Prototype of 6-DOF Magnetically Levitated Stage Based on Single Axis Lorentz force Actuator

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Abstract – We present a prototype of 6-DOF magnetically levitated stage based on single axis Lorentz force actuator is capable of positioning down to micron in several millimeter travel range with a simple and compact structure. The implementation of the Lorentz force actuators, instrument modeling, and motion controller of the maglev system are described. With the force-gap relationship of the actuator solved by Gaussian quadrature, 2-demsional (2D) lookup table is employed to store the result for the designing of control system. Based on the force-gap relationship, we choose the suitable stroke of each actuator to develop the stability of the maglev stage. Complete decoupling matrix is analyzed here to establish a decoupled dynamics between resulting six axis motion and eight outputs to the actuators. The design travel volume of the stage is 2mm×2mm×2mm in translation and 80mrad×80mrad×40mrad in rotation. With a constant gain and critical damping PID controller, the resolution of the translation is 2.8um and 4um root mean square in horizontal and vertical direction respectively. Experimental results are presented to illustrate the positioning fluctuations, step responds and multi axis motion. Some comparative tests are taken to highlight the advantage resulted from the accurate force-gap relationship, suitable stroke, and complete decoupling matrix.

Keywords: Magnetically levitated stage, Lorentz force actuator, Gaussian quadrature, Complete decoupling matrix, Critical damping PID controller

1. Introduction

High-precision positioning stages have been widely used in the measuring and detecting instruments, such as coordinate measuring machine, profile scanning device, spectrograph and microscope. The positioning stage can load an object, then move and rotate to a certain position and orientation. In manipulation, the evaluating indexes of the motion-control devices contain accuracy of position, range of motion, degrees of freedom, and bandwidth of response [1]. Traditional positioning stages usually employ rotary or linear motors assembled with some gearings. With the inherent friction and linkage, it is hard to improve the positioning accuracy and response speed. A promising solution for high-precision positioning is magnetically levitated technology. This technology has been successfully implemented in various systems, such as high-speed maglev train [2, 3], vibration isolation system [4], magnetic bearings [5-7], and electromagnetic launchers [8]. Because the translator and the stator is noncontact with each other, there is no friction, stiction, and backlash existed, which lead to a simple system dynamic, excellent repeatability, high positioning accuracy, and clean working environment. As the translator can produce all 6 DOF motions without bearing and guiding mechanism, the magnetically levitated

stage allows a high bandwidth control if we need a complex motion in 3-D space.

The actuator takes a significant part in the maglev positioning stage. In general, the force created by the actuators should not only balance the gravity in vertical direction but also actuate the stage in horizontal direction. Nowadays, Lorentz-force actuator [9, 10], in which the magnetic force is created by the interaction between the magnetic field due to the permanent magnet and the current in the current carrying coil, is widely used in the high-precision positioning device. This kind of actuator is always assembled in the magnetically levitated stage because of the various designing structure and the simple controlling method. There are plenty of designs proposed in the literature. In these reports, the maglev positioning stages are proved to be excellent in positioning accuracy and respond speed, but the small travel range is a shortcoming. Enlarging the stroke of actuator is effective to solve this issue [11-13], but the system will suffer from three challenges:

- 1). hard to obtain the accurate force-gap relationship;
- 2). open-loop instability in each actuator;
- 3). strong coupling between all actuators.

Aiming at the three challenges, different solutions are proposed in current research. Firstly, polynomial fitting method is always employed to model the force-gap relationship for the controlling system [14, 15]. Secondly, with the inherent open loop instability of long stroke

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actuator, many researchers use the advanced control method [16, 17]. Thirdly, to eliminate the strong coupling between each actuator, the wrench-current transforming matrix is constructed to decouple each actuating unit [18]. However, some problems are existed in practice: as the force model of actuator becomes complicated with the stroke enlarging of the actuator, many designers just fit the force-gap relationship into the approximate single-variable function; the hardware overhead is high for the complicated control method, so high standard hardware is necessary for the control of the maglev positioning system; in order to simplify the wrench-current transforming matrix, the coupling between vertical actuators and horizontal actuator is always ignored. These factors will affect the performance and the application of maglev positioning device in reality.

This paper presents a prototype of 6-DOF magnetic levitated stage in which the travel range is enlarged due to developing the stroke of single axis cylindrical Lorentz force actuator. Eight independent long-stroke Lorentz force actuators are utilized to actuate the stage, and three methods are used to overcome the shortcomings aforementioned:

- 1) Solving the force-gap relationship by Gaussian quadrature, and storing it in a 2-dimensional look up table [19] for the feedback linearity in the control system;
- 2) Analyzing the force model of the actuator to choose suitable air-gap where the open loop instability in ironless Lorentz-force actuator decreases significantly;
- 3) Based on the designing structure, constructing the complete wrench-current transforming matrix to decouple all of the actuators.

With these methods, the maglev stage realizes the stroke of $2\text{mm} \times 2\text{mm} \times 2\text{mm}$ in translation and $80\text{mrad} \times 80\text{mrad} \times 40\text{mrad}$ in rotation. The resolution of the translation is $2.8\mu\text{m}$ in vertical direction and $4\mu\text{m}$ in horizontal direction.

The content of the rest of the paper is as follows. We present the mechanical design and assembly of the maglev stage in Section II. In Section III we introduce the solving method of force-gap relationship and discuss the choosing of the air-gap of actuator. The model of the maglev stage is described in Sections IV, including the actuating unit, sensors, decoupling, and control method; the subsection of force distribution in Section IV introduce the constructing of wrench-current transforming matrix in details. The experiment results are shown in section V. Conclusion is made in section VI.

2. Maglev Stage and Instrument System

2.1 The structure of the maglev stage

The structure of magnetically levitated stage is depicted in Fig. 1. The basic parts including platen, Lorentz force actuator, magnet holders, sensors and foundation support

Fig. 1. Assembly diagram of the maglev stage: (a) without levitated platen; (b) with levitated platen

Fig. 2. Maglev positioning stage

have been labeled. The coordinate system in this figure will be defined in next section. Aluminum is used to fabricate the stage with its light weight and high stiffness. Fig. 1(a) shows the stator and sensors, and does not contain the levitated part. The laser sensors are used to measure the displacement in horizontal direction while the eddy gauges are used to measure the displacement in vertical direction. Fig. 1(b) shows the main direction of Lorentz force created by the actuators. At the two symmetrical corners of the platen, there are four horizontal actuators that make the stage move in three horizontal DOFs, i.e., x - and y -translation and γ -rotation about the z -axis. Four vertical actuators in the middle of the platen's four edges can generate forces in three vertical DOFs, i.e., z -translations and α - and β -rotation about the x and y axis. The

Fig. 3. Integration of system

Fig. 4. The structure of Lorentz force actuator

photograph of this system is shown in Fig. 2.

2.2 System integration

Fig. 3 describes the structure of this instrument. The three-axis position in the horizontal plane (x -, y -translation, and γ -rotation) is sensed by laser displacement sensor, and three-axis position in the vertical plane (z -translation, and α -, β -rotation) is sensed by eddy gauge. The digital signal processor (DSP) TMS320F28335 by Texas Instruments takes care of all the foreground computing tasks in real-time control. It gathers the position data from these six displacement sensors, applies the control law, and calculates the control outputs and transfers the position information to the computer. We sample the analog signals of displacement sensor and convert them to digital data by two 16-bit analog-to-digital converters (ADC) ADS8361. These data are used by the real-time digital controller to calculate the position of the stage. To reduce the affection from the inherent noise of the sensor and circuit, we use a Kalman filter to process the data obtained from the ADC. The position data is used as the input of the control unit, while the output of the control unit is magnitude of voltage. The output voltage is generated via two 16-bit digital-to-analog (DAC) converters AD5665. All the above tasks are accomplished in an interrupt service routine (ISR) initiated by a timer interrupt at every 500 μ s. We employ eight voltage controlled constant current source based on OPA659 to excite the actuator. The output current from the amplifier is linearly proportional to the input voltage. The user commands and position information used to display are

interacted by USB bus between the DSP and computer.

3. Actuator design

3.1 Analyze of force-gap relationship

The vertical and horizontal actuator has the same composition as shown in Fig. 4. It uses AWG #21 wire for winding and cylindrical NdFeB permanent magnet (PM) for the moving part. The current carrying coil is excited by the digital constant current source. The Lorentz force created by the interaction of the magnetic field and the current in the coil can actuate the magnet.

We calculate the force based on Lorentz law. The magnetic field distribution should be acquired at first and it can be solved conveniently in terms of the cylindrical coordinates. As shown in Fig. 5, we define a polar coordinate system. The origin of the system $\{m\}$ is the mass center ${}^m o$ of the PM and the central axis of cylindrical magnet is the ${}^m z$ -axis. An arbitrary point P1 on North Pole and an arbitrary point P2 on South Pole can contribute to the scalar potential at point $P(r, \theta, z)$ respectively. So the scalar potential at point $P(r, \theta, z)$ can be represented as:

$$\Phi(\vec{s}) = \frac{u_0 M}{4\pi} \left(\iint_N \frac{1}{|\vec{s} - \vec{s}_1|} d\vec{s} - \iint_S \frac{1}{|\vec{s} - \vec{s}_2|} d\vec{s} \right). \quad (1)$$

In this expression, the vectors have been shown in Fig. 5. Parameter M is remanence of the PM and u_0 is vacuum magnetic permeability. As magnetic flux density is the gradient of scalar potential, the magnetic flux density at point $P(r, \theta, z)$ is available. If there are no free currents, the magnetic flux density can be written as the gradient of scalar potential Φ . As the magnet is coaxial with ${}^m z$ -axis, the polar component is 0. The radial and axial components of magnetic flux density ${}^m B_r$ and ${}^m B_z$ can be written as:

$${}^m B_r(r, z) = \frac{u_0 M}{4\pi} \int_0^{2\pi} \int_0^R \left(\rho \cdot r - \rho^2 \cos \varphi \right) \left(\frac{1}{L_1^{3/2}} - \frac{1}{L_2^{3/2}} \right) d\rho d\varphi, \quad (2)$$

Fig. 5. Geometry of the cylindrical magnet

Fig. 6. Geometry of the actuator.

$${}^m B_z(r, z) = \frac{u_0 M}{4\pi} \int_0^{2\pi} \int_0^R \left(\frac{(z - H/2) \cdot \rho}{L_1^{3/2}} - \frac{(z + H/2) \cdot \rho}{L_2^{3/2}} \right) d\rho d\varphi. \quad (3)$$

Variables R and H , which means the radius and height of the PM respectively, can be found in Fig. 5. L_1 and L_2 are:

$$L_1 = \rho^2 + r^2 - 2 \cos \varphi \rho r + (z - H/2)^2, \quad (4)$$

$$L_2 = \rho^2 + r^2 - 2 \cos \varphi \rho r + (z + H/2)^2. \quad (5)$$

In general, when the levitated stage is working, the PM is always not coaxial with coil as shown in Fig. 6. Both the radial and axial components of magnetic flux density, ${}^m B_r$ and ${}^m B_z$, can affect the force acting on the current carrying coils. As mentioned above, the magnetic flux density $\vec{B}$ is written as:

$$\vec{B} = {}^m B_r \hat{r} + {}^m B_z \hat{z}. \quad (6)$$

As shown in Fig. 6, ${}^c o$ is the geometric center of the coil. Variable Δd and Δh represent the position of coils' geometric center, ${}^c o$, in the polar coordinate system $\{m\}$. The current density at volume element is:

$$\vec{J} = {}^m J_r \hat{r} + {}^m J_\theta \hat{\theta}. \quad (7)$$

${}^m J_r$ is the radial component, ${}^m J_\theta$ is the polar component and J is the magnitude of the current density in the coils.

$${}^m J_r(r, \theta) = \frac{\Delta d \cdot \sin \theta}{\sqrt{\Delta d^2 + r^2 - 2 \cdot \Delta d \cdot r \cos \theta}} \cdot J \quad (8)$$

$${}^m J_\theta(r, \theta) = \frac{r - \Delta d \cdot \cos \theta}{\sqrt{\Delta d^2 + r^2 - 2 \cdot \Delta d \cdot r \cos \theta}} \cdot J \quad (9)$$

We use three variables ${}^m F_\theta$, ${}^m F_z$ and ${}^m F_r$ to reflect three components of Lorentz force acting on the coils. Even though the actuator including the coils and the magnet in Fig. 6 is not axial symmetric about ${}^m z$ -axis, it is still

plane symmetric about the plane of this paper. Therefore ${}^m F_\theta$ is approximate to zero. ${}^m F_z$ and ${}^m F_r$ can be written as [20]:

$${}^m F_z(\Delta d, \Delta h) = \iiint_V {}^m J_\theta \times {}^m B_r dV$$

$$= J \int_0^{2\pi} \int_{z_0}^{z_h} \int_{r_0}^{r_h} r \frac{r - \Delta d \cos \theta}{\sqrt{\Delta d^2 + r^2 - 2 \cdot \Delta d \cdot r \cos \theta}} {}^m B_r dr dz d\theta, \quad (10)$$

$${}^m F_r(\Delta d, \Delta h) = \iiint_V {}^m J_\theta \times {}^m B_z dV$$

$$= J \int_0^{2\pi} \int_{z_0}^{z_h} \int_{r_0}^{r_h} r \frac{r - \Delta d \cos \theta}{\sqrt{\Delta d^2 + r^2 - 2 \cdot \Delta d \cdot r \cos \theta}} {}^m B_z \cos \theta dr dz d\theta. \quad (11)$$

In above formulas, z_0 , z_h , r_0 ' and r_h ' can be solved by (12). The variables r_0 , r_h and h represent the inner, outer radius and the height of the coils respectively.

$$\begin{cases} z_0 = -h/2 + \Delta h \\ z_h = h/2 + \Delta h \\ r_0' = \sqrt{r_0^2 - \sin^2 \theta \cdot \Delta d^2} - \cos \theta \cdot \Delta d \\ r_h' = \sqrt{r_h^2 - \sin^2 \theta \cdot \Delta d^2} - \cos \theta \cdot \Delta d \end{cases} \quad (12)$$

The design parameters of the actuator are shown in Fig. 6. Their dimensions are in millimeter. If the turns of coils is n and the current is I , the magnitude of current density $\vec{J}$ is:

$$J = n \cdot I / (r_h - r_0) h. \quad (13)$$

The accurate result is significant to design the control unit. It is hard to get their result directly, and we use Gaussian quadrature to solve ${}^m F_r(\Delta d, \Delta h)$ and ${}^m F_z(\Delta d, \Delta h)$. The numerical integral is expressed as:

$$\int_{-1}^1 f(\lambda) dx \approx \sum_{i=0}^{N-1} w_i f(\lambda_i). \quad (14)$$

We take 8 order Gaussian quadrature to solve (10) and (11).

$${}^m F_z(\Delta d, \Delta h) = \frac{\pi J h}{4} \sum_{i,j,k \in \mathfrak{Z}} w_i w_j w_k r_k \frac{(r_k - \Delta d \cos \theta_i) [r_h'(\theta_i) - r_0'(\theta_i)]}{\sqrt{\Delta d^2 + r_k^2 - 2 \cdot \Delta d \cdot r_k \cos \theta_i}} B_r(r_k, z_j) \quad (15)$$

$${}^m F_r(\Delta d, \Delta h) = \frac{\pi J h}{4} \sum_{i,j,k \in \mathfrak{Z}} w_i w_j w_k r_k \frac{(r_k - \Delta d \cos \theta_i) [r_h'(\theta_i) - r_0'(\theta_i)]}{\sqrt{\Delta d^2 + r_k^2 - 2 \cdot \Delta d \cdot r_k \cos \theta_i}} B_z(r_k, z_j) \cos \theta_i. \quad (16)$$

$\mathfrak{S}$ represents a set $\{0,1,\dots,7\}$. The magnetic flux density. mB_r and mB_z are also be solved by the Gaussian quadrature.

$${}^mB_r(r, z) = \frac{u_0 MR}{8} \sum_{m,n \in \mathfrak{S}} w_m w_n \left(\frac{\rho_n \cdot r - \rho_n^2 \cos \varphi_m}{L_1(\rho_n, \varphi_m, r, z)^{3/2}} - \frac{\rho_n \cdot r - \rho_n^2 \cos \varphi_m}{L_2(\rho_n, \varphi_m, r, z)^{3/2}} \right) \quad (17)$$

$${}^mB_z(r, z) = \frac{u_0 MR}{8} \sum_{m,n \in \mathfrak{S}} w_m w_n \left(\frac{(z - H/2) \rho_n}{L_1(\rho_n, \varphi_m, r, z)^{3/2}} - \frac{(z + H/2) \rho_n}{L_2(\rho_n, \varphi_m, r, z)^{3/2}} \right) \quad (18)$$

These five parameters, θ_i , z_j , r_k , φ_m , ρ_n , should be calculated via the upper and lower limit of corresponding integral. For example, the parameter r_k can be calculated by:

$$r_k = \frac{(r_h' + r_0')}{2} + \frac{(r_h' - r_0')}{2} \lambda_k, \quad (19)$$

While λ_k is the node of the Gaussian quadrature in (14).

We define a polar coordinate system of coils $\{c\}$ as shown in Fig. 6 too, and the origin of $\{c\}$ is ${}^c o$. Matrix ${}^c \mathbf{r} = [{}^c r, {}^c \theta, {}^c z]^T$ is the position of mass center of the magnet in $\{c\}$. As the force acting on coils and the force acting on PM is a pair of action and reaction, the force acting on the magnet would be expressed as:

$${}^c f_r({}^c r, {}^c z) I = -{}^m F_r({}^c r, {}^c z), \quad (20)$$

$${}^c f_z({}^c r, {}^c z) I = -{}^m F_z({}^c r, {}^c z). \quad (21)$$

I is the current in the coil. The result of ${}^c f_r$ and ${}^c f_z$ is the force-position relationship of the actuator, and defined in the coordinate system of coils $\{c\}$, their unit is N/A. ${}^c f_r$ and ${}^c f_z$ can be plotted in Fig. 7 refer to ${}^c z$ with different ${}^c r$. The force-gap relationship is related to the variables ${}^c r$, ${}^c z$ which represent the relative movement between coil and magnet in horizontal and vertical direction. Actually, the (15) and (16) is hard to solved in real time, so we store the force-gap relationship in a 2-D look-up table and employ the variables ${}^c r$, ${}^c z$ for indexing.

3.2. The gap for open loop stability

The maglev stage is open loop stable if the axial component ${}^c f_z({}^c r, {}^c z)$ and radial component ${}^c f_r({}^c r, {}^c z)$ meet (22).

Fig. 7. The force-position relationship: Axial force and Radial force.

$$\begin{cases} I^c f_z / d {}^c z < 0 \\ I^c f_z > 0 \end{cases} \quad \& \quad \begin{cases} I^c f_r < 0 \\ I^c f_r > 0 \end{cases} \quad (22)$$

The expression has two meanings: firstly, to the axial force, the change of $I^c f_z({}^c r, {}^c z)$ exert on the magnet would be opposite with the movement and its direction should be contrast to the gravity; secondly, the radial component $I^c f_r({}^c r, {}^c z)$ would force the magnet to be coaxial with coils, so the force should be minus. As illustrated in these two curves of Fig. 7, based on the monotonicity and positive-negative of the force, the movement region of magnet can be divided into four parts or six parts respectively. In order to meet (22), the axial position ${}^c z$ of magnet is equal to 10mm.

4. Modeling

4.1. Sensor system

The actual position measurements of the platen are essential for real-time control. Fig. 8 shows the arrangement of these sensors. We use three eddy gauges for vertical position sensing. These gauges face the bottom surface of the platen to sense its height at three different

Fig. 8. The distributing graph of the sensor.

locations. Their locations construct an isosceles right triangle as shown. Three laser displacement sensors are employed for horizontal position sensing. The working principle of these laser sensors is laser triangulation method, so the mirror is not needed. The side face of the laser sensor is used as the datum surface, while the measurement surface is one side face of the magnet holder in the horizontal Lorentz force actuator. With raw data of these sensors, we can calculate the six-axis position of the platen in vertical and horizontal plane.

Define the stator coordinate system $\{o\}$ and its origin oO is the midpoint of the hypotenuse of the isosceles right triangle. With the $^ox, ^oy$ axis described in this figure, the horizontal plane $^ox^oy$ coincide with the top surface of eddy gauges. The plane $^ox^oz$ and $^oy^oz$ are parallel with a certain datum surface of the laser sensor respectively. The rigid-body dynamics of the levitated platen can be characterized by a position and orientation vector $^o\mathbf{p} = [^ox, ^oy, ^oz, ^o\alpha, ^o\beta, ^o\gamma]^T$, which indicate the position and orientation of the levitated platen in the coordinate system $\{o\}$. The platen geometric center lO is on the bottom surface of the platen as depicted in the front view of Fig. 8. If position and orientation vector $^o\mathbf{p}$ is equal to $\mathbf{0}$, oO and lO is coincided with each other. Vectors $^o\mathbf{r} = [^ox, ^oy, ^oz]^T$ and $^o\mathbf{q} = [^o\alpha, ^o\beta, ^o\gamma]^T$ contain three displacements for translation and three Euler angles for

rotation respectively.

Vector $[dv_1, dv_2, dv_3]^T$ is the measured value of the eddy gauge in vertical direction. The center position of these three probe vs1, vs2 and vs3 are equal to

$(-L_2, -L_2, 0)$, $(-L_2, L_2, 0)$ and $(L_2, L_2, 0)$ respectively.

So the vertical parameter $\{^oz, ^o\alpha, ^o\beta\}$ can be calculated by:

$$\begin{bmatrix} dv_1 \\ dv_2 \\ dv_3 \end{bmatrix} = \begin{bmatrix} 1 & -L_2 & L_2 \\ 1 & -L_2 & -L_2 \\ 1 & L_2 & -L_2 \end{bmatrix} \begin{bmatrix} ^oz \\ ^o\alpha \\ ^o\beta \end{bmatrix}. \quad (23)$$

The range of laser displacement sensor is 50 ± 10 mm. Each laser beam is parallel with plane $^ox^oz$ or $^oy^oz$ respectively, and the distance between the laser beam and its corresponding plane is L_1 . The vector $[dh_1, dh_2, dh_3]^T$ is the raw data which is equal to actual distance subtract static distance (50mm) as shown in Fig. 8. If the horizontal parameter $\{^ox, ^oy, ^o\gamma\}$ is equal to $\{0, 0, 0\}$, the matrix $[dh_1, dh_2, dh_3]^T$ is equal to $\{0, 0, 0\}$. The horizontal parameter $\{^ox, ^oy, ^o\gamma\}$ can be calculated by (24). The value of parameters $L_1 \sim L_2$ is shown in the appendix A1.

$$\begin{bmatrix} dh_1 \\ dh_2 \\ dh_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -L_1 \\ 1 & 0 & -L_1 \\ 0 & 1 & L_1 \end{bmatrix} \begin{bmatrix} ^ox \\ ^oy \\ ^o\gamma \end{bmatrix} \quad (24)$$

4.2 Constructing of decoupling matrix

The properties of wrench-current transformation matrix are the force and torque acting on the platen due to each actuator. Analyzing the force distribution in the maglev positioning stage is solving the force and torque due to each actuator, which is the foundation of obtaining complete wrench-current transformation matrix

Fig. 9 shows the arrangement of translator (platen, magnets and magnet holders) and stator (coils). Define the levitated platen coordinate system $\{l\}$. The $^lx, ^ly$ axis and origin lO are described in Fig. 9(a). The horizontal plane $^lx^ly$ is the bottom surface of the platen. In order to obtain $f_r(r, z)$ and $f_z(r, z)$, we should get the relative position between the magnet and the corresponding coil. The position and orientation of the magnets are constant in the levitated platen, and we use the matrix $^l\mathbf{r}_i = [^l x_i, ^l y_i, ^l z_i]^T$ to reflect the mass center of magnet in the coordinate system $\{l\}$. The distance L_3 , L_4 and L_5 reflect the position of each magnet in the $^lx^ly$ plane. Parameter $^l z_i$ is equal to H_1 if $i = 1 \sim 4$,

Fig. 9. The distributing graph of magnets and coils in all actuators: (a) translator with magnets, (b) stator with coils.

and parameter ${}^l_m z$ is equal to H_2 if $i = 5 \sim 8$. The value of parameters $L_3 \sim L_5$, $H_1 \sim H_2$ are shown in the appendix A1.

The rigid-body dynamics of the levitated platen is characterized by one vector and one matrix: vector ${}^o_l \mathbf{t}(t)$ for translation and matrix ${}^o_l \mathbf{R}(t)$ for rotation. So the position of the mass center of the magnet i can be expressed as ${}^o_m \mathbf{r}(t)$ in the stator coordinate system $\{o\}$.

$${}^o_m \mathbf{r}(t) = {}^o_l \mathbf{R}(t) {}^l_m \mathbf{r}(t) + {}^o_l \mathbf{t}(t) \quad (25)$$

With the position and orientation vector ${}^o_l \mathbf{p}$ (consist of ${}^o_l \mathbf{r}$, ${}^o_l \mathbf{q}$) known, we solve the translation vector ${}^o_l \mathbf{t}(t)$ and rotation matrix ${}^o_l \mathbf{R}(t)$. Translation vector ${}^o_l \mathbf{t}(t)$ and rotation matrix ${}^o_l \mathbf{R}(t)$ are expressed as:

$${}^o_l \mathbf{t}(t) = {}^o_l \mathbf{r} = \begin{bmatrix} {}^o_l x, {}^o_l y, {}^o_l z \end{bmatrix}^T, \quad (26)$$

$${}^o_l \mathbf{R}(t) = \begin{pmatrix} 1 & -{}^o_l \gamma & {}^o_l \beta \\ {}^o_l \gamma & 1 & -{}^o_l \alpha \\ -{}^o_l \beta & {}^o_l \alpha & 1 \end{pmatrix}. \quad (27)$$

Thus, in the vertical and horizontal actuators, the matrix ${}^c_m \mathbf{r}(t) = \begin{bmatrix} {}^c_m x, {}^c_m y, {}^c_m z \end{bmatrix}^T$ can be obtained. In section III, we use polar coordinate to reflect the relative position between the coil and magnet. Here, as the matrix ${}^c_m \mathbf{r}(t) = \begin{bmatrix} {}^c_m x, {}^c_m y, {}^c_m z \end{bmatrix}^T$ in (28) is obtained in cartesian

coordinate system, we need calculate the matrix ${}^c_m \mathbf{r}_p(t) = \begin{bmatrix} {}^c_m r_p, {}^c_m \theta_p, {}^c_m z_p \end{bmatrix}^T$ in polar coordinate system.

The parameter ${}^c_m \theta_p$ is not necessary here, and the parameter ${}^c_m r_p$ and ${}^c_m z_p$ used to reflect the axial and radial position in the actuator can be expressed as:

$${}^c_m r_p = \sqrt{{}^c_m x^2 + {}^c_m y^2}, {}^c_m z_p = {}^c_m z, (i = 1, 2, 3, 4), \quad (29)$$

$${}^c_m r_p = \sqrt{{}^c_m y^2 + {}^c_m z^2}, {}^c_m z_p = {}^c_m x (i = 5) \text{ or } -{}^c_m x (i = 7), \quad (30)$$

$${}^c_m r_p = \sqrt{{}^c_m x^2 + {}^c_m z^2}, {}^c_m z_p = {}^c_m y (i = 6) \text{ or } -{}^c_m y (i = 8). \quad (31)$$

So the Lorentz force acting on the magnet i is ${}^c_i f_r({}^c_m r_p, {}^c_m z_p) i_i$ and ${}^c_i f_z({}^c_m r_p, {}^c_m z_p) i_i$. As we use a lookup table to store the axis and radial Lorentz force acting on the magnet, the force and torque acting on the translator can be obtained immediately as soon as the parameters ${}^c_m r_p$ and ${}^c_m z_p$ solved.

A wrench vector $\mathbf{w} = \begin{bmatrix} F_x, F_y, F_z, T_x, T_y, T_z \end{bmatrix}^T$ is defined, which contains the net force $\mathbf{F} = \begin{bmatrix} F_x, F_y, F_z \end{bmatrix}^T$ and net torque $\mathbf{T} = \begin{bmatrix} T_x, T_y, T_z \end{bmatrix}^T$ respecting to the mass center of the levitated stage acting on the levitated platen. The wrench vector is equal to:

¹ We use the right subscript to indicate the position matrix is expressed in the polar coordinate system.

$$\mathbf{w} = \begin{bmatrix} \mathbf{F} \\ \mathbf{T} \end{bmatrix} = \begin{bmatrix} \Gamma_{11}({}^o\mathbf{p}) & \Gamma_{12}({}^o\mathbf{p}) & \cdots & \Gamma_{18}({}^o\mathbf{p}) \\ \Gamma_{21}({}^o\mathbf{p}) & \Gamma_{22}({}^o\mathbf{p}) & \cdots & \Gamma_{28}({}^o\mathbf{p}) \\ \Gamma_{31}({}^o\mathbf{p}) & \Gamma_{32}({}^o\mathbf{p}) & \cdots & \Gamma_{38}({}^o\mathbf{p}) \\ \Gamma_{41}({}^o\mathbf{p}) & \Gamma_{42}({}^o\mathbf{p}) & \cdots & \Gamma_{48}({}^o\mathbf{p}) \\ \Gamma_{51}({}^o\mathbf{p}) & \Gamma_{52}({}^o\mathbf{p}) & \cdots & \Gamma_{58}({}^o\mathbf{p}) \\ \Gamma_{61}({}^o\mathbf{p}) & \Gamma_{62}({}^o\mathbf{p}) & \cdots & \Gamma_{68}({}^o\mathbf{p}) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_8 \end{bmatrix} = \Gamma({}^o\mathbf{p})\mathbf{i} \quad (32)$$

The decoupling matrix $\Gamma({}^o\mathbf{p})$ is crucial to the desired current vector $\mathbf{i}$, whose elements are the exciting current of all the coils. The complete decoupling matrix can be obtained due to the expression in appendix A2. As the matrix $\Gamma({}^o\mathbf{p})$ is a 6*8 and the matrix $\mathbf{i}$ is 8*1, the least norm solution would be utilized to solve the equation array. So, the current vector $\mathbf{i}$ would be expressed as:

$$\mathbf{i} = \Gamma^T({}^o\mathbf{p})(\Gamma({}^o\mathbf{p})\Gamma^T({}^o\mathbf{p}))^{-1} \mathbf{w} \quad (33)$$

4.3 Rigid-Body Dynamics and control

The rigid-body dynamics of the stage is characterized by vector ${}^o\mathbf{r} = [{}^o x, {}^o y, {}^o z]^T$ and ${}^o\mathbf{q} = [{}^o\alpha, {}^o\beta, {}^o\gamma]^T$. The equations of motion can be expressed as follows:

$$\mathbf{F}(t) - \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} = m {}^o\ddot{\mathbf{r}}(t) \quad (34)$$

$$\mathbf{T}(t) = \mathbf{I}({}^o\mathbf{q}) {}^o\ddot{\mathbf{q}}(t) \quad (35)$$

In (34) and (35) m is the mass of the levitated stage and $\mathbf{I}({}^o\mathbf{q})$ is the orientation-dependent inertia matrix. Inertia matrix $\mathbf{I}({}^o\mathbf{q})$ is constant, $\mathbf{I}_0 = \text{diag}\{I_x, I_y, I_z\}$. In order to counteract the gravity, a virtual net force can be defined to compute the desired net force:

$$\mathbf{F}'(t) = \mathbf{F}(t) - \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix}. \quad (36)$$

With $\mathbf{F}'(s)$ and $\mathbf{T}(s)$ as input, the system can be simplified to six two order differential equations. (37) represents three equations for x, y, z-displacement and (38) represents three equation for α, β, γ -rotation.

$$\frac{\mathbf{r}(s)}{\mathbf{F}'(s)} = \frac{1}{ms^2} \quad (37)$$

$$\frac{\mathbf{q}(s)}{\mathbf{T}(s)} = \frac{1}{Is^2} \quad (38)$$

Fig. 10 The diagram of control system

The platen is modeled as a pure mass of 0.2187kg, and the inertial matrix about the platen center of mass is

$$\begin{pmatrix} 399.1 & 21.68 & 0 \\ 21.68 & 399.1 & 0 \\ 0 & 0 & 759.4 \end{pmatrix} 10^{-6} \text{ kg.m}^3,$$

so $m = 0.2187$, $I_x = I_y = 399.1 \times 10^{-6}$ and $I_z = 759.4 \times 10^{-6}$.

A PID controller is designed in the control system. The controller is designed with damping ratio $\zeta = 1$ and the phase margin $\gamma = 75^\circ$ the crossover frequency of 34Hz:

$$G_{PID}(s) = \frac{K(s+3)(s+50)}{s}, \quad (39)$$

, where K is the gain of the controller. For x, y, and z-translation the value of K is 45.05N/m. The values of gain K are 0.0822N.m for α, β and 0.1564N.m for γ respectively. There is one free pole at the origin which can eliminate steady-state error. This continuous-time transfer function of the controller is converted to a discrete-time one by the zeroth-order-hold equivalence method with a 2-kHz sampling frequency, and implemented in the DSP.

The controller would solve the required forces and torques in terms of 6-DOF motion of the levitated translator. The results are achieved through eight forces of the Lorentz-force actuators. Therefore (33) is employed to calculate the desired current to excite the coils in each actuator. The control block of the maglev stage is depicted in Fig. 10.

5. Experimental Results

Several experiments have been conducted to illustrate the performance of the levitated stage in terms of positioning fluctuations of each axis, step respond, motion resolution, travel range, and multi-axis motion. Then some comparative experiments are taken to highlight the development resulted from the proposed three methods. We employ ${}^o\mathbf{p}$ to record the position and rotation of the platen. In these tests, the supply source for the controlling part and actuating part are separate. A 3.3v source is for the DSP, and a 6V source for actuating. The raw position data is from the laser displacement sensor LD50L and eddy

Table 1. Resolution and travel range of the stage

	Resolution	Travel range
$^o_i x$	4 μ m	-1mm, 1mm
$^o_i y$	4 μ m	-1mm, 1mm
$^o_i z$	2.8 μ m	1mm, 3mm
$^o_i \alpha$	116 μ rad	-40mrad, 40mrad
$^o_i \beta$	110 μ rad	-40mrad, 40mrad
$^o_i \gamma$	80 μ rad	-20mrad, 20mrad

Fig. 11 Position stability

gauge EV04T produce by Omron® Ltd. The resolution of laser sensor and eddy gauge is 1 μ m and 1.4 μ m respectively.

5.1. The performance of specification

5.1.1 Positioning fluctuations

With the controller above, the positioning fluctuations of each axis of the levitated stage are shown in Fig. 11. The desire position matrix $^o\mathbf{p}$ is equal to $[-1, 1, 1, 5, 5, -5]^T$. The unit of displacement and rotation is mm and mrad respectively. Under this circumstance, the coupling between horizontal and vertical actuator is significant. Those fluctuations determine the positioning resolution due to (40), where the Res means the resolution, M_i is the measuring data, and N is the measuring times. The resolution and travel range of the stage is summarized in Table 1.

$$Res = \sqrt{\frac{1}{N} \left(\sum_{i=1}^N M_i - \frac{1}{N} \sum_{i=1}^N M_i \right)^2} \quad (40)$$

5.1.2 Step respond

In order to examine the linearity of the stage, responds of four different steps in the $^o_i z$, three different steps in $^o_i \alpha, ^o_i \beta, ^o_i \gamma$ are normalized and shown in Fig. 12. To the step in $^o_i z$ -axis, the respond of the 20 μ m shows notable

Fig. 12. Normalized step respond of different axis with different step

Fig. 13. Step responds with various loads

noise while other responses do not. With the increase of the step, the respond is slower due to the limited output of constant current source. The raising time is nearly equal to 25ms when the magnitude of step is 20 μ m or 100 μ m and the raising time of 2mm step is near to 100ms. The same result is also appeared in the step respond in $^o_i \alpha, ^o_i \beta, ^o_i \gamma$, but not as obvious as the translation since the rotation is small. All of the step time of rotation is nearly to 25ms with a small step, and the step time become 40ms with the largest step. Because the damping ratio is 1, there is no overshoot in the step respond. The step responds in $^o_i x$ and $^o_i y$ are also recorded. The result is similar with the curves of $^o_i z$ as shown in this figure.

5.1.3 Load respond

The performance of this magnetically levitated positioning stage with four different payloads is shown in Fig. 13. The stage is controlled to take a 0.5mm step in vertical direction. An additional mass of 300g was able to be levitated and positioned. The payload decreases the natural frequency of the moving part, so the period becomes longer with the larger load. The period is near to 80ms

Fig. 14. Triangle trajectory tracking and the tracking error

Fig. 15. Circle trajectory tracking and the desired trajectory

Fig. 16. 10um-stepping in x and y axis

without the load, and is approximated to 150ms with a 300g load.

5.1.4 Trajectories tracking in the travel range

Fig. 14 illustrates motion tracking of a triangle reference trajectory in 0x and 0y , and the tracking error is also depicted in this figure. In this test, the trajectory covers the total stroke of the stage in horizontal direction.

On the other hand, we make the stage takes a motion along a circle with a radius of 1mm. The desired and real

Fig. 17. Large travel motion in each axis together

trajectories are shown in Fig. 15. In this test, the maximum error is less than 50um. The error range in this test is approximate to the triangle trajectory tracking.

5.1.5 Motion resolution

The motion resolution is tested in the 0x and 0y . With a positioning fluctuation is 4um, stepping of 10um per step can be clearly seen in Fig. 16.

5.1.6 Multi axis motion

Coupling between horizontal and vertical motions existed due to asymmetric location of the platen mass center. With the decoupling matrix proposed above, the affection between different axis motions decreased. The respond of different axis motion in the travel with the POV of maglev stage from ${}^0\mathbf{p} = [-0.5, -0.5, 1, -20, -20, -10]^T$ to ${}^0\mathbf{p} = [0.5, 0.5, 2, 20, 20, 10]^T$ is shown in Fig. 17.

5.2. Comparative testing

5.2.1 Force-gap relationship

We employ 2-D look up table to store the force-gap relationship of each linear actuator. In many proposed designs, the force-gap relationship is expressed as single variable function, which just cares about the main translation. Here, we make a comparison between the controlling systems which employ 2-D or 1-D look up table in the controlling system. The 1-D look up table omits the radial displacement, and likes fitting the force-gap relationship in single variable function. The stage takes a step in 0x , 0y , and 0z in horizontal. Fig. 18 shows the different step respond with the two control systems. With 1-D look up table, the creeping in the respond is more significant.

5.2.2 The choosing of air-gap in actuator

With the choosing method of the air gap above, the reference position of each actuator, ${}^c_m z$, is equal to 10mm.

Fig. 18. Step responds in horizontal direction with the force-gap relationship stored in (a) 1-D look up table; (b) 2-D look up table

Fig. 19. o_z with a sudden load change: (a) ${}^c_m z$ is 8mm, (b) ${}^c_m z$ is 10mm.

We change the reference position ${}^c_m z$ to 8mm, where the actuator is not open loop stability, to do the comparative tests. We place three different loads, 300g, 100g, and 50g, on the stage, which generates perturbations in o_z . After the stage return back to original position, the load is taken off. We record the position in o_z in Fig. 18. The deviation from the reference position is smaller if the reference position is equal to 10mm. That is because the change of magnetic force in actuator will help the translator return back to the original position. Thus, the actuator of a 10mm air gap is better than that of 8mm for the stability of the maglev positioning stage.

5.2.3 Decoupling method

Many proposed magnetically levitated stages ignore

Fig. 20. Fluctuations in vertical axes with a 100 μ m step in x : (a) incomplete decoupling, (b) complete decoupling

coupling between the vertical and horizontal actuators, which is an incomplete decoupling. In this paper, with the force and torque distribution analyzing aforementioned, the complete decoupling matrix between the current vector and wrench vector can be obtained. A comparative test between the controlling method employing different decoupling matrices is taken. We assume the radial force of actuator is identically equal to 0 to simulate the decoupling method that ignores the coupling between the vertical and horizontal actuators. Then, we make the stage make a 1mm step in o_x . The fluctuations in vertical direction, including o_z , ${}^o_\alpha$, and ${}^o_\beta$, are shown in Fig. 20. The figure shows that the vertical fluctuations are decreased with a complete decoupling matrix.

6. Conclusion

In this paper, the design and implementation of a compact magnetic levitated stage are presented. A cylindrical single axis Lorentz force actuator is analyzed and designed, while eight actuators are implemented to realize six axis actuation of the stage. Three eddy gauges and three laser displacement sensors are employed here to measure the translation and rotation of the stage, which are the object of the controller. The force-gap relationship is solved and stored by Gaussian quadrature and 2-D look up table respectively; based on the force model of the actuator, we choose the reasonable air-gap of each actuator; due to the rigid-body dynamics and the force and torque distribution of the stage we construct the complete decoupling matrix. The resolution of the translation is 2.8 μ m in vertical direction and 4 μ m in horizontal direction with the travel range is 2 $\times$ 2 $\times$ 2mm in translation and 80 $\times$ 80 $\times$ 40mrad in rotation. The solving process of the force-gap relationship, choosing method of air-gap, and

decoupling method are presented in this paper. The experiment result verifies the validity of these methods.

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Appendix

A.1 The value of $L_1 \sim L_8$, $H_1 \sim H_4$

$$\begin{aligned} L_1 &= 60\text{mm} & L_2 &= 23\text{mm} & L_3 &= 47\text{mm} & L_4 &= 49\text{mm} \\ L_5 &= 64\text{mm} & L_6 &= 47\text{mm} & L_7 &= 49\text{mm} & L_8 &= 74\text{mm} \\ H_1 &= -8\text{mm} & H_2 &= -11\text{mm} & H_3 &= -6\text{mm} & H_4 &= -19\text{mm} \end{aligned}$$

A.2 The details of force and torque acting on the levitated platen

$$\begin{aligned} F_x &= i_1 c_1 f_r \frac{c_1 x}{m_1 r_p} + i_2 c_2 f_r \frac{c_2 x}{m_2 r_p} + i_3 c_3 f_r \frac{c_3 x}{m_3 r_p} + i_4 c_4 f_r \frac{c_4 x}{m_4 r_p} \\ &+ i_5 c_5 f_z + i_6 c_6 f_r \frac{c_6 x}{m_6 r_p} - i_7 c_7 f_z + i_8 c_8 f_r \frac{c_8 x}{m_8 r_p} \end{aligned} \quad (41)$$

$$\begin{aligned} F_y &= i_1 c_1 f_r \frac{c_1 y}{m_1 r_p} + i_2 c_2 f_r \frac{c_2 y}{m_2 r_p} + i_3 c_3 f_r \frac{c_3 y}{m_3 r_p} + i_4 c_4 f_r \frac{c_4 y}{m_4 r_p} \\ &+ i_5 c_5 f_r \frac{c_5 y}{m_5 r_p} + i_6 c_6 f_z + i_7 c_7 f_r \frac{c_7 y}{m_7 r_p} - i_8 c_8 f_z \end{aligned} \quad (42)$$

$$\begin{aligned} F_z &= i_1 c_1 f_z + i_2 c_2 f_z + i_3 c_3 f_z + i_4 c_4 f_z \\ &+ i_5 c_5 f_r \frac{c_5 z}{m_5 r_p} + i_6 c_6 f_r \frac{c_6 z}{m_6 r_p} + i_7 c_7 f_r \frac{c_7 z}{m_7 r_p} + i_8 c_8 f_r \frac{c_8 z}{m_8 r_p} \end{aligned} \quad (43)$$

$$\begin{aligned} T_x &= -i_1 c_1 f_z L_3 + i_3 c_3 f_z L_3 - i_5 c_5 f_r \frac{c_5 z}{m_5 r_p} L_5 \\ &- i_6 c_6 f_r \frac{c_6 z}{m_6 r_p} L_4 + i_7 c_7 f_r \frac{c_7 z}{m_7 r_p} L_5 + i_8 c_8 f_r \frac{c_8 z}{m_8 r_p} L_4 \end{aligned} \quad (44)$$

$$\begin{aligned} T_y &= -i_2 c_2 f_z L_3 + i_4 c_4 f_z L_3 + i_5 c_5 f_r \frac{c_5 z}{m_5 r_p} L_4 \\ &+ i_6 c_6 f_r \frac{c_6 z}{m_6 r_p} L_5 - i_7 c_7 f_r \frac{c_7 z}{m_7 r_p} L_4 - i_8 c_8 f_r \frac{c_8 z}{m_8 r_p} L_5 \end{aligned} \quad (45)$$

$$\begin{aligned} T_z &= i_1 c_1 f_r L_3 \frac{c_1 x}{m_1 r_p} + i_2 c_2 f_r L_3 \frac{c_2 y}{m_2 r_p} - i_3 c_3 f_r L_3 \frac{c_3 x}{m_3 r_p} - i_4 c_4 f_r L_3 \frac{c_4 y}{m_4 r_p} \\ &+ i_5 \left(c_5 f_z L_3 - c_5 f_r L_3 \frac{c_5 y}{m_5 r_p} \right) - i_6 \left(c_6 f_z L_3 - c_6 f_r L_3 \frac{c_6 x}{m_6 r_p} \right) \\ &+ i_7 \left(c_7 f_z L_3 + c_7 f_r L_3 \frac{c_7 y}{m_7 r_p} \right) - i_8 \left(c_8 f_z L_3 + c_8 f_r L_3 \frac{c_8 y}{m_8 r_p} \right) \end{aligned} \quad (46)$$

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