of the right branch of the above tableau is a Hintikka formula, where the series consisting of a and b constitues a chain, while d also happens to be another consisting of only one name variable.

Before going to the construction of a model of first-order predicate logic with equality, through which the T-transform of this Hintikka formula (6) is falsified, it is remarked that the Hintikka formula itself is falsified by a model for L_1 in the following way.

With this in view, every atomic formula, which occurs in the Hintikka formula as a positive (negative) part, is made false (true). Such an atomic formula is certainly in existence by Lemma 5.1.

Thus,

5.41 $\in cb$ is made false,

5.42 $\in ab$, $\in dc$, $\in bb$, $\in ba$, $\in aa$ and $\in dd$ are made true with other atomic formulas assigned any truth value, say, falsity.

In terms of these truth value assignments, every positive (negative) part (of the Hintikka formula) takes the value false (true) as easily proved by induction on the length of the positive (negative) parts.

It is also observed on the basis of the properties of Hintikka formulas that the model thus constructed with the countable domain consisting of all the name variables constitutes a model for L_1. For example, if $\in ab$ happens to be true in the model, it occur as a negative part of the Hintikka formula. Then, by the property of Hintikka formulas $\in aa$ also occurs there as such, and this makes $\in aa$ true in the model. Thus, every formula of the form 2.21 is true in this model. The axioms of other forms are analogously shown true in this model.

By induction on the length of the branch which ends with the Hintikka formula, it is easily proved that all the constituent formulas of the branch including the given formula are made false since they respectively constitutes positive parts of the Hintikka formula.

In the proposed (countable) model every name variable is interpreted by itself, although some of them, say, a and b might be identical in the sense that $\in ab$ and $\in ba$ is true in the model. For example, all the members of a chain are identical in this sense.

On the basis of the modelling as described above, we are proceeding to construct a model for first-order predicate logic with equality, in which the T-transform of every positive (negative) part of the Hintikka formula is false (true).

The right branch of the tableau 5.34, it is recalled, ends with the Hintikka formula (6), which contains two chains, one consisting of a and b and the other of d only.

Now, we are assigning a set of natural number to the (monadic) predicates corresponding to the members of these chains in the following way :

To F_a and F_b we assign $\{0\}$,

To F_d we assign $\{1\}$,

To F_c, which is the predicate corresponding to a name variable not belonging to any chain, but constituting a name variable with which the second chain ends, we assign $\{1, 2\}$, where 2 is a number not used so far for the assignment to the member of any chain. To the predicates corresponding to all other name variables we are assigning ϕ, i.e., the empty set.

On the basis of such assignments we obtain a model M for first-order predicate logic with equality such that $M = \langle N, =, F_a, F_b, F_c, F_d, \ldots \rangle$, where N is the set $\{0, 1, 2\}$, = the identity relation between these natural numbers, and $F_a, F_b, F_c, F_d, \ldots$ the sets of natural numbers as above specified.

We wish to show that the T-transforms of positive (negative) parts of the given Hintikka formula is false (true) in the model M.

With this in view the T-transforms of $\in$ aa is evaluated on the basis of the model M and we have :

$$
\begin{aligned}
T{\in}aa &= F_a \,\iota\, x\bar{F}_a\, x = \exists x(F_a\, x \wedge F_a\, x) \wedge \forall x \forall y(F_a\, x \wedge F_a\, y. \supset x = y) \\
&= .(\{0\}0 \wedge \{0\}0) \vee (\{0\}1 \wedge \{0\}1) \vee (\{0\}2 \wedge \{0\}2). \\
&\wedge (\{0\}0 \wedge \{0\}0. \supset 0 = 0) \\
&\wedge (\{0\}0 \wedge \{0\}1. \supset 0 = 1) \\
&\wedge (\{0\}0 \wedge \{0\}2. \supset 0 = 2) \\
&\wedge (\{0\}1 \wedge \{0\}0. \supset 1 = 0) \\
&\wedge (\{0\}1 \wedge \{0\}1. \supset 1 = 1) \\
&\wedge (\{0\}1 \wedge \{0\}2. \supset 1 = 2) \\
&\wedge (\{0\}2 \wedge \{0\}0. \supset 2 = 0) \\
&\wedge (\{0\}2 \wedge \{0\}1. \supset 2 = 1) \\
&\wedge (\{0\}2 \wedge \{0\}2. \supset 2 = 2),
\end{aligned}
$$

where all the conjuncts are true, and this makes the T-transform of $\in$ aa true in the model M. The T-transforms of $\in$ ab, $\in$ bb and $\in$ ba are analogously shown true in M.

The evaluation of T $\in$ dd proceeds as follows:

$$T \in dd = F_d \iota x F_d x = \exists x(F_d x \wedge F_d x) \wedge \forall x \forall y(F_d x \wedge F_d y . \supset x=y).$$

$$= . \wedge(\{1\}0 \wedge \{1\}0 \vee (\{1\}1 \wedge \{1\}1) \vee (\{1\}2 \wedge \{1\}2).$$

$$\wedge(\{1\}0 \wedge \{1\}0. \supset 0 = 0)$$

$$\wedge(\{1\}0 \wedge \{1\}1. \supset 0 = 1)$$

$$\wedge(\{1\}0 \wedge \{1\}2. \supset 0 = 2)$$

$$\wedge(\{1\}1 \wedge \{1\}0. \supset 1 = 0)$$

$$\wedge(\{1\}1 \wedge \{1\}1. \supset 1 = 1)$$

$$\wedge(\{1\}1 \wedge \{1\}2. \supset 1 = 2)$$

$$\wedge(\{1\}2 \wedge \{1\}0. \supset 2 = 0)$$

$$\wedge(\{1\}2 \wedge \{1\}1. \supset 2 = 1)$$

$$\wedge(\{1\}2 \wedge \{1\}2. \supset 2 = 2)$$

which is obviously true in M.

T $\in$ dc is evaluated in M in the following way:

$$T \in dc = F_c \iota x F_d x = \exists x(F_d x \wedge F_c x) \wedge \forall x \forall y(F_d x \wedge F_d y . \supset x = y)$$

$$= (\{1\}0 \wedge \{1,2\}0) \vee (\{1\}1 \wedge \{1,2\}1) \vee (\{1\}2 \wedge \{1,2\}2)$$

$$\wedge(\{1\}0 \wedge \{1\}0. \supset 0 = 0)$$

$$\wedge(\{1\}0 \wedge \{1\}1. \supset 0 = 1)$$

$$\wedge(\{1\}0 \wedge \{1\}2. \supset 0 = 2)$$

$$\wedge(\{1\}1 \wedge \{1\}0. \supset 1 = 0)$$

$$\wedge(\{1\}1 \wedge \{1\}1. \supset 1 = 1)$$

$$\wedge(\{1\}1 \wedge \{1\}2. \supset 1 = 2)$$

$$\wedge(\{1\}2 \wedge \{1\}0. \supset 2 = 0)$$

$$\wedge(\{1\}2 \wedge \{1\}1. \supset 2 = 1)$$

$$\wedge(\{1\}2 \wedge \{1\}2. \supset 2 = 2),$$

which is true in M. We, next, evaluate in M the T-transform of $\in$cd, which is the only atomic formula occurring in the Hintikka formula as a positive part thereof:

$$T \in cb = F_b \iota x F_c\, x = \exists x(F_c\, x \wedge F_b\, x) \wedge \forall x \forall y(F_c\, x \wedge F_c\, y . \supset x = y)$$

$$= (\{1,2\}0 \wedge \{0\}0) \vee (\{1,2\}1 \wedge \{0\}1) \vee (\{1,2\}2 \wedge \{0\}2)$$

$$\wedge (\{1,2\}0 \wedge \{1,2\}0. \supset 0 = 0)$$

$$\wedge (\{1,2\}0 \wedge \{1,2\}1. \supset 0 = 1)$$

$$\wedge (\{1,2\}0 \wedge \{1,2\}2. \supset 0 = 2)$$

$$\wedge (\{1,2\}1 \wedge \{1,2\}0. \supset 1 = 0)$$

$$\wedge (\{1,2\}1 \wedge \{1,2\}1. \supset 1 = 1)$$

$$\wedge (\{1,2\}1 \wedge \{1,2\}2. \supset 1 = 2)$$

$$\wedge (\{1,2\}2 \wedge \{1,2\}0. \supset 2 = 0)$$

$$\wedge (\{1,2\}2 \wedge \{1,2\}1. \supset 2 = 1)$$

$$\wedge (\{1,2\}2 \wedge \{1,2\}2. \supset 2 = 2),$$

which is false in M.

Lastly, we wish to evaluate $T \in ef$, where e and f are any name variables not occurring in the given Hintikka formula (being not necessarily different):

$$T \in ef = F_f \iota x F_e\, x = \exists x(F_e\, x \wedge F_f\, x) \wedge \forall x \forall y(F_e\, x \wedge F_e\, y . \supset x = y)$$

$$= (\phi 0 \wedge \phi 0) \vee (\phi 1 \wedge \phi 1) \vee (\phi 2 \wedge \phi 2).$$

$$\wedge (\phi 0 \wedge \phi 0. \supset 0 = 0)$$

$$\wedge (\phi 0 \wedge \phi 1. \supset 0 = 1)$$

$$\wedge (\phi 0 \wedge \phi 2. \supset 0 = 2)$$

$$\wedge (\phi 1 \wedge \phi 0. \supset 1 = 0)$$

$$\wedge (\phi 1 \wedge \phi 1. \supset 1 = 1)$$

$$\wedge (\phi 1 \wedge \phi 2. \supset 1 = 2)$$

$$\wedge (\phi 2 \wedge \phi 0. \supset 2 = 0)$$

$$\wedge (\phi 2 \wedge \phi 1. \supset 2 = 1)$$

$$\wedge (\phi 2 \wedge \phi 2. \supset 2 = 2),$$

which is false in M.

Thus, the T-transform of every atomic formula (of L_1) has been evaluated in the model M, and every atomic positive (negative) part of the given Hintikka formula are evaluated false (true). By induction on the length of positive (negative) parts of the Hintikka formula it is, then, proved that the T-transform of every positive (negative) part of Hintikka formula is evaluated false (true) in M. Since the given formula constitutes a positive part of the Hintikka formula, its T-transform is false in M. In other words, the given formula, which is not proved by the tableau method for L_1 is falsified by the model M for first-order predicate logic with equality.

In this Hintikka formula, c, intuitively speaking, contains only one atom, namely, d, but itself does not constitute an atom. In other words, d is a unit set without being an atom. In Ishimoto [2], [3] and Kobayashi and Ishimoto [5], this singularity turned out to be an obstacle for constructing a model for predicate logic, and it was remedied by a rather complicated device. But, here, the singularity was overcome almost automatically.

In the above construction of the model, F_a and F_b were assigned $\{0\}$, and F_d assigned $\{1\}$, while F_c, which is not a member of any chain, assigned $\{1,2\}$.

Nevertheless, we could assign to F_c such sets (of intrgers) as $\{1,2,3\}$, $\{1,2,3,4\}$ and the like with the domain of the model consisting of all the numbers involved. Such models again falsify the T-transform of the given Hintikka formula. Another model which is easily envisaged would be that with the domain consisting of all the natural numbers, the predicates involved being properly defined.

We, next, wish to construct a model for the T-transform of 5.32, namely:

5.32 $\sim(\sim\in ab \vee \in aa) \vee \sim\sim\in ab \vee \in ba$,

which happens to be a Hintikka formula, but does not contain any atoms. Thus, there is not in existence any chain in this Hintikka formula.

As in the preceding example, every predicate corresponding to a name variable of L_1, whether it occur in the Hintikka formula or not, is given the value ϕ , i.e., the empty set (of natural mumbers). Nevertheless, the domain of the model for first-order predicate logic is defined to consit of any non empty set of natural numbers, and it is easily seen that T 5.32 is falsified alike in any of these models (for first-order predicate logic with equality).

In Ishimoto [2], [3] and Kobayashi and Ishimoto [5] we had some difficulties in taking care of such a singularity, namely, the absence of atoms. But, here, this kind of singularity is again solved almost automatically.

We are, now, in a position to generalize the model construction thus far exemplified.

With this in view a formal procedure will be described for constructing a model for first-order predicate logic with equality in which the T-transform of the given Hintikka formula is falsified.

Let us assume that the given Hintikka formula contains n chains $(0 \leq n)$ as defined in Definition 5.2:

$$5.41 \left.\left\{\begin{array}{l} a_1, a_2, \ldots, a_\ell \ (1 \leq \ell), \\ b_1, b_2, \ldots, b_m \ (1 \leq m), \\ c_1, c_2, \ldots, c_h \ (1 \leq h), \\ \qquad \ldots \end{array}\right.\right\} n$$

to which are associated the following atomic formulas occurring in the Hintikka formula as negative parts:

$$5.411 \quad \in a_i\, a_j,\ \in b_i\, b_j.\ \in c_i\, c_j,\ \ldots,$$

where a_i and a_j, b_i and b_j, c_i and c_j... are respectively ranging over chains $\{a_1, a_2, \ldots, a_\ell\}$, $\{b_1, b_2, \ldots b_m\}$, $\{c_1, c_2, \ldots, c_h\}$, ...

There could be name variables with which a chain ends without being a member of a chain. Such name variables called tails will be designated as:

5.42 $d_1, d_2, \dots, d_k \ (0 \le k)$,

where, it is remarked, some chains might not have such a name variable, i.e., a tail. The atomic formulas, which occur in the Hintikka formula as negaive parts involving these name variables, are:

$$5.421 \begin{cases} \in a_1 d_1, \in a_2 d_1, \dots, \in a_\ell d_1, \\ \in b_1 d_1, \in b_2 d_1, \dots, \in b_m d_1, \\ \in c_1 d_1, \in c_2 d_1, \dots, \in c_h d_1, \\ \dots \\ \in a_1 d_2, \in a_2 d_2, \dots, \in a_\ell d_2, \\ \in b_1 d_2, \in b_2 d_2, \dots, \in b_m d_2, \\ \in c_1 d_2, \in c_2 d_2, \dots, \in c_h d_2, \\ \dots \quad , \end{cases}$$

of which some could be not present.

With this, we conclude the listing up of all the atomic formulas occurring in the Hintikka formula as negative parts. In view of the properties of the Hintikka formual the listing up is exhaustive.

The atomic formulas, which occur in the Hintikka formula as positive parts, are as follows:

$$5.51 \begin{cases} \in a_i b_j \quad (1 \le i \le \ell, \quad 1 \le j \le m), \\ \in a_i c_j \quad (1 \le i \le \ell, \quad 1 \le j \le h), \\ \dots \quad , \\ \in b_i a_j \quad (1 \le i \le m, \quad 1 \le j \le \ell), \\ \in b_i c_j \quad (1 \le i \le m, \quad 1 \le j \le h), \\ \dots \quad , \\ \in c_i a_j \quad (1 \le i \le h, \quad 1 \le j \le \ell), \\ \in c_i b_j \quad (1 \le i \le h, \quad 1 \le j \le m), \\ \dots \quad , \end{cases}$$

i.e., every atomic formula listed above is made up of two name variables (along with $\in$) taken from different chains.

$$5.52 \begin{cases} \in a_i d_j \quad (1 \le i \le \ell, \ 1 \le j \le k), \\ \in b_i d_j \quad (1 \le i \le m, \ 1 \le j \le k), \\ \in c_i d_j \quad (1 \le i \le h, \ 1 \le j \le k), \\ \dots \quad , \end{cases}$$

where d_i $(1 \leq i \leq k)$ is a name variable not associated with a_i , b_i , c_i,...

5.53 $\in a_i$ e, $\in b_j$ e,... $(1 \leq i \leq \ell,\ 1 \leq j \leq m,\ \ldots)$,

where a_i , b_j, ... are the members of chains, but e is any name variable which is neither a member of a chain nor a tail.

5.54 $\in d_i$ f $(1 \leq i \leq k)$,

where d_i is a tail and f is any name variable occurring in the Hintikka formula.

5.55 $\in$ef,

where e constitutes neither a member of a chain nor a tail, while f is any name variable occurring in the Hintikka formula.

5.56 $\in$gf,

where g and f are the name variables ranging over the countablly infinite collection of name variable not occurring in the Hintikka formula.

Now, the assignments of values to the predicate corresponding to the name variables as above listed proceeds in the following way:

To each F_{a_1} , F_{a_2} ,..., F_{a_ℓ} is assigned $\{0\}$,
To each F_{b_1} , F_{b_2} ,..., F_{b_m} is assigned $\{1\}$,
To each F_{c_1} , F_{c_2} ,..., F_{c_h} is assigned $\{2\}$,

...

In other words, to each predicate associated with a member of a chain is assigned $\{n\}$, the unit set consisting of the natural number n (preferably, beginning with 0), with predicates corresponding to the members of different chains assigned different unit set. Thus, if there are five chins, $\{0\}$, $\{1\}$, $\{2\}$, $\{3\}$ and $\{4\}$ will be the unit sets assigned to the predicates associated with the respective members of the five different chains.

To the predicates corresponding to the tails $d_1, d_2, \ldots, d_k$, say, d_i, we are assigning a finite set consisting of numbers corresponding to the predicates F_{a_j} with a_j being a member of a chain which ends with d_i along with another number not used as such. Thus, if a, b, and c are members of different chains all ending with d_i and F_a, F_b and F_c are, respectively, assigned $\{2\}$, $\{4\}$ and $\{5\}$, then F_{d_j} is interpreted as $\{2,4,5,k\}$ with k different from any numbers used for the predicates corresponding to the members of a ch

There still remain a number of name variables, to the predicate logic correspondents of which have not been assigned any set (of natural numbers).

For this purpose, we wish to propose a very simple assignments. Namely, ϕ, i.e., the empty set (of natural numbers) is assigned to the predicate logic counterpart of any of these name variables.

On the basis of these assignments, a model $M = \langle N, =, F_a, F_b, \ldots \rangle$ is constructed in the following way:

5.61 N is a finite set of the natural numbers introduced so far in the proces of assignments,

5.62 F_a, F_b, ... are the sets (of natural numbers) assigned to the predicate logic counterparts of the name variables (of L_1) as described above.

5.63 = is the identity relation between natural numbers.

In the model M all the positive (negative) parts of the given Hintikka formula are made false (true).

With this in view it is noticed in the first place that the predicate logic counterparts of the atomic formulas belonging to

5.411 are all true, since

$$F_a \,\iota\, xF_a\ x = \{n\}\iota x\{n\}\ x$$

$$= (\exists x)\ (\{n\}\ x \wedge \{n\}\ x) \wedge \forall x \forall y (\{n\}\ x \wedge \{n\}\ y. \supset x = y),$$

is true in M with a being a member of a chain.

The predicate logic counterparts of the atomic formulas coming under 5.421 are also proved true, since

$$F_d \,\iota\, xF_a\ x = \{0, 1, \ldots, k\}\iota x\{n\}\ x$$

$$= (\exists x)\ (\{0, 1, \ldots, k\}\ x \wedge \{n\} x) \wedge \forall x \forall y (\{n\} x \wedge \{n\}\ y. \supset x = y)$$

is true in M with d being a tail and a a member of a chain which ends with d. This is because n is a member of $\{0, 1, \ldots, k\}$ with n different from k.

The predicate logic counterparts of the formulas belonging to 5.51 are proved false in M, since in this case the unit set corresponding to the members of different chains are all different as stipulated above.

The predicate logic correlates corresponding to the formulas listed in 5.52 are again falsified in M. In fact, F_{a_i} and F_{d_j} are disjoint, since the chain $\{a_1, a_2, \ldots, a_\ell\}$ does not end with d_j.

The predicate logic counterparts of the atomic formulas falling under 5.53 ~ 5.56 are easily proved false in M. More specifically, the predicate logic correlates of 5.53 are of the form $\phi \,\iota\, xF_{a_i}\ x$, which is false in M in view of the definition of definite descriptions.

Analogously, the correspondents of 5.55 and 5.56 are shown false, although the latter, which does not take place in the Hintikka formula, has nothing to do with the evaluation of the Hintikka formula.

On the other hand, the formulas of the form $F_f \,\iota\, xF_{d_i}\ x$ correlated to 5.54 are also false, since F_{d_i} is not a unit set.

On the basis of the truth values thus assigned, it is shown that the predicate logic counterparts of the positive (negative) parts of the given Hintikka formula is false (true) in M. Since the given formula constitutes a positive part of the Hintikka formula as stated in Theorem 3.2, the T-transform of the given formula comes to be false in M.

This concludes the proof of the completeness of L_1, which is stated as:

Lemma 5.7 _If TA is a thesis of first-order predicate logic with equality, then A is provable in L_1 in its tableau method version._

It is remarked in passing that use is made of the soundness result of predicate logic in passing from semantics to syntax.

Combining the soundness and completeness results (Lemma 4.3 and Lemma 5.7), we have:

Theorem 5.8 _A is a thesis of L_1 in its tableau method version iff TA is provable in first-order predicate logic with equality._

It is not difficult to see that a predicate logic model constructed before for falsifying the T-transform of the given Hintikka formula was defined following the general setting here described.

As mentioned earlier, this was the result obtained by Ishimoto [2], [3] and Kobayashi and Ishimoto [5] , the proofs of which were, however, more complicated.

Theorem 5.9 (Cut elimination theorem) _If $B \vee A$ and $\sim A \vee C$ are provable in L_1 in its tableau method version, then $B \vee C$ is also a thesis thereof. (Here B or C could be the empty expression.)_

By Lemma 4.3.(soundness theorem), $TB \vee TA$ and $\sim TA \vee TC$ are provable in first-order predicate logic with equality. From this follows the thesishood of $TB \vee TC$, namely, $T\, B \vee C$ in predicate logic. In view of Lemma 5.7 (Completeness theorem) $B \vee C$ is provable in L_1 in its tableau method. (The cut elimination theorem for L_1 was also proved by Mr. N. Kanai syntactically.)

From the completeness theorem (Theorem 5.7) and the construction of a Hintikka formula for a formula (of L_1) not provable there, it follows that for every formula (of L_1), if it is provable at all, it is proved by a nomal tableau. Suppose, if possible, there were such a formula. Then, by reducing it following the procedure as described in Theorem 3.2 (Fundamental theorem) there would obtain a Hintikka formula, on the basis of which we could falsify the T-transform of the formula against the soundness theorem (Theorem 4.3). This is also the result announced earlier.

On the basis of the cut elimination theorem, we can prove that any thesis of L_1 in its Hilbert-type version is provable in L_1 in its tableau method version. Conversely, any formula of L_1 in its tableau method version constitutes a thesis of L_1 in its Hilbert-type version. This is proved by the length of tableaux.

Theorem 5.10 (Separation theorem) _If a quantifier-free formula A is provable in L, i.e., Lesniewski's (elementary) ontology, A is already a thesis of_ L_1

If such an A is not provable in L_1, by Completeness theorem, a predicate logic model could be constructed falsifying its T-transform. On the basis of the model we could obtain a model for L which falsifies A. (Details are omitted.)

Before concluding this section we wish to remark that the construction of a predicate logic model is by no means unique. For example, the domain N of the model could be the infinite set of natural numbers, and to the tail we could assign any set (finite or infinite) which contains at least all the members of unit sets assigned to predicate logic counterpart of the members of a chain which ends with the tail.

6 Grammar We are going to take up in this section the relevancy to grammar or grammatical theory of our L_1, which has hitherto been developed only as a logical theory.

With this in view a number of lexicons of a fragment of a natural language, say English are identified with their counterparts of Lesniewski's higher-order ontology. The proposed identifications proceeds in the following way:

the	=	$\lambda x \lambda' y \in xy$
a (an, some)	=	$\lambda x \lambda' y \ (\exists z)\ (\in zx \wedge \in zy)$,
is (copula)	=	$\lambda' P \ \lambda x P\ (\lambda y\ x = y)$,
or	=	$\lambda' P \ \lambda y P\ (\lambda x\ x = y)$,
and	=	$\lambda' p \ \lambda' q\ (p \wedge q)$,
or	=	$\lambda' p \ \lambda' q\ (p \vee q)$,
not	=	$\lambda' p \ \sim p$,
...		
Socrates	=	$\lambda' y \ \in$ Socrates y,
Bill	=	$\lambda' y \ \in$ Bill y,
Mary	=	$\lambda' y \ \in$ Mary y,
...		
man	=	man
boy	=	boy
...		

Here, $\lambda x \ldots x \ldots$, intuitively, stands for the collection (or set) of atoms or atomic names x's such that ... x ..., while $\lambda' x$ .. x ... and the like represents the collection of x's satisfying ... x ... $\lambda x \ldots x \ldots$, it is emphasized, is always assumed to represents a name, while $\lambda' x \ldots x \ldots$ and the like do not necessarily stand for a name. In particular, $\lambda' P \ldots P \ldots$ is, if interpreted, a collection of P's such that ... P ... with P ranging over the denotations of noun phrases, in our case, those corresponding to the expressions of the form $\lambda' y \in a\ y$ or $\lambda' y\ (\exists z)\ (\in za \wedge \in zy)$, where a is constant name, not necessarily, atomic. (Our readenring of copula is after Montaque [8].) $a = b$, on the other hand, is an abbreviation of $\in ab \wedge \in ba$.

The presence of such variables as P and those ranging over propositions makes our logical system into Lesniewski's higher-order ontology with λ -conversion as a rule. And, the derivations in the sequel will take place in this higher-oder ontology.

The status of $\lambda x \ldots x \ldots$ as a name is characterized by axioms of the form:

6.1 $\in a\ \lambda x \ldots x \ldots \equiv_{\bullet} \ldots a \ldots \wedge \in aa$,

which is adjoined to L or Lesniewski's higher-order ontology as additional axioms. In fact, the expression $\in a\ \lambda x \ldots x \ldots$ is meaningful only if $\lambda x .. x \ldots$ is of the category of names, and the namehood of $\lambda x \ldots x \ldots$ is guaranteed by this axiom.

It is remarked in passing that proper names such 'Socrates' and 'Bill' are thought of as 'the Socrates', 'the Bill' and the like as seen from the above identifications. It is noted that we are following Cresswell [1] in identifying the expressions of natural language with those taken from a logical system unlike the case of PTQ, i.e., Montague [8], where the former are <u>translated</u> into their logical correlates.

Now, there are all in all enght possible constituent analyses for the simple sentences of the form:

Det + Noun + Copula + Det + Noun,

if Det is ranging over 'the' and 'a' as above identified. Namely, we have:

6.21 ((the Noun)(Copula (the Noun))),

6.22 (((the Noun) Copula)(the Noun)),

6.31 ((the Noun)(Copula (a Noun))),

6.32 (((the Noun) Copula)(a Noun)),

6.41 ((a Noun)(Copula (the Noun))),

6.42 (((a Noun) Copula)(the Noun)),

6.51 ((a Noun)(Copula (a Noun))),

6.52 (((a Noun) Copula)(a Noun)).

Hereby, Nouns are assumed synonimous with names in the sense of L_1 with the same category. Although the above list exhausts all the possible constituent analyses from the logical point of view, it remains a problem whether a simple sentence (of natural language) having the above forms is susceptible of one of these analyses from the linguistic point of view.

We are, now, in a position to present a number of sample sentences (taken from English) and transform them within Lesnewski's higher-order ontology. Each of these sentences coreesponds to one of the constituent analyses 6.21--6.52 as described above.

6.61 (Bill (is John))

$$\equiv ((\lambda' y \in \mathrm{Bill}\ y)$$

$$((\lambda' P\ \lambda x P(\lambda y\ x = y))\ (\lambda' y\ \in \mathrm{John}\ y)))$$

$$\equiv ((\lambda' y\ \in \mathrm{Bill}\ y)\ (\lambda x\ ((\lambda' y \in \mathrm{John}\ y)\ (\lambda y\ x = y))))$$

$$\equiv ((\lambda' y\ \in \mathrm{Bill}\ y)\ (\lambda x \in \mathrm{John}\ (\lambda y\ x = y)))$$

$$\equiv ((\lambda' y\ \in \mathrm{Bill}\ y)\ (\lambda x\ x = \mathrm{John}))$$

$$\equiv \in \mathrm{Bill}\ (\lambda x\ x = \mathrm{John})$$

$$\equiv \mathrm{Bill} = \mathrm{John}$$

$$\equiv .\ \in \mathrm{Bill}\ \mathrm{John} \wedge \in \mathrm{John}\ \mathrm{Bill},$$

where operators are allowed to operate not only from left to right, but also from right to left. This was alredy practiced in Gresswell [1].

6.62 $\vdash$ ((Bill is) John)

$\equiv$ (((λ'y $\in$ Bill y) (λ'P λy P (λx x = y))) (λ'y $\in$ John y))

$\equiv$ ((λy ((λ' z $\in$ Bill z) (λx x = y))) λ'y $\in$ John y))

$\equiv$ ((λy $\in$ Bill (λx x = y)) (λ'y $\in$ John y))

$\equiv$ (λy Bill = y) (λ'y $\in$ John y))

$\equiv$ $\in$ John (λy Bill = y)

$\equiv$ Bill = John

$\equiv$. $\in$ Bill John $\wedge$ $\in$ John Bill.

6.63 $\vdash$ (Bill (is (a man)))

$\equiv$ ((λ'y $\in$ Bill y)
((λ'P λxP(λy x = y))((λ'x λ'y($\exists$z)($\in$zx $\wedge$ $\in$zy)) man)))

$\equiv$ ((λ'y $\in$ Bill y)
((λ'P λxP(λy x = y))(λ'y($\exists$z)($\in$z man $\wedge$ $\in$zy))))

$\equiv$ ((λ'y $\in$ Bill y)
(λx(λ'y($\exists$z)($\in$z man $\wedge$ $\in$zy))(λy x = y))))

$\equiv$ ((λ'y $\in$ Bill y)(λx($\exists$z)($\in$z man $\wedge$ $\in$z (λy z = y))))

$\equiv$ ((λ'y $\in$ Bill y)(λx($\exists$z)($\in$z man $\wedge$ x = z)))

$\equiv$ ((λ'y $\in$ Bill y)(λx $\in$ x man))

$\equiv$ Bill (λx $\in$x man)

$\equiv$ $\in$Bill man.

Hereby, use is made of a lemma:

$\in$ ab $\equiv$ ($\exists$x) ($\in$xb $\wedge$ a = x),

which is easily proved in L. This and similar ones will be employed in what follows without mentioning them.

6.64 $\vdash$ ((Bill is) (a man))

$\equiv$ (((λ'y $\in$Bill y)(λ'P λyP(λx x = y)))
((λ'x λ'y($\exists$ z)($\in$ zx $\wedge$ $\in$zy) man))

$\equiv$ ((λy((λ'y $\in$Bill y)(λx x = y)))
(λ'y($\exists$z)($\in$z man $\wedge$ $\in$zy)))

$\equiv$ ((λy $\in$ Bill (λx x = y)) (λ'y($\exists$z) ($\in$z man $\wedge$ $\in$zy)))

$\equiv ((\lambda y\ Bill = y)\ (\lambda' y\ \ (\exists z)\ (\in z\ man \wedge \in zy)))$

$\equiv (\exists z)\ (\in z\ man \wedge \in z\ (\lambda y\ Bill = y))$

$\equiv (\exists z)\ (\in z\ man \wedge Bill = z)$

$\equiv \in Bill\ man.$

6.65 $\vdash$((a man) (is Bill))

$\equiv (((\lambda' x\ \ \lambda' y\ (\exists\ z)\ (\in zx \wedge \in zy))\ man)$

$((\lambda' P\ \ \lambda z P(\lambda y\ x = y))\ (\lambda' y\ \ \in Bill\ y)))$

$\equiv ((\lambda' y\ (\exists z)\ (\in z\ man\ \wedge \in zy))$

$(\lambda x((\lambda' y\ \ \in Bill\ y)\ (\lambda y\ x = y))))$

$\equiv ((\lambda' y(\exists z)\ (\in z\ man\ \wedge \in zy))\ (\lambda x\ \ \in Bill\ (\lambda y\ x = y)))$

$\equiv ((\lambda' y(\exists z)\ (\in z\ man\ \wedge \in zy))\ (\lambda x\ x = Bill))$

$\equiv (\exists z)\ (\in z\ man \wedge \in\ z\ (\lambda x\ x = Bill))$

$\equiv (\exists z)\ (\in z\ man \wedge z = Bill)$

$\equiv \in Bill\ man.$

6.66 $\vdash$ (((a man) is) Bill)

$\equiv ((((\lambda' x\ \ \ \lambda' y\ (\exists z)\ (\in zx \wedge \in zy))\ man)$

$(\lambda' P\ \ \lambda y P\ (\lambda x\ x = y)))\ (\lambda' y\ \ \in Bill\ y))$

$\equiv (((\lambda' y\ (\exists z)\ (\in z\ man \wedge \in zy))$

$(\lambda' P\ \ \lambda y P\ (\lambda x\ \ x = y)))\ (\lambda' y\ \ \in Bill\ y))$

$\equiv ((\lambda y((\lambda' w(\exists z)(\in z\ man \wedge \in zw))\ (\lambda x\ x = y))\ (\lambda' y\ \ \in Bill\ y))$

$\equiv ((\lambda y(\exists z)(\in z\ man \wedge \in z\ (\lambda x\ x = y)))\ (\lambda' y\ \ \in Bill\ y))$

$\equiv ((\lambda y(\exists z)(\in z\ man \wedge z = y))\ (\lambda y\ \ \in Bill\ y))$

$\equiv ((\lambda y \in y\ man)\ (\lambda' y\ \ \in Bill\ y))$

$\equiv\ \in Bill\ (\lambda y \in y\ man)$

$\equiv\ \in Bill\ man.$

6.67 $\vdash$ ((a man) (is (a student)))

$\equiv (((\lambda' x\ \ \lambda' y\ \ (\exists z)(\in zx \wedge \in zy))\ man)$

$((\lambda' P \lambda x P(\lambda y\ x = y))((\lambda' x\ \ \lambda' y(\exists z)(\in zx \wedge \in zy))\ student))))$

$\equiv ((\lambda' y(\exists z)\ (\in z\ man \wedge \in zy))$

$((\lambda' P\ \ \lambda x P(\lambda y\ x = y))(\lambda' y(\exists z)(\in z\ student \wedge \in zy))))$

$\equiv ((\lambda' y\ \ (\exists z)(\in z\ man \wedge \in zy))$

$(\lambda x((\lambda' y(\exists z)(\in z\ student \wedge \in zy))(\lambda y\ x = y))))$

$\equiv ((\lambda' y(\exists z)(\in z\ man \wedge \in zy))$

$(\lambda x(\exists z)(\in z\ student \wedge \in z\ (\lambda y\ x = y))))$

$\equiv ((\lambda' y\ (\exists z)\ (\in z\ \text{man} \wedge \in zy))\ (\lambda x (\exists z)(\in z\ \text{student} \wedge x = z)))$

$\equiv ((\lambda' y\ (\exists z)\ (\in z\ \text{man} \wedge \in zy))\ (\lambda x (\in x\ \text{student}))$

$\equiv (\exists z)\ (\in z\ \text{man} \wedge \in z\ (\lambda x \in x\ \text{student}))$

$\equiv (\exists z)\ (\in z\ \text{man} \wedge \in z\ \text{student}).$

6.68 $\vdash$(((a man) is) (a student))

$\equiv ((((\lambda' x\ \ \lambda' y\ \ (\exists z)\ (\in zx \wedge \in zy))\ \text{man})$

$\quad (\lambda' P \lambda y P(\lambda x\ x = y)))((\lambda' x \lambda' y (\exists z)(\in zx \wedge \in zy))\ \text{student}))$

$\equiv (((\lambda' y (\exists z)\ (\in z\ \text{man} \wedge \in zy))$

$\quad (\lambda' P \lambda y P(\lambda x\ x = y)))\ (\lambda' y (\exists z)(\in z\ \text{student} \wedge \in zy)))$

$\equiv ((\lambda y ((\lambda' y (\exists z)(\in z\ \text{man} \wedge \in zy))(\lambda x\ x = y)))$

$\quad (\lambda' y (\exists z)(\in z\ \text{student} \wedge \in zy)))$

$\equiv ((\lambda y (\exists z)(\in z\ \text{man} \wedge \in z\ (\lambda x\ x = y)))$

$\quad (\lambda' y (\exists z)(\in z\ \text{student} \wedge \in zy)))$

$\equiv ((\lambda y (\exists z)(\in z\ \text{man} \wedge z = y))$

$\quad (\lambda' y (\exists z)(\in z\ \text{student} \wedge \in zy)))$

$\equiv (\lambda y \in y\ \text{man})(\lambda' y (\exists z)(\in z\ \text{student} \wedge \in zy)))$

$\equiv (\exists z)(\in z\ \text{student} \wedge \in z\ (\lambda y \in y\ \text{man}))$

$\equiv (\exists z)(\in z\ \text{student} \wedge \in z\ \text{man})).$

On the basis of these derivations we have

Theorem 6.7 $\vdash$ 6.61 $\equiv$ 6.62,
$\vdash$ 6.63 $\equiv$ 6.64 $\equiv$ 6.65 $\equiv$ 6.66,
$\vdash$ 6.67 $\equiv$ 6.68.

The equivalence of 6.63, 6.64 and 6.65 might be rather surprising in view of their rather diverse constituent analyses. However, this holds good at least, from the logical voint of view, not from the linguistic point of view perhaps. It is also observed that excepting 6.67 and 6.68 all these sentences are eventually transformed into sentences belonging to L_1, namely, to those quntifier-free sentences of L. Among them 6.61 and 6.62 are, however, not reduced to a simple or atomic sentence of L_1 although they are for certain the sentences of L.

Lastly, it is observed that the T-transforms of these sample sentences are exactly the same as their translations in the sense of Montague [8] if we disregard intension.

§7 Conclusion There are two main results obtained in this paper. In the first place, it was shown that a fragment of English, a very tiny fragment though, could be accommodated to L_1, namely, the propositional fragment of Lesniewski's ontology. More specifically, it was found that such sentences as 'Bill is a man' and 'a man is Bill' are reduced to an atomic formula of L_1. Further the T-transforms of these sentences were found to be the translations in the sense of Montague [8].

Secondly, it was shown that the model for the sentences of L_1 is not uniquely determined. As has been repeated many times, there are an infinite number of models for the sentences of L_1 along with more diverse models for the T-transforms of these sentences of L_1. As a result, the notion of uniquely determined model or semantics for natural language does not make sense, and we are urgently requested to settle the matter if we wish to put the logical grammar upon more secure foundations. One of the rescues, which inmediately comes up to mind, would be to propose a canonical model for natural language as has been successfully proposed in a variety of mathematical studies.

In view of the failure of uniquely determining the model of natural language, we are requested to scrutinize the status of individuals in the model more carefully. For example, any noun, say, 'man' is represented by a variety of the sets of natural numbers, by no means, uniquely determined. By this, perhaps, we could explain the shadowy status of individuals. In other words, individuals are called for only to support nouns and other lexicons involved upon the world of anonymous individuals, and these individuals never come up to the surface. From this also follows the benefit to employ Lesniewski's ontology in place of traditional predicate logic, and there no privileged status is reserved for individuals. Further, it would be interesting to explore how far we could proceed in the logic of natural language confining ourselves to propositional logic like L_1.

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