A Simple Temperature Dependent Model to Predict the Bloom of *Aurelia Aurita* Polyps

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**Abstract:** Asexual benthic polyp reproduction plays a major role in the jellyfish bloom. Recent studies found that temperature is the most important factor to regulate the budding rate of the polyps. We established a simple dynamic model to count the number of polyps depending on the variation of temperature with two data sets from different places. The population of polyps was counted through the budding rate and the number of budding times by Fibonacci sequence. It is assumed that the budding rate depends on the temperature only. The budding rate of the asexual reproduction shows very sensitive to the distribution of the seawater temperature. The model was tested to the temperature data of Ansan located in the west sea of Korea. The results indicate that this model can be useful to predict the blooms of *Aurelia aurita* polyps, which may have considerable influence on the bloom of medusa. The shape of temperature curve plays a key role in the predicting the bloom of *Aurelia aurita* polyps.

**Keywords:** Bloom, *Aurelia aurita*, Asexual reproduction, Polyp, Fibonacci sequence

1. INTRODUCTION

*Aurelia aurita*, commonly referred to as moon jellyfish, is found in many coastal areas worldwide and can easily adapt to a wide range of habitats within which the degree of water depth, temperature, salinity, and trophic conditions can vary quite considerably [1-3]. Even the species has been known to thrive in polluted or even anoxic environments [4-5]. Global warming can lead to the acidification of seawater, the restriction of water exchange, eutrophication, and the loss of biodiversity, which can transform the marine ecosystem. Jellyfish blooms appear to be naturally suited to such environmental changes.

For a more in-depth understanding of jellyfish blooms it is necessary to review the life cycle of *A. aurita*. The life cycle of *A. aurita* consists of the medusa, planula, polyp, and ephyra...
Thiel found that the nudibranch Gröndahl found that nudibranch important predator in Kiel Fjord, Germany [11]. Hernroth and the bloom of results we proposed a temperature dependent model to predict regulate the budding rate of polyps [5, 7, 13-17]. Using their and light, and found that temperature is the major factor to consider environmental factors such as temperature, salinity, and time to predict the tendency of the dynamics.

2. MATERIALS AND METHODS

2.1. Fibonacci model for an asexual pattern of polyp reproduction

The lifecycle of A. aurita consists of the sexually reproduced planula, asexually reproduced benthic polyp, and medusa phasess. Benthic polyps are asexually reproduced in three ways: budding, stolon, and fission. Budding and stolon are quite different from the biological point of view, but the method for counting the number of new polyps can be similar. The three types of reproduction can be simplified into one loop for the counting the number of polyps. A grown polyp reproduces a daughter polyp, and the daughter polyp becomes a young polyp. After maturing, the polyp can develop a new bud.

The amount of time required for this morphological change can differ by stage in the real world. Sometimes it takes few days for a daughter polyp to be a young polyp while it takes few months for a mature polyp to make a daughter polyp by budding. It depends on temperature [7]. In Fig. 1, \( t_1 \), \( t_2 \) and \( t_3 \) show the temperature dependent times in the morphological change. It is impossible to predict the number of polyps reproduced from one polyp for one year. However we can forecast the A. aurita polyp's bloom by counting the minimum number of polyps on asexual reproduction. For simplicity, this study assumes that the same amount of time, \( t_d \), is required for the morphological change. We defined the duration time as \( t_d = \text{Max}\{t_1, t_2, t_3\} \). The longest period in the morphological change

![Fibonacci model of the asexual reproduction of A. aurita polyps.](image)

**Fig. 1.** Fibonacci model of the asexual reproduction of A. aurita polyps. \( t_1 \), \( t_2 \) and \( t_3 \) show the temperature dependent times in the morphological change. The longest time in the morphological change could make the possible budings minimum. It makes the reproduction cycle to resemble the Fibonacci sequence. The image of polyps was captured from the book of Yasuda [22].
could make the possible budings minimum. This assumption makes the reproduction cycle to resemble the Fibonacci sequence. Thus, this study uses the Fibonacci sequence to represent the reproduction rate for polyps. The Fibonacci sequence is widely used in biology [18-19]. It was proposed and solved by Leonardo Pisano, Fibonacci, in 1202 in the Liber Abaci [20]. It is the first mathematical model regarding the study of population dynamics.

Therefore the number of budding polyps can be expressed as

\[ a_n = a_{n-1} + a_{n-2} \]  

(1)

where \( a_n \) represents the total number of polyps at the \( n \)th budding stage.

The general expression of \( a_n \) is

\[ a_n = \frac{1}{\sqrt{5}} \left( \frac{1 + \sqrt{5}}{2} ight)^n - \frac{1}{\sqrt{5}} \left( \frac{1 - \sqrt{5}}{2} \right)^n \]  

(2)

The second term in Eq. (2) is always less than 0.5, and thus, one can just use the first term and round it to the nearest integer. Then we can define the function \( a(n) \) that shows the total number of polyps in terms of budding times \( n \):

\[ a(n) = \text{Round} \left[ \frac{1}{\sqrt{5}} \left( \frac{1 + \sqrt{5}}{2} \right)^n \right] = \text{Round} \left[ \frac{1}{\sqrt{5}} e^{0.4812n} \right] \]  

(3)

It means that the population dynamics of \( A. aurita \) polyps can be expressed in terms of asexual budding times. If we know the total number of possible budding times, then we can calculate total number of polyps by using Eq. (3).

2.2. Modeling method

We first suggested a time-dependent model of population dynamics of \( A. aurita \) polyps and then proposed a temperature-dependent model. For the notation, the function \( r(t) \) denotes the time-dependent budding rate, and \( r(T) \) denotes the temperature-dependent one. The function \( n(t) \) denotes the total number of budding times at time \( t \), and \( n(T) \) denotes the same number at temperature \( T \). We used the sinusoidal function to evaluate \( n(t) \) in the time dependent model using two data sets in Table 1. Those data sets are from the results of Wilcox et al. [13] and Liu et al. [16] and rescaled uniformly for 90 days. Their experiments showed that the temperature is the most important factor among many environmental factors to control the budding rate of \( A. aurita \) polyps.

To evaluate the temperature dependent \( n(T) \), we constructed the reference budding rate function \( r(T)_{\text{ref}} \). Using \( n(T) \) the number of polyps by asexual reproduction was counted by the Fibonacci sequence. We tested the proposed model by using temperature data from Ansan, Korea from 2002 to 2008.

2.3. Time-dependent model

We assumed four types of asexual budding rates for each of the four seasons. We assumed that spring and fall would be similar in terms of variations in temperature, and thus, we used three types of budding rate to simplify the asexual reproduction of polyps. Table 1 shows seasonal variations of the budding rate for a one-year period at two places, Tasmania of Australia and Tapong Bay of Taiwan. Because \( r(t) \) denotes the number of budding times per season, it is a periodic function in terms of time and can be expressed as a sinusoidal function regression in Eq. (4).

\[ r(t) = A \cos B + C \]  

(4)

We may regard the set of seasons (spring, summer, fall, and winter) as a set of intervals \((0, \pi/2, \pi, 3\pi/2)\), because one year is close to \(2\pi\). We can use the following scales:

\[ \text{budding times} \quad \text{season} = \frac{\text{budding times}}{\pi/2} = \frac{\text{budding times}}{3 \text{ months}} \]

Then the total number of budding times \( n(t) \) can be expressed as

\[ n(t) = \frac{2}{\pi} \int_{0}^{2\pi} (A \cos B + C) dt \]  

(5)

where \(2/\pi\) comes from the division by one season, \(\pi/2\), for one year, \(2\pi\). And the total number of polyps can be calculated using Eq. (3). Therefore, the total number of polyps from one polyp can be expressed as

\[ a(n) = \text{Round} \left[ \frac{1}{\sqrt{5}} e^{0.4812} \int_{0}^{2\pi} (A \cos B + C) dt \right] \]  

(6)

We calculated the possible number of polyps from one polyp

| Table 1. The budding rate \( r(t) \) and the number of polyps \( a(n) \) for a year from results of Wilcox et al. [13] and Liu et al. [16]. The data are rescaled uniformly for 90 days |
|---------------------------------|----------------|----------------|-------------|-------------|-------------|----------------|----------------|-------------|-------------|
|                              | Tasmania        | Tapong Bay      |
| 10°C (summer)                | 10°C (fall)     | 16°C (winter)   | 13°C (spring)| 20°C (winter)| 25°C (spring)| 30°C (summer)  | 25°C (fall)    |
| \( r(t) \)                  | 0.1             | 3               | 6            | 3           | 8           | 3              | 1              | 3           |
| \( a(n) \)                  | a(0.1) = 1      | a(3) = 3        | a(6)= 13     | a(3) = 3    | a(8) = 34   | a(3) = 3       | a(1)=1        | a(3) = 3    |
for two cases in Table 1.

The first case is the experiment of Liu et al. [16]. This experiment was conducted under three temperatures 20°C, 25°C, and 30°C, resulting in budding rates 0.09, 0.03, and 0.01 for one polyp per day, respectively. We assumed that those temperatures are representative for four seasons [21].

The second case is the experiment of Willcox et al. [13]. According to this experiment, after 32 days, the mean increase in population size was 8%, 93%, and 189% for 10°C, 13°C, and 16°C, respectively. The results were rescaled for a 90-day period, approximately one season.

2.4. Temperature-dependent model

In the Eq. (6) the total number of polyps depends on the number of budding times $n$, and the budding times $n$ depends on the budding rate $r$. Here the budding rate $r$ is for a season or 3 months. In the previous cases, $n$ and $r$ were the functions of time. For the data like Willcox et al. [13] we can’t calculate the temperature dependent budding rate by sinusoidal regression. Comparing to the seasonal time variation, temperature variation is not periodic, hence it is better to use other method than the sinusoidal regression.

To count the total number of polyps at various temperatures, we need to determine (T), the temperature dependent budding rate. The coastal water environment is regional, and thus, r(T) varies according to the location. Although it is difficult to determine the exact function r(T), it can generally be developed through lab experiments. To develop the temperature dependent budding rate we need at least five kinds of data points. It should cover smallest and intermediate budding rate in both high and low temperature zones and nearly maximum budding rate temperature zone. Then we can construct r(T) by mathematical tools such as regression or interpolation.

In this paper, we constructed of r(T) by using the data in Table 1. We assumed that the budding rate of A. aurita is identical only if the temperature is equal at any place. So we combine the two results although the data came from different experiments. Fig. 2 shows the budding rate depending on the temperature. The graph was constructed using the piecewise cubic Hermite interpolation through six data points in Table 1. It shows the minimum below 10°C and above 30°C and the maximum at 20°C. At 13°C and 25°C it shows the intermediate budding rate.

Fig. 2. The budding rate $r(T)_{ref}$ as a function of temperature. The graph was constructed using the piecewise cubic Hermite interpolation through six data points in Table 1. It shows the minimum below 10°C and above 30°C and the maximum at 20°C. At 13°C and 25°C it shows the intermediate budding rate.

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We took Ansan because there is ShiWha lake in Ansan. It is reported several times of moon jelly fish bloom. To clarify the notation we denote the budding rate function r(T) from Table 1 as $r(T)_{ref}$ and the approximated budding rate function for Ansan as $r(T)_{Ansan}$. The ref in the notation $r(T)_{ref}$ means the reference to induce the $r(T)_{Ansan}$ as an example, we calculated the budding rate $r(T)_{Ansan}$ from $r(T)_{ref}$ in Fig. 2 by using 2007 temperature data. The mean temperatures of seawater of Ansan, Korea, from 2002 to 2008 are applied for the modeling. These temperature data come from the on line data of Korea Hydrographic and Oceanographic Administration in Table 2.

![Fig. 2. The budding rate $r(T)_{ref}$ as a function of temperature.](image)

### Table 2. Mean temperature of seawater in Ansan, Korea, from 2003~2008 (on line data of Korea Hydrographic and Oceanographic Administration)

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<td>2007</td>
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<td>24.1</td>
<td>29.0</td>
<td>11.4</td>
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3. RESULTS

For case 1 :: Time dependent model for the data from Willcox et al. [13]

In this case, the regression function is calculated

\[ r(t) = 3.71 \cos t + 3.86 \] (7)

The number of budding times is

\[ n(t) = \frac{2}{\pi} \int_{0}^{\pi} (3.71 \cos t + 3.86) dt = 15.44 \] (8)

Finally, the number of new polyps was

\[ a(n) = \text{Round} \left[ \frac{1}{\sqrt{5}} e^{0.4812 \times 15.44} \right] = 750 \] (9)

Thus one polyp can produce 750 new polyps a year. For the budding rate function, we could have used other types of periodic functions, but for simplicity, we used the sinusoidal function.

For case 2 :: Time dependent model for the data from Liu et al. [16]

In this case, the number of new polyps was

\[ a(n) = \text{Round} \left[ \frac{1}{\sqrt{5}} e^{0.4812 \times 12.12} \right] = 153 \] (10)

Thus one polyp can produce 153 new polyps a year by asexual reproduction.

For case 3 :: Temperature dependent model

We expect that the temperature distribution pattern play a key role to predict the budding rate and total number of polyps. Fig. 3 shows the budding rate \( r(T)_{\text{Ansan}} \) for Ansan in 2007. The total number of budding times, \( n(T) \), was calculated by integrating the budding rate function \( r(T) \) and dividing by 3. The integration was performed for 12 months or 4 seasons. Thus, the integration was divided by 3 to get the total number of budding times. The “x” coordinate represents the month, and the “y” coordinate represents the budding rate, that is, budding times/3 months.

![Fig. 3. The approximated budding rate \( r(T)_{\text{Ansan}} \) for the year 2007. The total number of budding times, \( n(T) \), was calculated by integrating the budding rate function \( r(T) \). The integration was performed for monthly budding rate using the data for 90 days. Thus, the integration was divided by 3 to get the total number of budding times. The “x” coordinate represents the month, and the “y” coordinate represents the budding rate, that is, budding times/3 months.](image)

The “x” coordinate represents the month, and the “y” coordinate represents the budding rate, that is, budding times/3 months.

The total number of budding times in 2007 was 12.2333, and the total number of polyps from one polyp was 161 from the Eq. (4)

\[ a(n) = \text{Round} \left[ \frac{1}{\sqrt{5}} e^{0.4812 \times 12.2333} \right] = 161 \] (11)

Table 3 shows the budding rate from 2002-2008, which was calculated by using the temperature data and the reference budding rate function \( r(T)_{\text{ref}} \). By using \( r(T) \) for each year, we determined the total number of polyps for Ansan, Korea, from 2002-2008 shown in Table 4. Fig. 4 shows the total number of polyps from one polyp for 2002-2008 calculated from the model. The population could be increased almost 700-fold in 2003 and 161-fold in 2007 by the suggested model. There was no substantial difference in temperature between 2003 and 2007. More months were near 20°C in 2003 than in 2007. A high temperature does not necessarily indicate a large number of budding times. The more important factor may be the temperature distribution near 20°C.

4. DISCUSSION

A general expression of polyp reproduction as a function of time or temperature may not exist because \( A.aurita \) in different locations may exhibit different characteristics. However, because only the budding rate is different across different locations and temperatures, we can evaluate budding patterns at the local level. Although the result of this study looks satisfactory to predict the polyps’ asexual reproduction in terms of temperature, future research should use a wider range of data on the budding rate \( r(T) \) to provide a better understanding of the population dynamics of \( A.aurita \) polyps. The budding rate function has to be set by local experiment. In this paper, we calculated \( r(T) \) by using two data set from Taiwan and another from Tasmania, and thus, we were not able to get the real re-
ference budding rate function for the Ansan area. However, the results clearly indicate that it is possible to forecast A. aurita blooms by using only the pattern of seawater temperature variation. This model is useful for coastal region if we can construct suitable temperature dependent budding rate function. At least five points of data for one region is needed. Two points for near minimum, two for near intermediate budding rate in both high and low temperature and one point of temperature showing near maximum budding rate. Of course the bloom of polyps predicted through this model should be compared to the bloom of medusa of A. aurita.

The proposed model can be particularly useful when considering the effect of global warming: increases in the mean seawater temperature may lead to more moon jellyfish blooms. However, the more important factor looks like the pattern of increases in the mean temperature as shown in the example from Ansan using our model. In this model, the budding of A. aurita polyps does not occur below 10°C and above 30°C; it occurs mainly between 15~25°C. Thus, if global warming were to increase the mean seawater temperature to between 15~25°C, then there could be large blooms of A. aurita polyps. However, other factors can also influence the population dynamics of A. aurita polyps. If the predator does appear in the time of bloom of polyps [12] then the bloom of medusa does not follow the bloom of polyps. The pollution of sea water can make the scenario too bad. A. aurita polyps have been known to thrive in polluted or even anoxic environments [4-5]. If the predator and competitor dose not survive in the polluted water, then the bloom of polyps directly means the bloom of moon jellyfish, A. aurita. Thus, the model can be a useful tool to forecast A. aurita blooms.

Although we should still consider other factors precisely such as the mortality rate, the settling substrate and the presence of predators, prey, and competitors, the results of this study indicate that it can be used as a first step to forecast the bloom of A. aurita polyps by using only the temperature data of seawater. In summary, in this paper we would like to see the influence of temperature variation to the bloom of jellyfish in the qualitative point of view by calculating the number of benthic polyps of A. aurita through modeling. Increasing the sea water temperature does not mean the increasing the number of polyps. Rather the temperature distribution pattern affects more to the population of polyps. Although, further studies and experiments are needed to include other factors in the form of dyna-
mic equation systems, this model can be used to predict the bloom of the polyps. It is not our goal to calculate the number of polyps reproduced for a year precisely. We conclude that the shape of temperature distribution pattern curve play a key role in the predicting the bloom of A. aurita polyps. As the longer duration time near high budding rate temperature not near high temperature, the possibility of bloom of A. aurita polyps is getting higher. This paper is helpful to predict the possibility of jellyfish bloom in near future such as next year if we can have temperature dependent reference budding rate from the laboratory data.

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