Comparison of High Frequency Detailed Generator Models for Partial Discharge Localization

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Abstract – This paper presents partial discharge localization in stator winding of generators using multi-conductor transmission line (MTL) and RLC ladder network models. The high-voltage (HV) winding of a 6kV/250kW generator has been modeled by MATLAB software. The simulation results of the MTL and the RLC ladder network models have been evaluated with the measurements results in the frequency domain by applying of the Pearson's correlation coefficients. Two PD generated calibrator signals in kHz and MHz frequency range were injected into different points of generator winding and the signals simulated/measured at the both ends of the winding. For partial discharge localization in stator winding of generators is necessary to calculate the frequency spectrum of the PD current signals and then estimate the poles of the system from the calculated frequency spectrum. Finally, the location of PD can be estimated. This theory applied for the above generator and the simulation/measured results show the good correlation for PD Location for RLC ladder network and MTL models in the frequency range of kHz ($10\text{kHz} < f < 1\text{MHz}$) and MHz ($1\text{MHz} < f < 5\text{MHz}$) respectively.

Keywords: Generators, Partial discharge, Pearson's correlation coefficients, Multi-conductor transmission line model, RLC ladder network model.

1. Introduction

Generators are considered to be one of the essential and costly equipment's of power system. Identifying insulation faults in the best possible time prevents the appearance of irreparable damages. Partial discharge (PD) is one of the most important insulation faults. The first step in the study of partial discharge is precise modeling of transformer winding in high frequencies. In this paper two detailed RLC ladder network and MTL Models have been applied for partial discharge studies. The validity of frequency rang of this two models are different. The fundamental elements of the ladder network model are the lumped R, L and C elements and the frequency limitation for the validity of this model is in the range of a few hundred kHz. The fundamental elements of MTL model are distributed parameters. The model accuracy will be expanded over MHz frequency range. The published works on the machines is more focused on the RLC ladder network model in past [1-6]. While the published works on MTL modeling is mostly concentrated on modeling of electrical rotating machines and also on only the homogenous transformer windings ignoring the frequency dependency of the winding insulation parameters [7-12]. This paper

compares the two well-known MTL and RLC ladder network models and used these models for PD studies in high frequency range $10\text{kHz} < f < 5\text{MHz}$.

2. MTL Model

MTL theory deals with a network of N conductors coupled all together, characterized by its inductance matrix [L], and capacitance matrix [C]. In the MTL model, windings parameters are considered as distributed parameter and winding behavior is described by transmission line equations. MTL model for turns of one coil is depicted in the Fig. 1 [13].

For developing the model a single turn is considered as the basic element of analysis. Fig. 1 shows an electrical machine winding with three turns in series connection. In this figure, Z_r is the impedance for representing the

Fig. 1. Winding model with three turns in series connection

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terminal conditions, short and open circuit. The voltage V_{s1} is the surge at the beginning of the winding.

Surge impedances and coefficient of propagation can be estimated of the following Eqs. [7-10].

$$Z_i = \frac{1}{v_s \left[c_0^i + c_1^i + 2k \left\{ 1 - \cos\left(\frac{\omega a}{v_s}\right) \right\} \right]} \quad (1)$$

$$\gamma = \frac{1}{v_s d} \sqrt{\frac{\omega}{2\sigma\mu}} + \frac{\omega \tan \delta}{2v_s} + \frac{j\omega}{v_s} \quad (2)$$

where

k : inter-turn capacitance, a : turns average length
 d : disks gap, v_s : velocity

The first and second terms in the (2) are representing the skin effect and the dielectric losses respectively. σ , μ and d are the conductivity, permeability and the winding coils gap respectively. The details of modeling and the parameters estimation for an inhomogeneous winding (realizing frequency dependent parameters) are discussed in [10].

2.1 PD Analyses for MTL model

According to the Fig. 1 we have these telegraphs equations:

$$\frac{\partial V_t}{\partial x} = -L \left(\frac{\partial I_t}{\partial t} \right) \quad (3)$$

$$\frac{\partial I_t}{\partial t} = -C \left(\frac{\partial V_t}{\partial x} \right) + C_0 \frac{\partial E_0}{\partial t} \quad (4)$$

in Eqs. (3) and (4), V_t and I_t are the voltage and current vectors. The order is equal to the number of turns in a coil. L and C are square matrices of the inductances and capacitances in the coil while E_0 and C_0 denotes the excitation function and capacitance from one turn to the static plate. To study the PD phenomena the excitation function don't exist so in the (4):

$$\frac{\partial E_0}{\partial t} = 0 \quad (5)$$

By solve the Eqs. (3) and (4) and by insertion of (5), one can obtain following equations:

$$V_i(x) = A_i \exp(-\Gamma(\omega)x) + B_i \exp(\Gamma(\omega)x) \quad (6)$$

$$I_i(x) = \frac{1}{z_i} [A_i \exp(-\Gamma(\omega)x) - B_i \exp(\Gamma(\omega)x)] \quad (7)$$

Eqs. (6) and (7) are $2n$ equation and contain $2n$ undefined parameters (A_i and B_i). By using the terminal

conditions we have:

$$\begin{aligned} I_{r1} &= -I_{s2} \quad I_{r2} = -I_{s3} \quad V_{r1} = V_{s2} \quad V_{r2} = V_{s3} \\ \text{and} \quad -I_{r3} &= V_{r3} / Z_r \end{aligned} \quad (8)$$

$2n-2$ equations are available. For two other equation can be simulated by a capacitance C_B connected at the line-end. Then,

$$I_{(s1)} = -j\omega C_B V_{(s1)} \quad (9)$$

if the neutral end is at earth potential,

$$V_{r(N)} = 0 \quad (10)$$

If a PD current pulse I_{PD} is injected into the k^{th} turn of the winding, (8) is modified when $i=k-l$:

$$I_s(k) = I_r(k-1) + I_{PD} \quad (11)$$

With applying Eqs. (8) to (11) in Eqs. (6) and (7), the coefficients A_i and B_i have been calculated and then the current due to PD pulse can be calculated in the generator terminals.

$$\begin{bmatrix} I_{s(1)} \\ 0 \\ \vdots \\ 0 \\ I_{PD} \\ 0 \\ \vdots \\ 0 \\ I_{R(n)} \end{bmatrix} = [Y] \begin{bmatrix} V_{s(1)} \\ V_{s(2)} \\ \vdots \\ \vdots \\ V_{s(k)} \\ \vdots \\ \vdots \\ V_{s(n)} \\ V_{R(n)} \end{bmatrix} \quad (12)$$

$$\begin{bmatrix} I_{s(1)} \\ V_{s(2)} \\ V_{s(3)} \\ \vdots \\ V_{s(k)} \\ \vdots \\ \vdots \\ V_{s(n)} \\ V_{R(n)} \end{bmatrix} = [T] \begin{bmatrix} V_{s(1)} \\ 0 \\ \vdots \\ \vdots \\ I_{PD} \\ \vdots \\ \vdots \\ 0 \\ I_{R(n)} \end{bmatrix} \quad (13)$$

where: $[Y]$ is a $(n+1) \times (n+1)$ matrix. If matrix $[Y]$ is inverted and re-arranged, it is possible to get Eq. (13).

Hence, if the line-end voltage, the neutral-end current and the PD current are known all other voltages and currents can be calculated.

3. RLC Ladder Network

A PD generated within a generator winding can have an associated frequency range between a few kHz to hundreds MHz Fig. 2 shows an RLC ladder network model [3].

Fig. 2. RLC ladder network model for generator winding (PD occurs in node 2)

The PD occurrence can be simulated as a current pulse injected into the network. To obtain the output current through the neutral point in such a circuit, it is very convenient to use state-space equations as follows:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = C_1x + C_2\dot{x} \end{cases} \quad (14)$$

with

$$x = [i_1(t), i_2(t), \dots, i_N(t), v_1(t), v_2(t), \dots, v_N(t)]^T$$

$$A = \begin{bmatrix} -L^{-1}R & -L^{-1}N \\ \Theta^{-1}M & \Theta^{-1}G \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ \Theta^{-1}K \end{bmatrix}$$

$$C_1 = [1, 0, \dots, 0, -G_s, 0, \dots, 0];$$

$$C_2 = [0, 0, \dots, 0, -G_s, 0, \dots, 0];$$

$$K = [0, \dots, 0, 0, \dots, 1_{(r)}, \dots, 0];$$

Note that the position of 1 in vector K is decided by the input position. L , Θ and G are the inductance, capacitance and admittance matrices of this system. The frequency response can be calculated by taking the Laplace transform of Eq. (14).

$$\frac{Y(s)}{U(s)} = (C_1 + C_2s)(SI - A)^{-1}B \quad (15)$$

with regard to the unit distribution for the PD current source, the frequency spectrum of the system can be evaluated according the bellow equation:

$$\begin{aligned} Y(s) &= (C_1 + C_2s)(SI - A)^{-1}B \\ U(s) &\approx 1 \end{aligned} \quad (16)$$

In Eq. (16) only the matrix B is related to the position of the PD current source location. The matrix A is not affected by the position of the PD current source, also, the system structure does not affect by this discharge. Therefore, the poles of the system, which are the eigenvalues of the matrix A , are completely determined only by the system (matrix A). On the other hand the poles of frequency spectrum of the signal contain the information of the PD source location.

Based on the above theory, for the PD current detection and PD location, the three bellow steps will be down:

1. The frequency spectrum of $Y(s)$ has been calculated from PD current signal.
2. The poles of the system have been estimated from the frequency spectrum.
3. The location of PD source has been estimated from poles of the system.

Step 1 can be calculated easily through the use of a fast Fourier transform (FFT). The second step can be estimated from the transfer function of the winding model as following: [14-16]

$$Y(S) = \frac{\beta(S)}{\alpha(S)} \quad (17)$$

where $\alpha(S)$ and $\beta(S)$ are polynomials and defined by bellow:

$$\alpha(S) = \alpha_n S^n + \alpha_{n-1} S^{n-1} + \dots + \alpha_1 S + \alpha_0 \quad (18)$$

$$\beta(S) = b_m S^m + b_{m-1} S^{m-1} + \dots + b_1 S + b_0 \quad (19)$$

applying Eqs. (18) and (19) in Eq. (17), the following equation is obtained:

$$Y(S) = X(S) \times \theta \quad (20)$$

where

$$X(S) = [-S^n Y(S) \dots -SY(S)S^m \dots S \ 1] \quad (21)$$

and

$$\theta = [\alpha_n \dots \alpha_1 \ b_m \dots b_1 \ b_0] \quad (22)$$

After obtaining the coefficients, the poles of the system can be estimated from the roots of polynomial $\alpha(S)$. The third Step is a nonlinear optimization problem. The objective function is to minimize the difference between the eigenvalues of matrix A and the calculated pole. The eigenvalues and pole frequencies to be arranged in descending ascending

$$eig(A) = eig\left(\begin{bmatrix} -L^{-1}R & -L^{-1}N \\ \Theta^{-1}M & \Theta^{-1}G \end{bmatrix}\right) = poles = roots(\alpha(S)) \quad (23)$$

Finally, with the poles of the system, the location of the PD can be estimated

4. Fast Transient Over-voltage's

In this section the winding of generator has been modeled once using MTL and next with RLC ladder networks. Then the fast transient over-voltages have been computed between turns. The analysis was carried out for a 6 kV/250 kW test generator. Table 1 and 2 show the generator specification and coil parameters. At this paper the coil parameters have been evaluated from two methods, the first by using the COMSOL software and the geometry and custom shape of coils with FEM methodology and the second by measuring the parameters using conventional methods in the laboratory.

The results of the fast transient over-voltage between coils have been shown in the Figs. 3, 4 and 5 for measured MTL and RLC models respectively. This figures show the absolute value of voltage between coils with respect to applied input voltage.

4.1 Pearson's correlation coefficient

Correlation is a useful technique to investigate the

Table 1. Specification of the under the test generator

Rated voltage	6000	V
Rated power	250	kW
Rated speed	1500	r.p.m
Rated frequency	50	Hz
Winding Connection	star	---
Number of turns per phase	176	---
Number of coils per phase	16	---
Number of turns in a coil	11	---
Conductor dimensions	11.5*236	mm

Table 2. Parameters of the generator coils

	Li(mH)	R(Ω)	Ki (PF)	Ci (PF)
Real	17.2	0.35	30	1331
Estimated	17.7	0.4	27.94	1253

Fig. 3. Ratio of measured voltage to the applied input voltage (10 kHz < f < 5MHz)

Fig. 4. Ratio of MTL model computed voltages to the applied input voltage (10 kHz < f < 5MHz)

Fig. 5. Ratio of RLC ladder network model computed voltages to the applied input voltage (10 kHz < f < 5MHz)

relationship between two quantitative, continuous variables. It measures the strength and direction of a linear relationship between two variables. This correlation coefficient is presented by the following equation:

$$r = \frac{\sum_{i=1}^n (U_{mi} - \bar{U}_m)(U_{si} - \bar{U}_s)}{\sqrt{\sum_{i=1}^n (U_{mi} - \bar{U}_m)^2 \sum_{i=1}^n (U_{si} - \bar{U}_s)^2}} \quad (24)$$

where U_m is the measured value of variable and U_s is its simulated value. r could be a number between 1 and -1. A positive value of r means a positive linear relationship; a negative value of r means a negative linear relationship. The correlation is ideal as approaches 1 and it is not desirable to approach -1[17]. Considering the measurement results shown in Fig. 4 and using SAS software to compute the correlation coefficient, the accuracy of the MTL and RLC ladder network model can be studied. Tables 3 and 4 show these coefficients for the RLC ladder network model and the MTL model simulations for respectively. These coefficients are the best means for evaluation of the models. Considering the last row of Table 3 it is obvious that the RLC ladder network model can present the best results for the frequency range of 10 kHz < f < 1MHz. The last row of Table 4 indicates that the MTL model can represent the

Table 3. Pearson correlation coefficient for RLC ladder network and MTL model for $10 \text{ kHz} < f < 1 \text{ MHz}$

Coil No.	RLC	MTL
1	0.261	-0.171
2	0.332	-0.14
3	0.247	-0.108
4	0.25	0.106
5	0.257	0.101
6	0.281	0.04
7	0.284	0.48
8	0.305	0.237
9	0.329	0.197
10	0.385	0.347
11	0.534	0.421
AVR.	0.315	0.105

Table 4. Pearson correlation coefficient for RLC ladder network and MTL model for $1 \text{ MHz} < f < 5 \text{ MHz}$

Coil No.	RLC	MTL
1	-0.04	0.122
2	0.022	0.402
3	-0.28	0.334
4	-0.093	0.303
5	-0.096	-0.389
6	-0.1	0.426
7	0.097	0.443
8	0.034	-0.026
9	-0.023	0.368
10	-0.133	0.344
11	-0.157	-0.257
AVR.	-0.069	0.188

response of generator better than next model for the frequency range of $1 \text{ MHz} < f < 5 \text{ MHz}$. The above-mentioned conclusions can be drawn from Figs. 3, 4 and 5 as well.

5. Result of PD Localization

In order to estimate the location of PD in generators with more reliability, it is better to verify the accuracy of the models that will be used in PD localization first. So it is recommended to measure and record the frequency spectrum of the generator in the first step. Then the simulation results must be compared with the measurements. If simulation results were satisfactory, the models can be used for PD localization. Otherwise the models must be modified or the parameters of the model must be optimized before PD localization.

At the first, PD pulses have been injected by the PD calibrator in the different part of the winding. Then the current signals measured/simulate in the end of the winding. These PD signals have been generated in kHz and MHz frequency range that used for RLC ladder network and MTL models respectively. Fig. 6 shows the PD pulse studies circuit in a laboratory. The PD pulse to be generated in the kHz range has been shown in Fig. 7. Figs. 8, 9 show the comparison between the current

Fig. 6. PD pulse measured in circuit in a laboratory

Fig. 7. The PD pulses measured in the kHz range (the PD has been injected in node 6)

Fig. 8. The comparison of current and the spectrum (PD has been injected in node 1 for RLC ladder network model)

Fig. 9. The comparison of current and the spectrum (PD has been injected in node 6 for RLC ladder network model)

spectrum for RLC model and measurements results. The PD pulse to be generated in the MHz range has been shown in Fig. 10. The Figs. 11, 12 show the comparison between the current spectrum for MTL simulated model and measurements results.

Fig.10. The PD pulses measured in the MHz range (the PD has been injected in node 6)

Fig. 11. The comparison of current and the spectrum (PD has been injected in node 1 for MTL model)

Fig. 12. The comparison of current and the spectrum (PD has been injected in node 6 for MTL model)

Table 5. Comparison between real position and the simulation estimated position of PD

Real position	PD(10)	PD(6)	PD(1)
Estimated (RLC)	11	6	2
Estimated (MTL)	10	6	2

In this section from the frequency spectrum of the current signals, and poles estimate of the system, the location of PD can be estimated along the winding. Then these results compared with real position. Table 5 shows the comparison between the real position and the simulated estimated position of PD (Depending to the node).

6. Conclusion

This paper compared the measurement and simulation results of transient over voltages and then partial discharge localization in the windings of a 6kV/250kW generator. Voltages along the generator windings were simulated by applying MTL and RLC ladder network theory. The results of the transients voltage calculated in the coils of the winding were compared with laboratory measurement. A relatively good correlation observed between measurement and RLC Ladder Network model simulation results for the frequency range of $10\text{kHz} < f < 1\text{MHz}$. However in the range of the frequency $1\text{MHz} < f < 5\text{MHz}$, the MTL model, provides a better agreement between the simulated and the measured voltages.

An algorithm for partial discharge localization has been developed in this paper. The approach is based upon frequency spectrum analysis of PD current signals. PD pulses have been injected by the PD calibrator in the different part of the winding. Then the PD signals measured or simulate in the end of the winding. These PD signals have been generated in kHz and MHz frequency range that used for RLC ladder network and MTL models respectively. A relatively good correlation have been observed between measurement and RLC ladder network model simulation results for the frequency range of kHz ($10\text{kHz} < f < 1\text{MHz}$), however in the frequency range of MHz ($1\text{MHz} < f < 5\text{MHz}$), the MTL model have been provided the better agreement between the simulated and the measured for PD Location.

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