A Method to enhance the Performance of Spectrum Sensing Under a Random Traffic of Primary User

Hyung-Yun Kong

Abstract This paper focuses on analyzing the effects of primary user (PU) signal arrival on the local spectrum sensing performance. The probability for signal arrival at a sample in the sensing time is uniformly distributed in the sensing time. We first analyze the main factor that causes the degradation in the detection results in the conventional energy detection (CED) under the uniformly random arrival of the PU-signal. Thus we propose an approach in order to enhance the detection performance, in which an estimator which detects the arrival of the PU signal cooperates with a composite energy detection. The mathematical analysis and numerical simulation has validated the outperformance of the proposed approach compared to the CED.

Key Words : Cognitive Radio, estimation, composite detection, spectrum sensing, UMP test.

1. Introduction

As being the primitive form of Cognitive Radio [1], spectrum sensing allows the CR to be aware of and sensitive to the environment’s variations. Thank to this special characteristic, CR is able to accommodate to the wireless environment changes, specially the fluctuation come from PU activities, by the spectrum holes detection and opportunistic occupation. Nevertheless, Spectrum Sensing confronts several crucial challenges to guarantee its tasks of having a quick and accurate sensing, well synchronization as well as the requirement of uncomplicated hardware. Among these specifications, the sensing accuracy emerges as the most important issue due to its contribution to the cognitive radio networks’ functioning.
In single user detection scheme, the three detection approaches being extensive explored are matched filter detection, energy detection and feature detection \[2\]. The energy detection is, in fact, much simpler than the rest of those methods and it is mostly optimal in the sense of detecting any unknown zeros mean constellation signal. However, the conventional energy detection is not practical due to the assumption of the two null hypotheses: the \(H_0\) presents the case of noise-only signal in the sensing period, and \(H_1\) is for the PU signal presence case \[3\].

Thus, in this paper, we focus on analyzing the performance of conventional detection in a such practical context: the PUs signal’s arrival can occur at any point during the sensing time, and once if having occurred, it will be last at least until the end of this interval. In addition, we propose the combining approach which engages the estimation of the time of arrival (ToA) of PU signal to co-operate with the conventional energy detection in order to deal with the degraded performance. The following parts of the article are arranged in the such orders: part II mentions about the System Model, part III is the Performance Analysis, part IV illustrates the Numerical Results, and the final is the Conclusions.

II. System Model

1. The conventional detection

Under the two null hypotheses of PU operation, the received signal is modeled as:

\[
\begin{align*}
H_0 : x(n) &= z(n) \\
H_1 : x(n) &= s(n) + z(n)
\end{align*}
\]

(1)

where \(x(n)\) is the received signal, \(s(n)\) is the transmitted PU signal and \(z(n)\) is Gaussian noise with zero mean and variance \(\sigma_n^2\). The both hypothesis tests are the energy of the received signals, and being given as:

\[
\begin{align*}
H_0 : r &= \sum_{n=1}^{W} |z(n)|^2 \\
H_1 : r &= \sum_{n=1}^{W} |s(n) + z(n)|^2
\end{align*}
\]

in which \(r\) presents as the received signal energy, \(W\) is the number of samples in sensing interval. Being proved by \[3,4\], the false alarm and detection probabilities are induced accordingly as:

\[
\begin{align*}
P_f^{H_0} &= \Pr\{r > \gamma|H_0\} = \frac{\Gamma(W/2, \sigma_n^2 \gamma)}{\Gamma(W/2)} \\
P_d^{H_0} &= \Pr\{r > \gamma|H_0\} = Q_{W/2}\left(\sqrt{\frac{\sigma_n^2 \gamma}{\sigma_n^2}}, \sqrt{\frac{\sigma_n^2 \gamma}{\sigma_n^2}}\right)
\end{align*}
\]

(2)

where \(Q(\ldots)\) is the Marcum’s Q function being identified in \[4\], \(\Gamma(.)\) is the Gamma function \[5\].

2. Effect of traffic Model

For a practical case of PU arrival, in this article, we consider the case that PU signal is possibly “born” during the sensing period and it “dies” before the next sensing interval start. In the circumstance in which the PU signal birth occurs during the sensing time, we make an assumption that the signal lasts over the end of the sensing interval (Fig.1b.). Due to the equal role of each sample, the probability of the event which the PU signal arrive at the specific sample is uniformly.

\[
\begin{align*}
\text{그림 1. 기존의 가설 } H_1 \text{과 실제의 차이}
\end{align*}
\]

Fig. 1. The difference between the conventional hypothesis \(H_1\) and the reality case.

3. Performance of the Conventional Energy Detection (CED)

In this subsection, we discuss more about the errors in detection of the CED. We denote \(\xi_j\) being the event that the PU signal emerges at the \(j\)-th sample, \(1 \leq j \leq W\), in the sensing duration, \(\xi_0\) is for the event of noise-only received signal. Thus, the energy signal in this case is
The Journal of the Institute of Internet, Broadcasting and Communication, Vol. 13, No. 6, December 2013

The non-central chi-square distribution with the non-centrality parameter being

\[ s^2 = x^2 = \sum_{n=j}^W s^2(n) = W - j + 1 \]  

as we have made the assumption of \( s(n) = 1 \), and the variance \( W x^2 + 4 x^2 s^2 \). It is clearly that this distribution is different from the above-mentioned null hypothesis \( H_0 \) of the CED or, in the other words, the null hypothesis cannot cover all situations of PU appearance during sensing time. To deal with this problem, the alternative detection method named as composite method [6] should be employed to replace the conventional detection. As a result, the alternative hypothesis can be model as the below:

\[
\begin{align*}
H_0: r(j) &= \sum_{n=1}^W \left| z(n) \right|^2 \\
H_1: r(j) &= \sum_{n=1}^W \left| z(n) \right|^2 + \sum_{n=j}^W \left| s(n) + z(n) \right|^2
\end{align*}
\]  

(3)

This alternative method has a null hypothesis \( H_0 \) and the detection criterion region for each value of \( j \) is \( \psi_j = \{ r(j) \in (R^+, 0) \mid r(j) > \tau \} \). From (2), we can find that if the threshold \( \tau \) is identified by a given false alarm probability, it is independent from the value of \( j \). Hence, the criterion region is independent from \( j \) and this satisfies the condition to employ the uniform most powerful (UMP) test [6, section II.E]. The false alarm and detection probability of the alternative detection method are:

\[
\begin{align*}
L_0(x) &= \prod_{n=1}^{(k-1)M} \frac{1}{\sqrt{2\pi \sigma_n}} \exp \left( -\frac{x^2(n)}{2\sigma_n^2} \right) \\
&\quad \times \prod_{n=(k-1)M+1}^{kM-1} \frac{1}{\sqrt{2\pi \sigma_n}} \exp \left( -\frac{x^2(n)}{2\sigma_n^2} \right) \\
&\quad \times \prod_{n=kM+1}^{NM} \frac{1}{\sqrt{2\pi \sigma_n}} \exp \left( -\frac{(x(n) - s(n))^2}{2\sigma_n^2} \right)
\end{align*}
\]

where \( \hat{A}_0, \hat{A}_1 \) are the region containing all possibly occurring cases in the above two hypotheses: for \( H_0 \), \( \hat{A}_0 = \{ \xi_0 \} \) and, for \( H_1 \), \( \hat{A}_1 = \{ \xi_j \mid j \in [1, W] \} \).

Because the ToA at the \( j \)-th sample is uniformly distributed (as each sample has the same role to get the ToA), we can easily obtain \( p_{\theta}(\xi_0 \mid \hat{A}_0) = 1 \) and \( p_{\theta}(\xi_j \mid \hat{A}_1) = \frac{1}{W} \). It is easy to find that the null hypothesis \( H_1 \) of the CED is the special case of the composite hypothesis when \( j = 1 \).

4. Performance of the Time of Arrival (ToA) Operation

However, the composite detection does not have enough powerfulness to cope with the practical case of PU arrivals. Thus, in this paper, we propose the method which is able to reduce limitation. Fig.2 is shown to illustrate the proposed model in which the time of arrival (ToA) estimator is added to cooperate with the energy detection for the purpose of gaining the accuracy. In this scheme, the ToA estimator coarsely implements the ToA searching procedure by dividing the sensing period into \( N \) \( M \)-sized blocks. Then, the composite energy detector uses the found ToA to implement its detection.

In the ToA estimation, the estimator uses likelihood
A Method to enhance the Performance of Spectrum Sensing Under a Random Traffic of Primary User

detection method to consider each block. Without loss of generality, we suppose that the PU arrival is at k-th block. The likelihood ratio (LR) test of the hypothesis \( H_k \) which assumes the ToA is occur at k-th block against the (k+1)-th block \( H_{k+1} \) is expressible as:

\[
L_k(x) = \prod_{n=1}^{M} \frac{1}{\sqrt{2\pi} \sigma_n} \exp\left(-\frac{x^2(n)}{2\sigma_n^2}\right)
\]

\[
= \prod_{n=(k-1)M+1}^{kM} \frac{1}{\sqrt{2\pi} \sigma_n} \exp\left(-\frac{(x(n) - s(n))^2}{2\sigma_n^2}\right)
\]

We denote \( LLR_k \) as the Log-likelihood test of (5) and the statistic test is given as:

\[
LLR_k = 2\sigma_n^2 \log(L_k)
\]

\[
= \sum_{n=(k-1)M+1}^{kM} (2x(n)s(n) - s^2(n))
\]

Errors in the estimator are categorized into the early false alarm and the arrival miss detection, whose probabilities are alternatively expressible as (7) with \( \eta \) is the preset threshold.

\[
\begin{align*}
P_{f}^\text{Est} &= P\{LLR_k > \eta \mid H_0\} \\
1 - P_{d}^\text{Est} &= P\{LLR_k < \eta \mid H_1\}
\end{align*}
\]

\( P_d^\text{Est} \) is denoted as the arrival detection probability, \( \overline{H}_0 \) is the hypothesis representing the case of noise only portion which is described as below:

\[
\{x(n) \sim N(0,\sigma_n^2), (k-1)M+1 \leq n \leq (k)M\}
\]

and \( \overline{H}_1 \) is for the portion being fully occupied by PU signal as the following description:

\[
\{x(n) \sim N(s(n),\sigma_n^2), (k-1)M+1 \leq n \leq kM\}
\]

For being more convenient, we make assumption that \( s(n) = 1 \). Additionally, from (6), it is clearly to recognize that the \( LLR_k \) has the Gaussian distribution in which under \( \overline{H}_0 \): \( LLR_k \sim N(-M,2M\sigma_n^2) \) and under \( \overline{H}_1 \): \( LLR_k \sim N(M,2M\sigma_n^2) \). The threshold \( \eta \) is determined based on either a given early false alarm probability \( \beta \) or the miss arrival detection probability \( \gamma \) in such way that \( \eta_{\beta} = Q_G^{-1}(\beta) \sqrt{2M\sigma_n^2} - M \) or \( \eta_{\gamma} = Q_G^{-1}(\gamma) \sqrt{2M\sigma_n^2} + M \) with \( Q_G(.) \) is the complementary Gaussian distribution function, \( Q_G(x) = 1/2\pi \int_x^{+\infty} \exp(-t^2/2) dt \).

### III. Simulation

The performance evaluation is conducted to show the performance of the proposed method in comparison with the conventional method. Fig.3. surveys the performance of the conventional energy detection and composite energy detection. The number of samples taken for this investigation is \( W = 180 \) samples and the \( SNR = \left| s^2(n)/\sigma_n^2 \right| = [0,1] dB \). For the case of the two null hypotheses \( H_0, H_1 \) only, the detection probability is higher than the composite ED (we mark as comED in the figure). Though, due to the fact that there are many cases of the arrival of PU signal during sensing time, this detection does not reflect the detection in reality. Meanwhile, the composite ED performance has shown that it matched with the simulation result. From Fig.3, we can easily to find that actual detection probability is much lower than the performance characteristic of the conventional detection.
The Journal of The Institute of Internet, Broadcasting and Communication, Vol. 13, No. 6, December 2013

Early False Alarm Probability
ToA Detection Probability
ToA Estimation Performance when M = 10

CToA SNR = -10dB
CToA, SNR = -7dB
CToA, SNR = -4dB

Fig. 3. The theoretic, practical performance of CED versus composite energy detection method

Fig. 4. describes the detection performance of ToA estimation in the different scenarios of SNR within each block size M is 10 samples. As SNR increases, the detection performance of the CED and composite ED is improved.

False Alarm Probability
Detection Probability
Comparision between the proposed method and Composite ED

prop,0dB
prop,1dB
prop,2dB
comED,0dB
comED,1dB
comED,2dB

Fig. 5. The performance of detection of the proposed method versus the composite energy detection method.

In the Fig. 5, we evaluate the performance of the proposed method in which the composite energy detection and ToA estimator are employed to co-operate. Furthermore, we make the comparison between two methods, the proposed method and composite ED only, over different SNRs: 0, 4, 8 dB within the early ToA false alarm probability is set at 0.01 for each block sizing at M = 10. The graph shows a significant improvement when using the proposed scheme in comparison with employing the composite energy detection only. Generally, as the SNR increasing, the both methods provide the improvement in detection. However, for the distinct comparison of each case of SNR, for example in case of SNR being at 0dB, the proposed method always supplies the superior detection to the method using the composite energy detection only.

IV. Conclusion

In the scope of this paper, we analyzed the downsides of the conventional energy detection when dealing with the practical cases. From the analysis, as
being affected by the practical arrival of PU signal, the conventional energy detection performance does not reflect the actual performance in reality while the composite detection does well match. However, the performance characteristic of the composite energy detection is fairly low. In addition, we also have proposed the scheme of detection by combining composite ToA of PU signal estimation and composite energy detection. The results show that the proposed method provides the much better performance than the method using composite energy detection only.

References


