Reduced-Order Unscented Kalman Filter for Sensorless Control of Permanent-Magnet Synchronous Motor

Cheol Moon* and Young Ahn Kwon[†]

Abstract – The unscented Kalman filter features a direct transforming process involving unscented transformation for removing the linearization process error that may occur in the extended Kalman filter. This paper proposes a reduced-order unscented Kalman filter for the sensorless control of a permanent magnet synchronous motor. The proposed method can reduce the computational load without degrading the accuracy compared to the conventional Kalman filters. Moreover, the proposed method can directly estimate the electrical rotor position and speed without a back-electromotive force. The proposed Kalman filter for the sensorless control of a permanent magnet synchronous motor is verified through the simulation and experimentation. The performance of the proposed method is evaluated over a wide range of operations, such as forward and reverse rotations in low and high speeds including the detuning parameters.

Keywords: Unscented Kalman filter, Reduced order model, Unscented transformation, Sensorless speed control, Permanent-magnet synchronous motor

1. Introduction

Many studies on the vector control of permanent-magnet synchronous motor (PMSM) drives without position sensors have been reported [1]. Sensorless control using an extended Kalman filter (EKF) facilitates the best estimation of state values, regardless of system and sensor noise [2]. However, the linearization process of this EKF often involves a difference between the modified model and the original system model.

Julier and Uhlman presented a new type of Kalman filter called the unscented Kalman filter (UKF), which employs unscented transformation (UT) [3-4]. Unlike the EKF, which has a linearization process method, the UKF features a direct mapping process involving UT, without a modified system model. Despite these advantages, the algorithm of such Kalman filters has a heavy computational load, which is a major drawback. Therefore, it is necessary to minimize the state variables in order to reduce the computational time.

The sensorless control of a PMSM with a reduced-order Kalman filter was previously reported [5-8]. A third-order EKF with a back-electromotive force for interior PMSM sensorless control was proposed in [5]. A third-order linear Kalman filter was also proposed; however, its implementation is difficult [6]. Another study introduced a parallel reduced-order EKF with a second-order parallel form at a low speed and a third-order parallel form at a high speed [7]. Recently, a square-root EKF with state

and output variables in the form of a 2×1 matrix was introduced [8]. However, with the previously reported paper of the Kaman filter using reduced form, an additional algorithm is needed to estimate the electrical-rotor position and speed, and the state estimation values are inaccurate in the low-speed region (lower than 10% of the rated speed). And practically no research has been conducted to reduce the order of UKF algorithms in this motors control field.

The type of UT depends on the scaling-parameter values [9-10]. However, control is difficult to apply owing to the limited information about the scaling-parameter values, how the observer is affected by the scaling parameters, and how the scaling parameters should be selected. Various types of UT, such as basic UT, general UT, and scaled UT can be specified according to the distribution of sigma points and weights.

Experimental results for basic UT ($\alpha = 1, \beta = 0, \kappa = 0$) and general UT ($\alpha = 1, \beta = 0, \kappa = 0.5, 1, 36$) based on scaling parameters have been presented [10]. A previous work summarizes the scaling parameters and explains how to select and perform a simple comparison simulation of basic UT ($\alpha = 1, \beta = 0, \kappa = 0$), general UT ($\alpha = 1, \beta = 0, \kappa = 0.1-0.9$). Several experimental results for various types of UT, such as basic UT and general UT ($\kappa = 1$), have been presented [11]. The application of a UKF with basic UT having the parameter β ($\alpha = 1, \beta = 2, \kappa = 0$) was reported [12].

This paper investigates the application, design, and implementation of a reduced-order unscented Kalman filter (ROUKF) for state estimation of the motor drive without a position sensor. Experimental results for a wide range of operations, such as forward and reverse rotation at low and rated mechanical speeds; detuning parameters;

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and load tests demonstrated the quality of the estimation performance.

2. Reduced-order Model of Permanent Magnet Synchronous Motor

Motor mathematical modeling using the $\alpha\beta$ stationary reference frame serves to reduce the complexity of the three-phase voltage equation. If the change in the electrical speed is assumed to be constant for one sampling period, the differential value is considered to be zero. The general full-order form is as follows [1].

$$\frac{d}{dt} \begin{bmatrix} i_{\alpha s} \\ i_{\beta s} \\ \omega_r \\ \theta_r \end{bmatrix} = \frac{1}{L_s} \begin{bmatrix} V_{\alpha s} - R_s i_{\alpha s} + \lambda_m \omega_r \sin(\theta_r) \\ V_{\beta s} - R_s i_{\beta s} - \lambda_m \omega_r \cos(\theta_r) \\ 0 \\ L_s \omega_r \end{bmatrix} \quad (1)$$

Here, $V_{\alpha\beta s}$ are the $\alpha\beta$ axis stator voltages, $i_{\alpha\beta s}$ are the $\alpha\beta$ axis stator currents, L_s is the stator inductance, and R_s is the stator resistance. λ_m is the flux linkage from the permanent magnet, ω_r is the electrical speed, and θ_r is the electrical-rotor position.

The state matrix of the reduced-order form is a 3×1 matrix, and the output matrix is a 2×1 matrix. The state matrix of the reduced-order model is $\mathbf{x} = [i_{\alpha s} \ \omega_r \ \theta_r]^T$, and the output matrix is $\mathbf{y} = [i_{\alpha s} \ i_{\beta s}]^T$. The reduced-order discrete forms are as follows.

$$x_{1(k)} = ax_{1(k-1)} + bx_{2(k-1)} \sin(x_{3(k-1)}) + cu_{1(k-1)} \quad (2)$$

$$x_{2(k)} = x_{2(k-1)} \quad (3)$$

$$x_{3(k)} = x_{3(k-1)} + \Delta t x_{2(k-1)} \quad (4)$$

$$y_{1(k)} = x_{1(k)} \quad (5)$$

$$y_{2(k)} = ay_{2(k-1)} + bx_{2(k-1)} \sin(x_{3(k-1)}) + cu_{2(k-1)} \quad (6)$$

Here, Δt is the sampling time, $a = (1 - \Delta t R_s / L_s)$, $b = \Delta t \lambda_m / L_s$, and $c = \Delta t L_s$.

3. Unscented Kalman Filter

Compared with the EKF, the UKF has the same structure, including the prediction, Kalman gain, and correction step. The only difference is the UT method instead of the linearization process. Moreover, unlike Monte Carlo simulation, the proper number of samples for state estimation can be determined according to the state variables. The purpose of the UT method is to generate sigma points χ and weights ω using mean and covariance values of the function $f(\mathbf{x})$. Such a UT is affected by the scaling parameters α , β , κ , and λ which specify the distribution of sigma points and weights.

In scaled UT [3, 4], α , β , and κ are nonzero, and α is less than 1. Generally, the sum of the weights equals 1, except in the case of scaled UT, because of the β parameter, which is used to create a different weight W_0 for an error covariance $\mathbf{y}$. The sigma points and weights are defined as follows [3, 4].

First, the λ scaling parameter value is selected.

Second, the upper triangular matrix $\mathbf{u}_i$ is calculated using Cholesky factorization.

Third, the sigma points and weights are determined.

$$\lambda = \alpha^2 \kappa + (1 - \alpha^2)n \quad (7)$$

$$\mathbf{U}^T \mathbf{U} = (n + \lambda) \mathbf{P}_x \quad (8)$$

$$\chi_0 = \mathbf{x}, \quad W_0 = \lambda / (n + \lambda)$$

$$W_0^{[P_y]} = W_0 + 1 - \alpha^2 + \beta,$$

$$\chi_i = \mathbf{x} + \mathbf{u}_i, \quad W_i = 1 / 2(n + \lambda),$$

$$\chi_{i+n} = \mathbf{x} - \mathbf{u}_i, \quad W_{i+n} = W_i, \quad i = 1, 2, \dots, n. \quad (9)$$

Here, n is the number of state variables.

The case where $\lambda = 0$ ($\alpha = 1$ and $\kappa = 0$) and $\beta = 0$ is called basic UT [10]. The 0th sigma point χ_0 and weight W_0 are removed, and the sum of the weights is 1. The number of state variables is n , and the number of sigma points is $2n^2$. The case where $\alpha = 1$ and $\beta = 0$, is called general UT. If κ equals zero point five ($\kappa = 0.5$), the general UT [10] has the same weight values. The number of sigma points is $2n^2 + n$, and the sum of the weights is 1.

The UKF but for the UT has three steps: prediction, Kalman gain, and estimation. This procedure is as follows [3, 4].

Initialization;

Calculation of sigma points and weights;

Prediction of the estimation value and error covariance;

Prediction of the measurement value and error covariance;

Calculation of the Kalman gain;

Correction of the estimation state and covariance.

$$\hat{\mathbf{x}}_{k|k-1} = \sum_{i=0}^{2n} W_i \mathbf{f}(\chi_i). \quad (10)$$

$$\mathbf{P}_{x,k|k-1} = \sum_{i=0}^{2n} W_i (\mathbf{f}(\chi_i) - \hat{\mathbf{x}}_{k|k-1})(\mathbf{f}(\chi_i) - \hat{\mathbf{x}}_{k|k-1})^T + \mathbf{Q}. \quad (11)$$

$$\hat{\mathbf{y}}_{k|k-1} = \sum_{i=0}^{2n} W_i \mathbf{h}(\chi_i). \quad (12)$$

$$\mathbf{P}_{y,k|k-1} = \sum_{i=0}^{2n} W_i (\mathbf{h}(\chi_i) - \hat{\mathbf{y}}_{k|k-1})(\mathbf{h}(\chi_i) - \hat{\mathbf{y}}_{k|k-1})^T + \mathbf{R}. \quad (13)$$

$$\mathbf{P}_{xy,k|k-1} = \sum_{i=0}^{2n} W_i (\mathbf{f}(\chi_i) - \hat{\mathbf{x}}_{k|k-1})(\mathbf{h}(\chi_i) - \hat{\mathbf{y}}_{k|k-1})^T. \quad (14)$$

$$\mathbf{K}_k = \mathbf{P}_{xy} \mathbf{P}_y^{-1}. \quad (15)$$

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k (\mathbf{y}_k - \hat{\mathbf{y}}_k). \quad (16)$$

$$\mathbf{P}_k = \mathbf{P}_{k|k-1} - \mathbf{K}_k \mathbf{P}_{k|k-1} \mathbf{K}_k^T. \quad (17)$$

4. Simulation

To evaluate the performance of the proposed method with the UT, a computer simulation model was constructed using the language C. The stator resistance and inductance values of a PMSM (eight-pole 750-W class) are 1.5Ω and 4.87 mH . And the rated speed and torque of this motor are 2,000 rpm and 3.58 Nm .

The overall block diagram for a PMSM drive without a position sensor for vector control is shown in Fig. 1. This motor model is analyzed in the simulation by using the

Fig. 1. Vector-control PMSM drive without a position sensor.

Fig. 2. Simulation results for the proposed method: (a) load test (500 rpm, $0 \rightarrow 100 \rightarrow 0\%$ torque load), (b) detuning-parameter test (2000 $\rightarrow$ -2000 rpm)

fourth-order Runge-Kutta method (numerical analysis) at intervals of $1 \mu\text{s}$. The current and speed sampling time are $200 \mu\text{s}$ and 1 ms , respectively. The switching frequency is 2.5 kHz , according to the symmetric space vector pulse-width modulation method.

Fig. 2 shows the simulation results for the motor-drive algorithm without position sensors using the proposed reduced-order form with scaled UT ($\alpha = 1, \beta = 0.1, \kappa = 0.5$). Each figure presents the performance evaluation for the estimated mechanical speed. Fig. 2(a) shows the results of a rated-load test (3.58 Nm), which indicate that the proposed scheme gives satisfactory results. Fig. 2(b) presents the speed response according to each detuning-parameter value according to the rated reference speed ($2,000 \rightarrow -2,000 \text{ rpm}$). If the detuning stator resistance is far lower than the real value, the estimated speed near the zero point is unclear. And if the detuning stator inductance is far higher than the real value, the estimated speed oscillates during the steady state.

5. Experimental Result and Discussion

Fig. 3 shows the experimental equipment used to analyze an eight-pole 1-horsepower (HP) PMSM. A Texas Instruments 150-MHz TMS320F28335 module was used for the main arithmetic unit. The PMSM was equipped with a 2000 PPR incremental encoder to measure the actual speed and position.

To verify the performance of the proposed method, the full-order form with basic UT ($\alpha = 1, \beta = 0, \kappa = 0$) and the reduced-order form with scaled UT ($\alpha = 1, \beta = 0.1, \kappa = 0.5$) were compared.

Fig. 4 shows experimental waveforms of the forward and reverse rotations at a high speed (2,000 to -2,000 rpm, 100% of rated speed). Figs. 4(a) and (b) show experimental results for the full and reduced-order forms. The figures sequentially present the real electrical-rotor position (blue line), the estimated electrical-rotor position (green line), the error (orange line) during the initial transient state, the real mechanical speed (yellow line), the estimated mechanical speed (red line), and the error between the real and estimated mechanical speeds (orange line). The motor runs at zero speed, increases to 2,000 rpm for 2 s, and then decreases to -2,000 rpm. In the case of the full-order form, this indicates that the initial rising time is less than 150 ms and means that the speed response is 13.333 rpm per 1 ms

Fig. 3. 1-HP PMSM with load.

Fig. 4. Experimental results for the rated forward and reverse rotations (2,000 → -2,000 rpm): (a) full-order form; (b) reduced-order form

Fig. 5. Real and estimated electrical position and mechanical speed at low speeds (100 → -100 rpm, 5% of rated speed): (a) full-order form; (b) reduced-order form

(here, 1 ms is the speed controller sampling time). For the reverse rotation, Figures show the exact speed estimation near the zero point. In the case of the reduced-order form, this method performs well in the initial transient and steady state. However, it does not meet performance standards near the zero point in a moment (~40 ms). The appropriate system model variable values Q and R should have been determined according to the new reduced-order model.

Fig. 5 presents the forward and reverse rotations at low speeds (100 to -100 rpm, 5% of rated speed). As shown in the experimental results, the full-order form performed relatively well. The proposed method presents some differences between the actual and estimated mechanical speeds in the reverse transient state; however,

they converge quickly. The estimated electrical-rotor position experiences a slight overshoot near the zero point. This indicates that the proposed method has proper estimation and control performance for low speed.

To ensure robust performance, a load experiment was conducted. Fig. 6 shows the experimental results obtained with 2.7 Nm (75% of the rated load) applied to the PMSM and removed after 1.6 s at 500 rpm. This method has a slightly inaccurate estimated value when a load is applied. This estimated state affects the covariance value, and as a result, the performance of the other estimated values deteriorates. However, because the estimated values are at an acceptable level, this is tolerable. Thus, the proposed method has adequately robust performance

Fig. 6. Behavior and comparison of load test (500 rpm, 0 → 75 → 0% of rated load): (a) full-order form; (b) reduced-order form

Fig. 7. Experimental waveforms from the detuned-parameter test (2,000 → -2,000 rpm, 90% of stator resistance and inductance values): (a) full-order form, (b) reduced-order form

when experiencing a disturbance such as a load.

Fig. 7 presents the influence of the motor detuning parameters on the experimental waveform. The experimental conditions are set as 90% of the stator inductance and resistance values when the motor runs with forward and reverse rotation from 2,000 to -2,000 rpm. In the case of 90% detuning parameter values, the performance of this method is similar to the full-order form with UT, but one has a slight oscillation in certain sections. It can be seen that the response speed is relatively slow according to the reduced-order form. This indicates that the proposed method has excellent performance regardless of the wrong parameter values.

The computation time under the same experimental conditions was compared. The computation time and code size for each form are listed in Table 1. In the case of the full-order form, there are slight differences between the

Table 1. Comparison of computational times

Type		Size of state matrix	Code Size (byte)	Computation time (μ s)
Basic	Full-order	4×1	11,627	75.34
	Reduced-order	3×1	8,529	49.61
Scaled	Full-order	4×1	12,219	88.26
	Reduced-order	3×1	9,191	55.44

scaled UT and basic UT. The presence or absence of the zeroth sigma point had an important effect on the computation time, which is reduced by $\sim 13 \mu$ s. The reduced-order form can reduce the computation time by 26μ s (basic UT) and 33μ s (scaled UT). Thus, this method can reduce the computation time by $\sim 30 \mu$ s regardless of the UT type, while maintaining constant performance.

This paper shows how to reduce the code size and computation time of the Kalman filter. When a Kalman

filter is used to estimate state values, the computational load is the main drawback. We significantly resolved this problem. Furthermore, according to the experimental results, the proposed Kalman filter exhibited proper estimation performance comparable to that of the conventional Kalman filters.

6. Conclusion

This paper investigates the application, design, and implementation of a reduced-order unscented Kalman filter for the state estimation of a vector-control motor drive without a position sensor. The performance of the proposed method was investigated over a wide range of operations, such as forward and reverse rotations at low and high speed, detuning, and disturbance tests. Compared with the full-order form, the proposed scheme greatly reduces the computational time by approximately 34.15% and 37.19% for basic UT and scaled UT, respectively. The accuracy and performance of this method are not significantly different from those of the full-order form. Moreover, we studied how the scaling parameters determine the type of UT. The experimental results demonstrate the validity of the proposed algorithm for the sensorless speed control of the PMSM. In future research, we will explore the reduction of the speed error near the zero point.

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