Effect of Circuit Parameters on Stability of Voltage-fed Buck-Boost Converter in Discontinuous Conduction Mode

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Abstract – The state transition matrix are obtained by solving state equations in terms of Laplace inverse transformation and Cayley-Hamilton theorem, and an establishment of a precise discrete-iterative mapping of the voltage-fed buck-boost converter operating in discontinuous conduction mode is made. On the basis of the mapping, the converter bifurcation diagrams and Lyapunov exponent diagrams with the input voltage, the resistance, the inductance and the capacitance as the bifurcation parameters are obtained, and the effect of the parameters on the system stability is deeply studied. The results obtained show that they have a great influence on the stability of the system, and the general trend is that the increase of either the voltage-fed coefficient, input voltage or the load resistance, or the decrease of the filtering inductance, capacitance will make the system stability become poorer, and that all the parameters have a critical value, and when they are greater or less than the values, the system will go through stable 1T orbits, stable 2T orbits, 4T orbits, 8T orbits and eventually approaches chaos.

Keywords: Buck-Boost converter, Precise discrete-iterative map, Bifurcation, Lyapunov exponent, Stability

1. Introduction

DC/DC power converters have found wide applications in industrial automation, power electronics, communications and many other fields. With them operating, some irregular phenomena often arise, such as a sudden collapse of the running state, an unknown electromagnetic noise, instability of the running system. These behaviors greatly affect the stability of the system [1-4]. However, they have been often ignored or treated as a fault by practical circuit designers for a long time. A buck-boost converter, as an important kind of DC/DC converter, has been widely used because of its wide range of input voltage, and advantages with both buck and boost converters [5, 6]. However, a variety of nonlinear behaviors, such as sub harmonics, bifurcation and chaos, are easily caused due to the cyclic switching of the circuit topologies [7-9]. Up to now, their studies are mainly focused on modeling or analyzing the nonlinear phenomena [10-12]. The methods adopted can be approximately divided into two kinds: numerical method and approximate model analytical method. In the former, the equations characterizing nonlinear behaviors of converters are solving by numerical iterative algorithms [13]. In the latter, the analytical solutions are obtained by making some approximate disposal [14]. Because of the discrete solutions obtained in numerical method, it is not readily to understand the dependence

of the circuit parameters on the stability of converters clearly. Although the analytical solutions can be achieved by the approximate model analytical method, some useful information about these systems is ignored. Subtle differences in the parameters can lead to very different results and the truth of conclusion will be questioned, especially for the nonlinear system. Therefore, it is very necessary to establish a precise discrete-iterative mapping. In this paper, precise discrete mapping which operates in the discontinuous conduction mode (DCM) has been established by a series of mathematical deduction without any approximation. Based on the model, the influence of the circuit parameters on the stability of the converter is analyzed.

2. Precise Discrete Mapping for the Buck-Boost Converter

The circuit model of a buck-boost converter is shown in Fig. 1, in which a switch S , a diode VD , a capacitor C , an inductor L , a load resistor R , an input voltage source E and a control-loop are included. The control-loop comprises two comparators and an amplifier, of which the given values of the comparators are respectively the desired output voltage V_o and the duty cycle D in steady-state, and the gain of the amplifier is voltage-fed coefficient k . In the model, the equivalent-series-resistance r_c of C is considered. Obviously, the circuit is of second-order. There are three kinds of switch states in DCM: ① S on and VD off; ② S off and VD on; ③ S and VD off.

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Fig. 1. Circuit model of voltage mode controlled buck-boost converter

The converter can be described by the following sequence of state equations

$$\begin{cases} \dot{x} = A_1 x + B_1 E & t_n \leq x < t'_n \\ \dot{x} = A_2 x + B_2 E & t'_n \leq x < t''_n \\ \dot{x} = A_3 x + B_3 E & t''_n \leq x < t_{n+1} \end{cases} \quad (1)$$

where

$$A_1 = \frac{1}{C(R+r_c)} \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (2)$$

$$A_2 = \frac{1}{C(R+r_c)} \begin{bmatrix} -1 & R \\ -\frac{CR}{L} & -\frac{CRr_c}{L} \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (3)$$

$$A_3 = \frac{1}{C(R+r_c)} \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}, B_3 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (4)$$

Where, x is the state vector of the circuit, i.e., $x = [V_C \ i_L]^T$, subscript n denotes the value at the beginning of the n th cycle, T is the switching period, and t_c, t_d and t_e are the running times in different stages of an switching period, which are given by:

$$t_c = t'_n - t_n, t_d = t''_n - t'_n, t_e = t_{n+1} - t''_n \quad (5)$$

$$\begin{cases} t_c = d_n T \\ t_d = \frac{E}{V_c(t_n)} \cdot t_c \\ t_e = T - t_c - t_d \end{cases} \quad (6)$$

Where d_n is the duty cycle during the n th period, defined as

$$d_n = D - k[V_c(t_n) - V_o] \quad (7)$$

Using a successive iterative method, the value of x_{n+1} can be expressed in terms of x_n .

$$\begin{aligned} x_{n+1} = & e^{A_3 t_e} e^{A_2 t_d} e^{A_1 t_c} \left[x_n + \int_{t_n}^{t'_n} e^{A_1(t'_n - \tau)} B_1 E d\tau \right] + \\ & e^{A_3 t_e} e^{A_2 t_d} \int_{t'_n}^{t''_n} e^{A_2(t''_n - \tau)} B_2 E d\tau + \\ & e^{A_3 t_e} \int_{t''_n}^{t_{n+1}} e^{A_3(t_{n+1} - \tau)} B_3 E d\tau \end{aligned} \quad (8)$$

Where, $e^{A_i t}$ is the transition matrix corresponding to $A_i (i=1,2,3)$, and they affect the accuracy of the discrete model.

The state transition matrixes are obtained by solving Eq. (1) in terms of Laplace inverse transformation and Cayley-Hamilton theorem,

$$e^{A_1 t} = e^{A_1 t} = \begin{bmatrix} e^{-mt} & 0 \\ 0 & 1 \end{bmatrix} \quad (9)$$

$$e^{A_2 t} = a_0 I + a_1 A_2 = \begin{bmatrix} a_0 - ma_1 & Rma_1 \\ -\frac{CR}{L}ma_1 & a_0 - \frac{CRr_c}{L}ma_1 \end{bmatrix} \quad (10)$$

where

$$m = 1/C(R+r_c) \quad (11)$$

$$a_0 = \frac{e^{-\alpha mt}}{\beta} [\alpha \sin(\beta mt) + \beta \cos(\beta mt)] \quad (12)$$

$$a_1 = \frac{e^{-\alpha mt}}{\beta m} \sin(\beta mt) \quad (13)$$

$$\alpha = \frac{1}{2} \left[1 + \frac{CRr_c}{L} \right] \quad (14)$$

$$\beta = \frac{1}{2} \sqrt{\frac{4R}{Lm} - \left[1 + \frac{CRr_c}{L} \right]^2} \quad (15)$$

and I is a second-order identity matrix.

Using Eqs. (2, 3) and (4), Eq. (8) can be simplified as

$$x_{n+1} = e^{A_3 t_e} e^{A_2 t_d} e^{A_1 t_c} x_n + e^{A_3 t_e} e^{A_2 t_d} \int_{t_n}^{t'_n} e^{A_1(t'_n - \tau)} B_1 E d\tau \quad (16)$$

The precise discrete mapping of the voltage-fed buck-boost converter operating in DCM can be obtained by combining Eq. (9) with Eq. (10), and Eq. (16) can be written as

$$x_{n+1} = e^{-m(t_c+t_e)} (a_0 - ma_1) x_n + e^{-mt_e} \frac{REma_1 t_c}{L} \quad (17)$$

In DCM of operation, the inductance current i_L is zero at the beginning of every switching cycle, that is, $i_L(t_n) = i_L(nT) = 0$, and only the capacitance functions. Therefore, the iterative mapping is changed as first-order precise discrete mapping.

In the ideal situation, a buck-boost converter can be

considered as a piecewise-linear circuit, but it constitutes a nonlinear system with voltage feedback incorporated. When the circuit parameters and topology are changed, the stability of the system will be affected, and bifurcation and chaotic behavior may be presented.

A usual way to determine the stability of a fixed point X is to consider the Taylor series expansion of Δx_{n+1} around the fixed point

$$\Delta x_{n+1} = \sum_{k=1}^{\infty} \frac{1}{k!} \frac{\partial^k f(x)}{\partial x^k} \Big|_{x=X} (\Delta x_n)^k \quad (18)$$

where $\Delta x_{n+1} = x_{n+1} - X$.

If the disturbance $-\Delta x_n$ is small, the higher order terms in Eq. (18) can be negligible. Thus, the stability criterion of DC/DC converter can be simplified as

$$\left| \frac{\partial x_{n+1}}{\partial x_n} \right| = \left| \frac{\partial f(x)}{\partial x} \Big|_{x=X} \right| \leq 1 \quad (19)$$

When the stability criterion can not be satisfied, the circuit will eventually evolve into chaos.

On the other hand, Eq. (17) can be represented as by a little transformation

$$x_{n+1} = \eta_1 x_n + \eta_2 \quad (20)$$

where

$$\eta_1 = e^{-m(t_c+t_e)} (a_0 - ma_1) x_n \quad (21)$$

$$\eta_2 = e^{-mt_e} \frac{REma_1 t_c}{L} \quad (22)$$

Taking the derivative of Eq. (20), gives

$$\frac{dx_{n+1}}{dx_n} = \eta_1 + \eta'_1 x_n + \eta'_2 \quad (23)$$

where

$$\eta'_1 = \frac{mET}{\beta x_n} e^{-m(t_c+t_e+\alpha t_d)} \left(\frac{d_n}{x_n} + k \right) \cdot ((1-\alpha)^2 + \beta^2) \sin(\beta m t_d) \quad (24)$$

$$\eta'_2 = \frac{RE}{\beta L} e^{-m(t_c+\alpha t_d)} [\beta m t'_d t_c \cos(\beta m t_d) + (m t_c (t'_c + t'_d) - \alpha m t'_c t'_d + t'_c) \cdot \sin(\beta m t_d)] \quad (25)$$

3. Stability Analysis of Buck-Boost Converter

On the basis of the model above, an analysis of the effect of all the main circuit parameters on the stability of a buck-

boost converter operating in DCM is made.

3.1 Effect of voltage-fed coefficient k on system stability

When the voltage-fed coefficient k is chosen as the bifurcation parameter, the stability criterion can be expressed as Eq. (26) in terms of Eq. (23)

$$\left| \frac{dx_{n+1}}{dx_n} \right| = |-11.4001k + 0.7887| \leq 1 \quad (26)$$

The circuit parameters are given as follows: $T=333.33\mu s$, $E=16V$, $L=208\mu H$, $C=222\mu F$, $V_o=25V$, $R=12.5\Omega$ and $T/(C(R+r_c))=0.12$. From Eq. (26), the critical value k_c of k can be solved as 0.1569, and the bifurcation diagram can be obtained as shown in Fig. 2. It can be seen that when $k < 0.1569$, the buck-boost converter is in the stable state 1T regions, but when $k > 0.1569$, it is firstly in the stable 2T and 4T, and then stable 8T, and eventually approaches chaos, whose Lyapunov exponent diagram is shown in Fig. 3, of which, the positive values on the Lyapunov exponent diagram mean chaos, and at the point of tangential bifurcation, the Lyapunov exponent is less than zero, and the simulation result is well agreement with bifurcation diagram. Actually, tangential bifurcation triggers intermittent chaos. In Fig. 4 is shown the phase portrait around the critical of k .

3.2 Effect of input voltage E on system stability

Due to its wide input voltage range, a buck-boost

Fig. 2. Bifurcation diagram with the variation of k

Fig. 3. Lyapunov exponent with the variation of k

Fig. 4. Phase portraits of sampled i_L versus v_o for $k=0.1569$.

Fig. 5. Bifurcation diagram with the variation of E

Fig. 6. Variation of Lyapunov exponent with E

Fig. 7. Phase portraits of sampled i_L versus v_o for $E=32.2V$.

converter has been widely used. Theoretically, a stable output voltage can be obtained by the pulse width modulation (PWM) control, but in fact, the system can remain stable only when the input voltage is in a certain range. In order to study the effect of the input voltage E on the stability of the converter, the bifurcation diagram and Lyapunov exponent diagram with E as bifurcation parameter can be obtained under the condition of $T=333.33 \mu s$, $L=208 \mu H$, $C=222 \mu F$, $V_o=25V$, $R=12.5\Omega$ and

$k=0.08$, which are respectively shown in Figs. 5 and Fig. 6. It can be seen that the critical stable value of E is approximately $32.2V$, that is, the system is stable when $E \leq 32.2V$, and when E is greater than the value, it will go through stable 1T orbits, stable 2T orbits, 4T orbits, 8T orbits as E increases, and eventually approaches chaos. In Fig. 7 is shown the phase portrait around the critical of E .

3.3 Effect of inductance L on system stability

Inductance L is one of the most important parameters determining the circuit operation modes, namely, DCM or continuous conduction mode (CCM). Its critical value L_c can be determined as:

$$L_c = \frac{(1-D)^2 RT}{2} \quad (27)$$

When $L < L_c$, the circuit operates in DCM, otherwise, it operates in CCM. $L_c=556 \mu H$ can be achieved by substituting the circuit parameters into Eq. (27). In order to investigate the effect of the inductance L on the stability of the converter, the bifurcation diagram and Lyapunov exponent diagram with L as bifurcation parameter can be drawn under the condition of $T=333.33 \mu s$, $E=16V$, $C=222 \mu F$, $V_o=25V$, $R=12.5\Omega$ and $k=0.08$, which are respectively shown in Figs. 8 and 9. It can be seen that the critical stable value of L is approximately $50 \mu H$, and when L is smaller than the value, the system will be through stable 1T orbits, stable 2T orbits, stable 4T orbits as L decreases, and eventually approaches chaos. In Fig. 10 is shown the phase portrait around the critical of L .

Fig. 8. Bifurcation diagram with the variation of L

Fig. 9. Lyapunov exponent with the variation of L

Fig. 10. Phase portraits of sampled i_L versus v_o for $L=50\mu\text{H}$.

Fig. 13. Phase portraits of sampled i_L versus v_o for $C=122\mu\text{F}$.

Fig. 11. Bifurcation diagram with the variation of C

Fig. 14. Bifurcation diagram with the variation of R

Fig. 12. Lyapunov exponent with the variation of C

Fig. 15. Lyapunov exponent with the variation of R

3.4 Effect of capacitance C on system stability

The capacitor C is used to filter the output voltage to reduce the ripple. The circuit parameters are given as follows: $T=333.33\mu\text{s}$, $L=208\mu\text{H}$, $E=16\text{V}$, $V_o=25\text{V}$, $R=12.5\Omega$ and $k=0.08$. In order to investigate the relationship between C and the stability of the converter, the bifurcation diagram and Lyapunov exponent diagram with C as bifurcation parameter can be made with the other parameters being invariant, which are respectively shown in Figs. 11 and 12. From them, it can be seen that capacitance has a great effect on the stability of the converter, namely, the smaller the capacitance value, the poorer the stability of the converter, and the critical value is $122\mu\text{F}$, the circuit will be stable when $C \geq 122\mu\text{F}$. With the decrease of the value of the capacitor, the system will be through stable 1T orbits, stable 2T orbits, stable 4T orbits, and eventually to chaos. The phase portrait around the critical of C is shown in Fig. 13.

Fig. 16. Phase portraits of sampled i_L versus v_o for $R=14.5\Omega$.

3.5 Effect of load R on system stability

In order to study the effect of the load on the system stability, according to the stability criterion Eq. (19), the critical value of the load resistance has been found as 14.5Ω by solving Eq. (23) with the other parameters being invariant, and the bifurcation diagram has been also obtained, as shown in Fig. 14, along with the corresponding Lyapunov exponent diagram as shown in Fig. 15. From

them, it can be seen that the effect of the load resistance on the system stability is: the bigger the resistance value, the poorer the system stability and the circuit will be stable when $R \leq 14.5\Omega$ and the phase portrait around the critical of R is shown in Fig. 16.

4. Conclusions

The precise iterative map of the voltage-fed buck-boost converter which is operating in DCM is proposed in this paper. Based on the map, the effect of the main circuit parameters on the system stability has been investigated, and the results obtained show that they have a great influence on the stability of the system, and the general trend is that the increase of k , E or R , or the decrease of L or C will make the system stability poorer, and that all the parameters have a critical value, and when they are greater or less than the value, the system will go through period-doubling bifurcation and eventually approaches chaos. It can be determined that the critical values of k , E , R , L and C are respectively 0.1569, 32.2V, 14.5 Ω , 50 μ H and 122 μ F, with the other parameters being constant.

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