Robust Position Control of One DOF Mechanical Systems Using Dual PIOs Without Velocity Measurement

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Abstract – This paper presents a robust position controller for a one degree-of-freedom (DOF) mechanical system using only position measurement. In order to alleviate the performance degradation owing to various uncertainties, a two-stage design method is studied by employing a proportional integral observer (PIO). In the first stage, a baseline backstepping controller is designed for a nominal system without accounting for uncertainties. The PIO is developed for estimating both the velocity information for the backstepping controller and an equivalent input disturbance for a feedforward compensation using the estimated uncertainty. It is shown that the estimation errors with the proposed PIO can be made arbitrarily small in a finite time. If the system suffers from undesirable actuator nonlinearities, however, it might be necessary to estimate the velocity and the disturbance with different rates of convergence. The proposed method combines the predesigned backstepping controller and dual PIOs to reduce mechanical vibrations as well as steady-state errors. The performance of the proposed method is tested through comparative computer simulations and experiments using a laboratory prototype.

Keywords: Robust position control, Backstepping control, Dual observers, Proportional integral observer

1. Introduction

This paper deals with a robust position control problem of a mechanical system actuated by a permanent magnet DC motor. Since most of practical systems are subject to parametric uncertainties, unmodeled dynamics, and external disturbances, it is not easy to design a high-performance controller for the system. In order to cope with adverse effects of the uncertainties, efficient controller design has been a central issue in control society [1, 2]. As a tool for disturbance attenuation, disturbance observer-based control (DOBC) has been widely employed because of its powerful ability to attenuate disturbances and compensate plant uncertainties [3-12].

Among the disturbance observer approaches, the use of observers to estimate an equivalent lumped disturbance is widespread [10-16]. The main idea is to view the disturbance input as an additional state variable and augment it with the plant. The observer is designed for the extended system. By using an additional integral of the estimation error to the Luenberger observer, the extended state observer (also known as proportional integral observer) successfully estimates a slow-varying disturbance. An interesting feature of the proportional integral observer (PIO) is that it can effectively estimate both system states and disturbance in the presence of plant uncertainties.

DOBC is a two-stage design process, that is, a baseline controller is designed first followed by a feedforward compensator incorporating the estimate from a disturbance observer [11]. This paper adopts the backstepping control (BC) in the first stage as a baseline controller [1]. BC has been widely used under the assumption that all the system states are measurable and the bounds of uncertainties are known [17]. In addition, BC has been combined with disturbance observers to compensate for uncertainties in a feedforward way without using high-gain control loops [18-20].

However, since the velocity measurement is not always available in most actual systems, it should be estimated for implementing BC-based controllers. This paper considers the PIO to estimate the system states accurately for the predesigned BC. It is shown that the estimation errors with the PIO can be made arbitrarily small in a finite time. Although a single PIO can estimate both the system states and the uncertainty at the same time, the proposed method constructs dual PIOs to estimate the states and the disturbance independently.

As the observer gains become larger to reduce the estimation errors, the rates of convergence become faster at the same time. While the fast convergence is necessary for the velocity estimation, the estimation of uncertainty does not always need to be so fast. For example, when the estimated uncertainty contains a fast-varying detrimental signal, the feedforward compensation may not achieve improved performance or even cause vibrations and noises in the presence of undesirable actuator nonlinearities such

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as backlash and delay. By using the proposed dual PIOs, the estimates of velocity and uncertainty can be obtained separately. The PIOs have different rates of convergence from each other. One fast PIO estimates the system states for the baseline controller. The other slow PIO is used for the feedforward compensation to remove the steady-state error owing to various uncertainties.

Comparative computer simulations and experiments using a laboratory prototype [21] have been conducted to test the effectiveness of the proposed method. In both simulation and experimental results, the proposed controller is compared with two other controllers: (i) the first one is a BC using estimated system states via a single PIO; (ii) The other controller is an integral backstepping controller (IBC) for removing the steady-state error with the BC. The results show enhanced performance of the proposed scheme when dual PIOs are applied.

The remainder of the paper is organized as follows. Section 2.1 introduces the problem formulation and the mathematical model of the system. The baseline backstepping controller (BC) is designed in Section 2.2 when the velocity information is available for feedback without accounting for uncertainties. In Section 2.3, the proposed PIO is presented, along with theoretical analysis that proves the estimation errors can be made arbitrarily small in a finite time. The proposed control scheme with dual PIOs is presented in Section 2.4. The performance of the proposed controller is tested in Section 3. Finally, Section 4 presents the conclusions.

Notations: For a matrix A , $(A)_{ij}$ denotes the element of row i and column j . The i -th element of a vector x is denoted by $(x)_i$.

2. Main Results

2.1 Problem formulation

This paper considers a position control problem of the mechanical system shown in Fig. 1. The system is actuated by a voltage-controlled permanent magnet DC motor. Under parametric uncertainties and disturbances, the system is described by

Fig. 1. One-link manipulator

$$\frac{d\theta_m}{dt} = \omega_m \quad (1a)$$

$$\frac{d\omega_m}{dt} = -\frac{B_m}{J_m}\omega_m + \frac{K_t}{J_m}i_a + d \quad (1b)$$

$$\frac{di_a}{dt} = -\frac{R_a}{L_a}i_a - \frac{K_b}{L_a}\omega_m + \frac{1}{L_a}u \quad (1c)$$

$$y = \theta_m \quad (1d)$$

where θ_m and ω_m are respectively the link position (angle) and velocity; i_a is the armature current; u and y are the control input (voltage) and the measurable output, respectively; and d represents unmodeled dynamics and external disturbances. J_m and B_m are the inertia and the friction coefficient, respectively; K_t and K_b are the torque and the back-emf constant, respectively; L_a and R_a denote the armature inductance and resistance, respectively.

Since the fast current dynamics described by (1c) can be disregarded, the system is represented by

$$\dot{x}_1 = x_2 \quad (2a)$$

$$\dot{x}_2 = -\theta_{1n}x_2 + \theta_{2n}u + \Delta \quad (2b)$$

$$y = x_1 \quad (2c)$$

where $x_1 = \theta_m$ and $x_2 = \omega_m$; θ_{1n} and θ_{2n} are the nominal values of $\theta_1 = (B_m R_a + K_t K_b)/(J_m R_a)$ and $\theta_2 = K_t/(J_m R_a)$, respectively; and $\Delta = -(\theta_1 - \theta_{1n})x_2 + (\theta_2 - \theta_{2n})u + d$.

In the presence of uncertainties, it is not easy to design a high-performance controller for the system. Among various robust control methods, the BC(backstepping controller) has been widely used under the assumption that the state x is measurable with the known bound of Δ [17]. This paper presents a PIO-based robust position controller that incorporates a BC as the baseline controller.

2.2 BC using velocity measurement

Assuming that the velocity is measurable¹, a BC is designed for the nominal system in (2). Let $r(t)$ be a desired trajectory which has bounded derivatives up to the second order. When $e_1 = r - x_1$, the time derivative of e_1 can be represented by

$$\dot{e}_1 = -k_1 e_1 + e_2 \quad (3)$$

where $e_2 = x_2^* - x_2$ and $x_2^* = \dot{r} + k_1 e_1$ with $k_1 > 0$. Hence, the tracking problem is equivalent to make e_2 as small as possible.

The dynamics of e_2 is given by

$$\dot{e}_2 = \dot{x}_2^* + \theta_{1n}x_2 - \theta_{2n}u - \Delta \quad (4)$$

¹ This assumption is removed in Section 2.3.

where $\dot{x}_2^* = \ddot{r} + k_1 \dot{e}_1 = \ddot{r} + k_1 (\dot{r} - x_2)$. When the control input is chosen as $u = u_c$ with

$$u_c = \theta_{2n}^{-1} (k_2 e_2 + \dot{x}_2^* + \theta_{1n} x_2) \quad (5)$$

where $k_2 > 0$, the closed-loop system is obtained by

$$\dot{e}_2 = -k_2 e_2 - \Delta. \quad (6)$$

This implies $e_2 \rightarrow 0$ and $e_1 \rightarrow 0$ if $\Delta = 0$.

When $\Delta \neq 0$, the high tracking accuracy might be obtained by employing large feedback gains [17, 19]. However, large gains are prone to cause undesirable noise and vibrations if the system has actuator nonlinearities (e.g. backlash and delay) and/or measurement noise. Though an integral backstepping control (IBC) can be adopted to deal with the uncertainty [1], there exists a contradiction between the uncertainty compensation and the nominal control performance [2]. Moreover, since the velocity is not available for the control input in most cases, this paper considers the PIO to estimate x_2 and Δ , respectively. A quantitative analysis is presented on the performance of the proposed PIO.

2.3 Performance Analysis of PIO

Defining $x_3 = \Delta$, the PIO is given by

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 + L(x_1 - \hat{x}_1) \\ \dot{\hat{x}}_2 &= -\theta_{1n} \hat{x}_2 + \hat{x}_3 + \theta_{2n} u + L_2(x_1 - \hat{x}_1) \\ \dot{\hat{x}}_3 &= L_3(x_1 - \hat{x}_1) \end{aligned} \quad (7)$$

where the observer gains are designed as $L_1 = 3\omega_o - \theta_{1n}$, $L_2 = 3\omega_o^2 - L_1\theta_{1n}$ and $L_3 = \omega_o^3$ with $\omega_o \geq 1$. Unlike the extended state observer in [10], the PIO described by (7) utilizes the nominal parameter θ_{1n} .

When $h = \dot{\Delta}$, the estimation error dynamics is given by

$$\begin{aligned} \dot{\tilde{x}} &= \begin{bmatrix} -3\omega_o + \theta_{1n} & 1 & 0 \\ -\theta_{1n}^2 + 3\omega_o\theta_{1n} - 3\omega_o^3 & -\theta_{1n} & 1 \\ -\omega_o^3 & 0 & 0 \end{bmatrix} \tilde{x} + \begin{bmatrix} 0 \\ 0 \\ h \end{bmatrix} \\ &=: A\tilde{x} + B h \end{aligned} \quad (8)$$

where $\tilde{x} = x - \hat{x}$ and $\hat{x} = [\hat{x}_1 \ \hat{x}_2 \ \hat{x}_3]^T$. Since the characteristic polynomial is $(s + \omega_o)^3$, the matrix A is Hurwitz. The solution of (8) is given by

$$\tilde{x}(t) = e^{At} \tilde{x}(0) + \int_0^t e^{A(t-\tau)} B h(\tau) d\tau =: \tilde{x}_{z,i,r} + \tilde{x}_{z,s,r} \quad (9)$$

where $\tilde{x}_{z,i,r}$ and $\tilde{x}_{z,s,r}$ represent the zero-input-response and the zero-state-response, respectively.

In many mechanical and electromechanical applications, a slowly varying disturbance is approximated by a constant

[11-13]. For the performance analysis of the PIO, we need the following assumption [2, 10, 12, 13, 19].

Assumption 1: The time derivative of Δ is bounded by a positive constant δ , i.e., $|h| \leq \delta$.

Remark 1: If the variable h is not bounded, Assumption 1 can be relaxed by using multiple integrals of the estimation error as in [14, 15, 20].

Lemma 1: Under Assumption 1, there exist positive constants σ_i and a finite time $T > 0$ such that

$$|\tilde{x}_i(t)| \leq \sigma_i, \quad \sigma_i = O\left(\frac{1}{\omega_o}\right), \quad \forall t \geq T, \quad (10)$$

for $i = 1, 2, 3$, where $\omega_o \geq \max\{1, \theta_{1n}/2\}$.

Proof: From (9), the proof considers the zero-input-response $\tilde{x}_{z,i,r}$ and the zero-state-response $\tilde{x}_{z,s,r}$ in sequence. As in [10], since the matrix A is Hurwitz, it can be shown that there exists a finite time $T > 0$ such that

$$|(e^{At})_{ij}| \leq \frac{1}{\omega_o^3}, \quad \forall t \geq T, \quad i, j = 1, 2, 3. \quad (11)$$

Let $\tilde{x}_{0s} = \sum_{i=1}^3 |\tilde{x}_i(0)|$, then it gives

$$|(\tilde{x}_{z,i,r})_i| \leq \frac{\tilde{x}_{0s}}{\omega_o^3}, \quad \forall t \geq T, \quad i = 1, 2, 3. \quad (12)$$

For the analysis of $\tilde{x}_{z,s,r}$, it is noted that

$$e^{At} B = \begin{bmatrix} \frac{1}{2} t^2 \\ t + \frac{1}{2} (2\omega_o - \theta_{1n}) t^2 \\ 1 + \omega_o t + \frac{1}{2} \omega_o^2 t^2 \end{bmatrix} e^{-\omega_o t}. \quad (13)$$

Hence, when $2\omega_o \geq \theta_{1n}$, $(e^{At} B)_i \geq 0$, $\forall t \geq 0$, for $i = 1, 2, 3$. This yields

$$\begin{aligned} |(\tilde{x}_{z,s,r})_i| &\leq \left(\int_0^t e^{A(t-\tau)} B |h(\tau)| d\tau \right)_i \leq \delta \left(\int_0^t e^{A(t-\tau)} B d\tau \right)_i \\ &= \delta \left(e^{At} [-A^{-1} e^{-A\tau} B]_0^t \right)_i = \delta (-A^{-1} B + e^{At} A^{-1} B)_i. \end{aligned} \quad (14)$$

Since

$$A^{-1} = \begin{bmatrix} 0 & 0 & -\omega_o^{-3} \\ 1 & 0 & -(3\omega_o - \theta_{1n})\omega_o^{-3} \\ \theta_{1n} & 1 & -3\omega_o^{-1} \end{bmatrix}, \quad (15)$$

it can be seen that

$$\left| (A^{-1}B)_i \right| \leq \frac{3}{\omega_o^{4-i}} \quad (16)$$

for $i = 1, 2, 3$. When $\omega_o \geq 1$, by (11) and (16),

$$\left| (e^{At} A^{-1} B)_i \right| \leq \frac{1}{\omega_o^3} \left(\frac{1}{\omega_o^3} + \frac{3}{\omega_o^2} + \frac{3}{\omega_o} \right) \leq \frac{7}{\omega_o^4} \quad (17)$$

for $i = 1, 2, 3$. From (16) and (17),

$$\left| (\tilde{x}_{z,s,r})_i \right| \leq \delta \left(\frac{3}{\omega_o^{4-i}} + \frac{7}{\omega_o^4} \right), \quad \forall t \geq T, \quad (18)$$

for $i = 1, 2, 3$. Finally, it can be shown that

$$\left| \tilde{x}_i(t) \right| \leq \frac{\tilde{x}_{0s}}{\omega_o^3} + \delta \left(\frac{3}{\omega_o^{4-i}} + \frac{7}{\omega_o^4} \right), \quad \forall t \geq T, \quad (19)$$

for $i = 1, 2, 3$. This completes the proof. $\square$

Remark 2: An extended state observer was constructed for an n -dimensional system in [10]. Unlike the previous result, this paper provides a more complete proof with consideration given to a nonzero θ_{1n} . For the sake of simplicity, the proof does not use the error state transformation $\varepsilon_i = \tilde{x}_i / \omega_o^{i-1}$ in [10].

By using an additional integral of the estimation error, the PIO estimates the system states more accurately than the Luenberger observer. From Lemma 1, the velocity estimation error can be made arbitrarily small with a sufficiently large ω_o . However, the estimated uncertainty ($\hat{x}_3$) using the large observer gain, would not be helpful for the feedforward compensation if the actuator has uncertain nonlinearities such as backlash and delay. In order to avoid undesirable vibrations owing to $\hat{x}_3$, the estimations of x_2 and x_3 need to be carried out separately with different

gains. The next subsection proposes a dual PIO-based control method.

2.4 Proposed controller: BC + Dual PIOs

The proposed controller is designed by incorporating the BC (5) with dual PIOs (PIO₁ and PIO₂) as depicted in Fig. 2. Each PIO estimates the state x and the uncertainty Δ , respectively, with different convergence speeds. The rates of convergence are determined by the observer parameter ω_o . Table 1 illustrates the notations on the PIOs.

The estimated state variables $\hat{x}^f = [\hat{x}_1^f \ \hat{x}_2^f \ \hat{x}_3^f]^T$ and $\hat{x}^s = [\hat{x}_1^s \ \hat{x}_2^s \ \hat{x}_3^s]^T$.

Since exact state estimations are necessary for the BC, a sufficiently large ω_o^f is selected for PIO₁ based on Lemma 1. Using $\hat{x}_1^f$ and $\hat{x}_2^f$, the proposed BC ($\hat{u}_c$) is given by

$$\hat{u}_c = \theta_{2n}^{-1} (k_2 \hat{e}_2 + \hat{x}_2^* + \theta_{1n} \hat{x}_2^f) \quad (20)$$

where $\hat{e}_2 = \hat{x}_2^* - \hat{x}_2^f$, $\hat{x}_2^* = \dot{r} + k_1(r - \hat{x}_1^f)$ and $\hat{x}_2^s = \dot{r} + k_1(\dot{r} - \hat{x}_2^f)$.

When a large value of k_2 in (20) causes vibrations, the feedforward compensation for Δ is performed by adopting another PIO (PIO₂) instead of using $\hat{x}_3^f$ of PIO₁. In order to avoid including undesirable signals in $\hat{x}_3^s$, ω_o^s for PIO₂ is designed through experimental tests as explained in the next section. Since a large ω_o^s for $\hat{x}_3^s$ can make undesirable vibrations and noises, the value of ω_o^s is determined to lessen the influence of Δ with a small k_2 .

Finally, the proposed control input is as follows:

$$\begin{aligned} u &= \hat{u}_c - \theta_{2n}^{-1} \hat{x}_3^s \\ &= \theta_{2n}^{-1} \{ \theta_{1n} \hat{x}_2^f + \dot{r} + (k_1 + k_2)(\dot{r} - \hat{x}_2^f) \\ &\quad + k_1 k_2 (r - \hat{x}_1^f) - \hat{x}_3^s \}. \end{aligned} \quad (21)$$

A step-by-step procedure to implement the controller (21) is as follows: (i) design of PIO₁ for $\hat{x}_1^f$ and $\hat{x}_2^f$; (ii) selection of k_1 and k_2 for the BC (20); and (iii) design of PIO₂ for $\hat{x}_3^s$.

In order to test the performance of the proposed method, various experimental tests are carried out in the next subsection with the system of Fig. 1 [21].

3. Performance Tests

The system and observer parameters are listed in Table 2. The real values of θ and the disturbance $d=1+2\text{sgn}(x_2)$ have been used in simulations. It can be seen that the disturbance does not satisfy Assumption 1 when the sign of x_2 changes. Although the disturbance could not satisfy

Fig. 2. Proposed controller structure

Table 1. Parameters of PIOs

	Fast Speed PIO	Slow Speed PIO
Name	PIO ₁	PIO ₂
State	$\hat{x}^f$	$\hat{x}^s$
ω_o	ω_o^f	ω_o^s

Table 2. System parameters

θ_{1n}	8.8889	θ_1	11.2592
θ_{2n}	12.4678	θ_2	9.9743
PIO_1	$\omega_0^f = 100$	PIO_2	$\omega_0^s = 20$

Table 3. Control gains

Controller	k_1	k_2	k_3
(i) BC (20)	50	50	-
(ii) IBC (22)	40	7	30
(iii) Proposed (21)	40	7	-

Assumption 1, the sign function has been included to test whether a practical disturbance such as Coulomb friction force would deteriorate the performance of the proposed method. The position angle is measured by using an incremental encoder whose resolution is 4000 ppr. The actuator gear has a little backlash that can make bad influences on the control performance. For the experiments, the proposed control algorithms are implemented by using CEMWARE [21]. The sampling period is 3ms.

In order to test the performance, three different controllers are compared: (i) BC described by (20), (ii) IBC(integral BC) described by

$$u = \hat{u}_c + k_3 \int_0^t (r - \hat{x}_1^f) d\tau \quad (22)$$

where $k_3 > 0$, and (iii) Proposed controller (BC+PIOs) described by (21).

Based on Lemma 1, the state estimations, $\hat{x}_1^f$ and $\hat{x}_2^f$, have been carried out by PIO_1 . The uncertainty estimation $\hat{x}_3^s$ has been performed by PIO_2 . The control gains are listed in Table 3.

Figs. 3 and 4 show the test results where the reference r is the square wave whose amplitude is 1rad and the period is 9.9sec. The controller BC described by (20) has employed large feedback gains to compensate for the uncertainty Δ . Since the gains are larger than the other two methods, the initial errors of BC are smaller than the other ones. Whilst much larger gains could reduce the steady-state errors in the simulations, this scheme was not successful because of unacceptable vibrations and noises in the experiments. In order to cope with steady-state errors and undesirable vibrations, the IBC described by (22) has been tested with low feedback gains (see Table 3). Though the vibrations and steady-state errors can be reduced with the IBC, the transient response shows a contradiction between the uncertainty compensation and the regulation performance. The results of the proposed controller described by (21) show improved performances without steady-state errors and undesirable vibrations.

The 2-norms of the error signals of Fig. 3 and 4 are compared in Table 4. It can be noticed that the initial errors of the IBC and the proposed method are bigger than the

(a) Link position (simulation)

(b) Link position error (simulation)

(c) Control input (simulation)

Fig. 3. Simulation results
Table 4. Norms of position error

Controller	Simulation	Experiment
(i) BC (20)	0.7142	0.4264
(ii) IBC (22)	3.4293	1.1629
(iii) Proposed (21)	1.1022	0.4073

result of the BC. Though 2-norm of the error signal from the proposed method is larger than that of the result of the BC in the simulations, 2-norms from real experimental results show a quantitative enhancement of the proposed method.

(a) Link position (experiment)

(b) Link position error (experiment)

(c) Control input (experiment)

Fig. 4. Experimental result

4. Conclusion

In the presence of parametric uncertainties and unmodeled dynamics, this paper presents a robust position controller for a one DOF mechanical system by using dual PIOs together with a backstepping controller based on position measurement. A detailed theoretical analysis has shown that the estimation errors using the proposed PIO can be made arbitrarily small in a finite time. Since many actual systems suffer from undesirable actuator non-

linearities and measurement noise, it is necessary to estimate the velocity and uncertainties with different rates of convergence. Through comparative computer simulations and experimental tests, the performance of the proposed method has been illustrated.

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