한국통신학회 1984년도 추계학술발표회 논문집 (1984. 11. 3)

Input Density에 대한 Compandor 특성에 관한 연구

84338

°
박 찬 경　　韓 榮 烈
漢陽大學校 電子通信工學科

A Study On The Compandor Characteristics For the Various Input Probability Density Funtions.

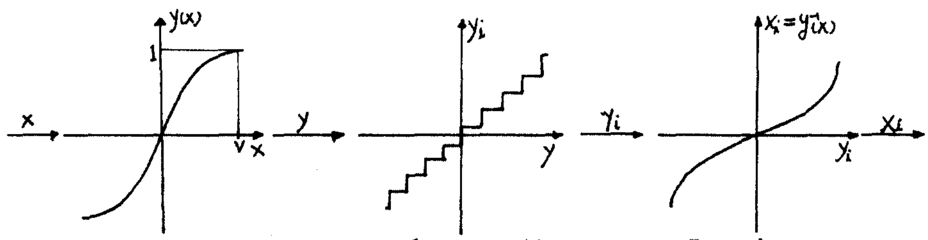
°
Chan Kyung, PARK.　　Young Yeul, HAN.
Dept. of Electronic-Comm. Eng.　　HAN-YANG University

Abstract

This paper describes the output P.D.F. of various Compandors, optimum, μ-law and A-law for the Gaussian and Laplacian Density as an input analog signal. Also we consider the truncated densities compensated by weighted impulse or density coefficient.

1. Introduction.

The random variable X is first passed through a nonlinear memoryless transformation Y(X) to yield another random variable Y at transmitter. This random variable is uniformly quantized to give Y_i, which is nonlinearly inverse transformed by X(Y) to give output X_i as in Fig.1 [1]



(4)

Compressor　　Uniform Quantizer.　　Expandor.

Fig.1 Companding Model of Nonuniform Quantization.

2. Compandor Characteristics.

We obtain the optimum compandor Y(x) function to minimize the mean-square-error $E[\varepsilon^2]$, where ε represents the error voltage between the instantaneous actual signal and its quantized equivalent.

Laplacian P.D.F. of x is given by $(-\infty \le x \le \infty)$

$$P(x)=\frac{1}{2x_0}\exp(-|x|/x_0) \quad , \quad x_0=\sigma/\sqrt{2} \qquad (1)$$

for which optimum compandor is [1]

$$Y(x)=\frac{1-\exp(-x/3x_0)}{1-\exp(-V/3x_0)} \qquad (2)$$

where x is bounded by maximum value V.

$(0 \le x \le V \ , \ 0 \le Y \le 1 \ )$

Gaussian optimum compandor function is expressed as follows;

$$Y(x)=\int_0^x K \cdot p^{1/3}(x)\, dx$$

$$p(x)=\frac{1}{\sqrt{2}}\exp(-x^2/2\sigma_x^2 \ ) \qquad (3)$$

K is a constant determined to satisfy boundary value. Thus,

$$K=\frac{1}{\int_0^V p^{1/3}(x)\, dx}$$

Therefore,

$$Y(x)=\frac{\int_0^x (1/2\pi\sigma_x^2)\exp(-x^2/6\sigma_x^2)}{\int_0^V (1/2\pi\sigma_x^2)\exp(-x^2/6\sigma_x^2)}$$

$$Y(x)=\frac{1-\mathrm{erfc}(x/\sqrt{6}\sigma_x)}{1-\mathrm{erfc}(V/\sqrt{6}\sigma_x)} \qquad (4)$$

where , $0\leq x\leq V$, $0\leq Y\leq 1$.

Logarithmic companding yields an improvement in SNR relative to a uniform quantization for a wider range of input variance (=volume levels) than did the optimum for a specific input variance.

For a μ-law compandor function [1]

$$Y(x)=\frac{\ln(1+\mu x/V)}{\ln(1+\mu)} \qquad (0\leq x\leq V,\ 0\leq Y\leq 1.) \qquad (5)$$

A-law compandor function is expressed by [2]

$$Y(x)=\frac{1+\ln(AV/x)}{\ln(1+A)} \qquad (6)$$

$$\left(\frac{1}{A}\leq x\leq V,\ \frac{1+\ln(1/V)}{1+\ln A}\leq Y\leq 1\right)$$

3.Truncated Densities.

In the previous section we took V as a maximum value of x. We now truncate the P.D.F.at x=V. Then we can compensate the truncated area replacing by coefficient and weighted impulse as in Fig. 2.

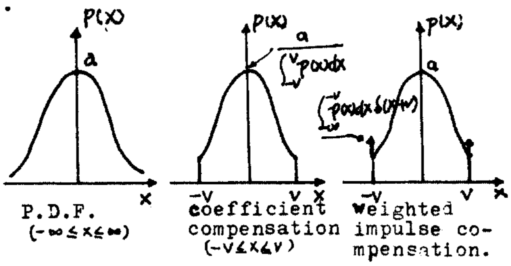


Fig.2. Truncated Densities.

(1) coefficient compensation

For Gaussian , from (3)

$$p_t(x)=\int_{-V}^{V} Cp(x)\ dx = 1$$

Therefore,

$$C=1/\mathrm{erf}(V/\sqrt{2}\sigma_x)$$

Thus we can obtain a compensated P.D.F.

$$p_t(x)=\frac{1}{\mathrm{erf}(V/\sqrt{2}\sigma_x)}\frac{1}{\sqrt{2\pi}\,\sigma_x}\exp(-x^2/2\sigma_x^2) \qquad (-V\leq x\leq V) \qquad (7)$$

For Laplacian

$$C=1/(1-\exp(-V/x_0))$$

Thus

$$p_t(x)=\frac{1}{1-\exp(-V/x_0)}(1/2x_0)\exp(-|x|/x_0) \qquad (8)$$

$$(-V\leq x\leq V)$$

(2) weighted impulse compensation

For Gaussian, Cumulative Density Function of truncated area lower than x=V is, from (3)

$$F(x\leq -V)=\int_{-\infty}^{-V} p(x)\ dx = \frac{1}{2}\mathrm{erfc}(V/\sqrt{2}\sigma_x) = F(V\leq x)$$

From which we have

$$p_t(x)=\frac{1}{\sqrt{2\pi}\sigma_x}\exp(-x^2/2\sigma_x^2)+\frac{1}{2}\mathrm{erfc}(V/\sqrt{2}\sigma_x)\left[\delta(x-V)+\delta(x+V)\right] \qquad (9)$$

For Laplacian

$$F(x\leq -V)=F(V\leq x) = \int_V^{\infty} p(x)\ dx = \frac{1}{2}\exp(-V/x_0)$$

$$p_t(x)=\frac{1}{2}\exp(-|x|/x_0)+\frac{1}{2}\exp(-V/x_0)\left[\delta(x-V)+\delta(x+V)\right] \qquad (10)$$

4. The Probability Density Function of Compandor Output.

It is random variable Y that the output of compressor when the input P,D.F of x is compressed by compandor characteristic Y(x). From probability transformation [3]

$$p_y(Y)=p_x(X)\left|\frac{dx}{dY}\right|_{x=y^{-1}} \qquad (11)$$

From (3) (4) (11), the $p_y(Y)$ of optimum compandor output for truncated Gaussian is

$$p_y(Y)=\frac{1}{\sqrt{2\pi}\sigma_x}\exp(-x^2/2\sigma_x^2)\left|\frac{\sqrt{6\pi}\sigma_x}{2}\frac{\mathrm{erf}(V/\sqrt{6}\sigma_x)}{\exp(-x^2/6\sigma_x^2)}\right|_{x=y^{-1}} = \frac{\sqrt{3}}{2}\mathrm{erf}(1/\sqrt{6}\sigma_x)\exp(-x^2/3\sigma_x^2)\Big|_{x=y^{-1}} \qquad (0\leq x\leq V\ ,\ 0\leq Y\leq 1\ )$$

where x will be replaced by Y calculated inversely from (4).

In the table 1 we tabulate $p_y(Y)$ of the optimum ,μ-law and A-law compandor for a truncated,coefficient and weighted impulse compensated densities at each case.

4. Conclusion.

We have shown the 18 output P.D.F.,p(Y). Here only 8 output P.D.F.s are shown in Fig.3—10. We have results that optimum compandor is useful for the high input signal levels. (Fig.3 Fig.6), while μ-law and A-law compandor has more precise

Table 1. Output P.D.F of the various compandors for the various Densities

Compandor / Densty	output P.D.F of optimum compandor	output P.D.F of μ - law compandor	output P.D.F. of A- law compandor
Truncated Gaussian	$P_y(Y) = \frac{\sqrt{3}}{2}\mathrm{erf}\left(\frac{v}{\sqrt{6}\sigma x}\right)\bar{e}^{x^2/3\sigma x^2}\Big\vert_{x=y^{-1}}$ (12) $y = \frac{\mathrm{erf}(x/\sqrt{6}\sigma x)}{\mathrm{erf}(v/\sqrt{6}\sigma x)}$ $\quad 0 \leqq x \leqq v$, $0 \leqq y \leqq 1$	$P_y(Y) = \frac{1}{\sqrt{2\pi}\sigma x}\bar{e}^{\frac{x^2}{2\sigma x^2}}\frac{(v+\mu x)}{\mu}\ln(1+\mu)\cdot\Big\vert_{x=\frac{v}{\mu}[(1+\mu)^y-1]}$ (15) $\quad 0 \leqq x \leqq v$, $0 \leqq y \leqq 1$	$P_y(Y) = \frac{(1+\ell n A)x}{\sqrt{2\pi}\sigma x}\bar{e}^{\frac{x^2}{2\sigma x^2}}$ $X = V(Ae)^{y-1}$ (18) $\frac{1}{A} \leqq x \leqq v$ $\quad \frac{1+\ell n\frac{1}{v}}{1+\ell n A} \leqq y \leqq 1$
Gaussian compensated by coefficient	$P_y(Y) = \frac{\sqrt{3}}{2}\bar{e}^{\frac{x^2}{3\sigma x^2}}\cdot\frac{\mathrm{erf}(\frac{v}{\sqrt{6}\sigma x})}{\mathrm{erf}(\frac{v}{\sqrt{2}\sigma x})}\Big\vert_{X=y^{-1}}$ (13) $y = \frac{\mathrm{erf}(x/\sqrt{6}\sigma x)}{\mathrm{erf}(v/\sqrt{6}\sigma x)}$	$P_y(Y) = \frac{(v+\mu x)\,\ell n(1+\mu)}{\mu\sqrt{2\pi}\sigma x\,\mathrm{erf}(\frac{v}{\sqrt{2}\sigma x})}\bar{e}^{\frac{x^2}{2\sigma_x^2}}$ $\Big\vert_{X=\frac{v}{\mu}(1+\mu)^y-1}$ (16)	$P_y(Y) = \frac{(1+\ell n A)X}{\sqrt{2\pi}\sigma x\,\mathrm{erf}(\frac{v}{\sqrt{2}\sigma x})}\cdot$ $\bar{e}^{x^2/2\sigma x^2}\Big\vert_{X=v(Ae)^{y-1}}$ (19)
Gaussian compensated by weighted impulse	$P_y(Y) = \frac{\sqrt{6\pi}\sigma x}{2}\cdot\frac{\mathrm{erf}(\frac{v}{\sqrt{6}\sigma x})}{\bar{e}^{(x^2/6\sigma x^2)}}\Big\{$ $\frac{1}{\sqrt{2\pi}\sigma x}\bar{e}^{\frac{x^2}{2\sigma x^2}}+\frac{1}{2}\mathrm{erfc}[\frac{v}{\sqrt{2}\sigma x}]$ $\delta[X-V]\Big\}\Big\vert_{X=y^{-1}}$ (14)	$P_y(Y) = \frac{(v+\mu x)\,\ell n(1+\mu)}{\mu}$ $\Big\{\frac{1}{\sqrt{2\pi}\sigma x}\bar{e}^{\frac{x^2}{2\sigma x^2}}+\frac{1}{2}\mathrm{erfc}(\frac{V}{\sqrt{2}\sigma x})\cdot$ $\delta(X-V)\Big\}\Big\vert_{X=\frac{v}{\mu}[(1+\mu)^y-1]}$ (17)	$P_y(Y) = (1+\ell n A)\cdot X\cdot\frac{1}{\sqrt{2\pi}\sigma x}\cdot$ $\bar{e}^{\frac{x^2}{2\sigma x^2}}+\frac{1}{2}\mathrm{erfc}(\frac{v}{\sqrt{2}\sigma x})\cdot$ $\delta(X-V)\Big\vert_{X=V(Ae)^{y-1}}$ (20)
Truncated Laplacian	$P_y(Y) = \frac{3}{2}\bar{e}^{\frac{\sqrt{2}x}{\sigma x}}(1-\bar{e}^{\frac{\sqrt{2}v}{3\sigma x}})\cdot$ $e^{\frac{\sqrt{2}x}{3\sigma x}}\Big\vert_{X=\frac{3\sigma x}{\sqrt{2}}\ell n[1-y\cdot(1-\bar{e}^{\frac{\sqrt{2}v}{3\sigma x}})]}$ (21) $0 \leqq x \leqq v$ $0 \leqq y \leqq 1$	$P_y(Y) = \frac{(v+\mu x)\,\ell n(1+\mu)}{\sqrt{2}\mu\sigma x}\cdot$ $\bar{e}^{\sqrt{2}x/\sigma x}\Big\vert$ (24) $X = \frac{v}{\mu}[(1+\mu)^y-1]$	$P_y(Y) = \frac{x(1+\ell n A)}{\sqrt{2}\sigma x}\bar{e}^{\frac{\sqrt{2}x}{\sigma x}}\Big\vert$ $X = V(Ae)^{y-1}$ $\quad \frac{1}{A} \leqq x \leqq v$ (27) $\frac{1+\ell n\frac{1}{v}}{1+\ell n A} \leqq y \leqq 1$
Laplacian compensated by coefficient	$P_y(Y) = \frac{3}{2}\bar{e}^{\frac{2\sqrt{2}x}{3\sigma x}}\frac{1-\bar{e}^{\sqrt{2}v/3\sigma x}}{1-\bar{e}^{\sqrt{2}v/\sigma x}}$ $X = -\frac{3\sigma x}{\sqrt{2}}\ell n[1-y(1-\bar{e}^{\frac{\sqrt{2}v}{3\sigma x}})]$ $0 \leqq x \leqq v$ $0 \leqq y \leqq 1$ $\quad$ (22)	$P_y(Y) = \frac{(v+\mu x)\,\ell n(1+\mu)}{\sqrt{2}\mu\sigma x(1-\bar{e}^{\sqrt{2}v/\sigma x})}\Big\vert$ $X = \frac{v}{\mu}[(1+\mu)^y-1]$ (25)	$P_y(Y) = \frac{x(1+\ell n A)}{\sqrt{2}\sigma x(1-\bar{e}^{\sqrt{2}v/\sigma x})}\cdot$ $\bar{e}^{\sqrt{2}x/\sigma x}\Big\vert$ $X = V(Ae)^{y-1}$ $\quad$ (28)
Laplacian compensated by weighted impulse	$P_y(Y) = \frac{3}{2}(1-\bar{e}^{\frac{\sqrt{2}v}{3\sigma x}})\cdot$ $[\bar{e}^{\frac{2\sqrt{2}X}{3\sigma x}}+\frac{\sigma x}{\sqrt{2}}\bar{e}^{\frac{\sqrt{2}}{\sigma x}}(\frac{x}{3}-V)$ $\delta(X-V)]\Big\vert$ $\quad$ (23) $X = -\frac{3}{\sqrt{2}}\ell n[1-y(1-\bar{e}^{\frac{\sqrt{2}v}{3\sigma x}})]$	$P_y(Y) = \frac{(v+\mu x)\,\ell n(1+\mu)}{\mu}\Big\{$ $\frac{1}{\sqrt{2}\sigma x}\bar{e}^{\frac{\sqrt{2}x}{\sigma x}}+\frac{1}{2}\bar{e}^{\sqrt{2}v/\sigma x}\delta(X-V)]$ (26) $X = \frac{v}{\mu}[(1+\mu)^y-1]$	$P_y(Y) = X(1+\ell n A)\Big\{$ $\frac{1}{\sqrt{2}\sigma x}\bar{e}^{\frac{\sqrt{2}v}{\sigma x}}+\frac{1}{2}\bar{e}^{\sqrt{2}v/\sigma x},$ $\delta(X-V)\Big\}\Big\vert$ $X = V(Ae)^{y-1}$ $\quad$ (29)

here we let for practical use

$\sigma x = 1$

$V = 3\sigma x = 3$

and $\mu = 225$ for μ − law, $A = 100$ for A − law.

quantization at low input signal .(Fig. 4.5.7.8.) Improvement by coefficient compensation is not significant of 0.08 dB for Laplacian and 0.01 dB for Gaussian.(Fig.9) The weighted impulse compensation will contribute to represent the truncated area. (Fig.10)

References

[1] James J .SPILKER Jr. "Digital Communications By Satellite" Prentice-Hall 1977 p.p. 26-51.

[2] Mischa Schwartz. "Information Transmission , Modulation & Noise " McGraw-hill . 1980 p.p. 108—128.

[3] Ziemer and Tranter " Principles of Communications " Houghton Mifflin company . 1976.

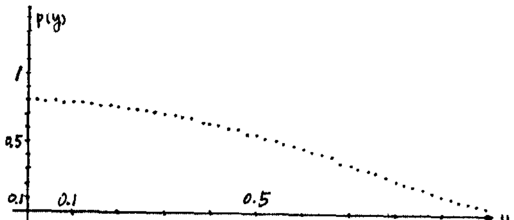


Fig.3 P.D.F.of optimum compandor output for truncated Gaussian.(12)

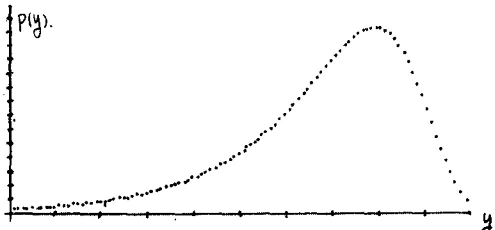


Fig.4 P.D.F.of μ-law compandor output for truncated Gaussian (15)

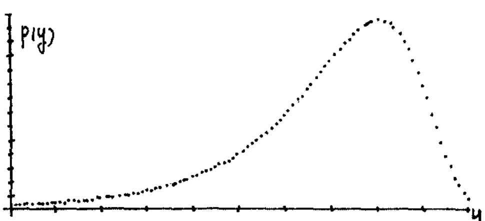


Fig.5 P.D.F.of A-law compandor output for truncated Gaussian(18)

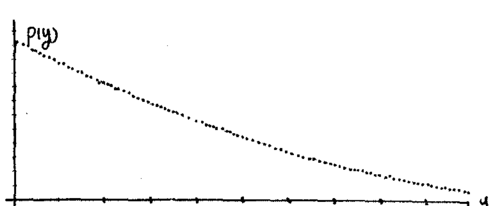


Fig.6 P.D.F.of optimum compandor output for truncated Laplacian(21)

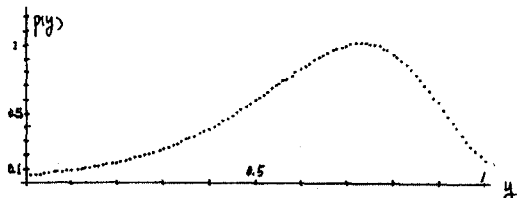


Fig.7 P.D.F.of μ-law compandor output for truncated Laplacian. (24)

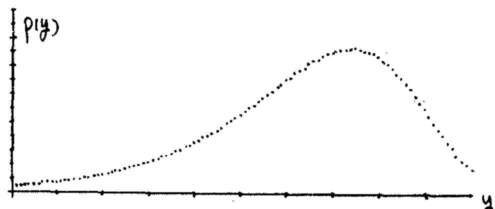


Fig.8 P.D.F.of A-law compandor output for truncated Laplacian (27)

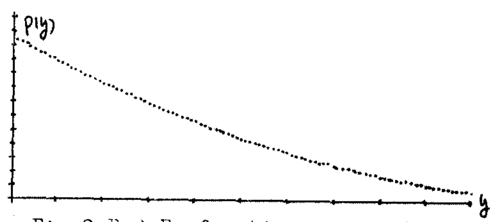


Fig.9 P.D.F.of optimum compandor output the coefficient compensated Laplacian. (22)

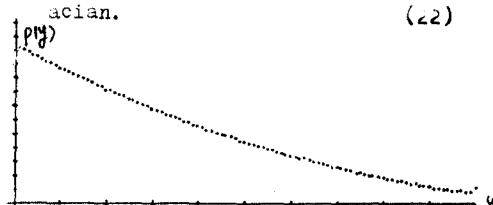


Fig.10 P.D.F.of optimum compandor output the weighted impulse compensated Laplacian (23)