Computational Speed Comparison between FFT Convolution and Recursive Filtering

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Performances of three computational algorithms for one-dimensional frequency filtering are compared and tradeoffs are studied. If the size of the filter impulse response is small, it is well-known that the conventional convolution is superior than the FFT convolution. If the size of the impulse response is large, it was suggested that the recursive filter might be competitive in terms of speed to the FFT convolution. We, therefore, have developed an computational algorithm for the recursive filter to compare the speed advantages over the FFT convolution and the results are presented.

In the reconstructive image processing, a reconstruction filter is required to obtain a reconstruction of high accuracy and resolution. Most of the previous reconstruction filtering methods employed either direct convolution or the FFT convolution to implement the filtering.[1] Since recursive filtering has potential of high speed convolution (filtering) over the conventional convolution or the FFT convolution[2], there is a genuine interest to investigate the use of the recursive filtering for the C.T. (Computerized Tomography) image reconstruction.

The number of complex computations required by using a FFT convolution is approximately

$$N_{FFT} = N\,(1 + 2\log_2 N) \qquad (1)$$

The other method to realize a digital filter is by use of recursive partial difference equation. The one-dimensional recursive relation is defined as,

$$g(m) = \sum_{j=1}^{J} a(j)\, f(m-j+1) - \sum_{k=2}^{k} b(k)\, g(m-k+1) \qquad (2)$$

where $f(n)$ is an input sequence, $g(m)$ is an output sequence, and $a(j)$ and $b(k)$ are weighting constants. The number of real computational operations required to implement the recursion relationship given above is,

$$N_{REC.} = (J + K)\ N \tag{3}$$

Equating the number of operations specified by Eqs.(1) and (3), it is indicated that the recursive filtering is much more efficient than the FFT convolution for a large size impulse response if the recursive filter window area satisfies the following inequality,

$$J + K \leq c \log_2 N \tag{4}$$

where c is a constant that depends on the type of FFT algorithm employed.

From the above discussion, we expect that the recursive filter could be used to perform the convolution in C.T. image reconstruction for the purpose of saving the computation time. It should be noted, however, that the recursive filtering might lead to somewhat decreased accuracy unless the frequency characteristics of the recursive filter is optimally designed.

Using a mathematical phantom, we have compared the speeds and image qualities obtained by the FFT convolution and recursive filtering. The results of the simulations will be presented and tradeoffs of the speeds versus image qualities are discussed.

REFERENCES

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