Equivalent Transmission-Line Sections for Very High Impedances and Their Application to Branch-Line Hybrids with Very Weak Coupling Power

Hee-Ran Ahn · Bumman Kim

Abstract

As operating frequency is raised and as more integration with active and passive elements is required, it becomes difficult to fabricate more than $120\ \Omega$ characteristic impedance of a microstrip line. To solve this problem, an equivalent high impedance transmission-line section is suggested, which consists mainly of a pair of coupled-line sections with two shorts. However, it becomes a transmission-line section only when its electrical length is fixed and its coupling power is more than half. To have transmission-line characteristics(perfect matching), independently of coupling power and electrical length, two identical open stubs are added and conventional design equations of even- and odd-mode impedances are modified, based on the fact that the modified design equations have the linear combinations of conventional ones. The high impedance transmission-line section is a passive component and therefore should be perfectly matched, at least at a design center frequency. For this, two different solutions are derived for the added open stub and two types of high impedance transmission-line sections with $160\ \Omega$ characteristic impedance are simulated as the electrical lengths of the coupled-line sections are varied. The simulation results show that the determination of the available bandwidth location depends on which solution is chosen. As an application, branch-line hybrids with very weak coupling power are investigated, depending on where an isolated port is located, and two types of branch-line hybrids are derived for each case. To verify the derived branch-line hybrids, a microstrip branch-line hybrid with $-15\ \text{dB}$ coupling power, composed of two 90° and two 270° transmission-line sections, is fabricated on a substrate of $\epsilon_r = 3.4$ and $h = 0.76\ \text{mm}$ and measured. In this case, $276.7\ \Omega$ characteristic impedance is fabricated using the suggested high impedance transmission-line sections. The measured coupling power is $-14.5\ \text{dB}$, isolation and matching is almost perfect at a design center frequency of $2\ \text{GHz}$, showing good agreement with the prediction.

Key words : High Impedance Transmission-Line Sections, Coupled-Transmission Line Sections with Two Shorts and Branch-Line Hybrids with Very Weak Coupling Power.

I. Introduction

One of main requirements for a suitable transmission-line structure is that the structure be planar in configuration. A planar configuration implies that the characteristics of the element can be determined by the dimension in a single plane. For example, the width of a microstrip line on a dielectric substrate can be adjusted to control its impedance. This results in convenient circuit fabrication through the techniques of photolithography and photoetching of thin films. Use of these techniques at microwave frequencies has led to the development of MICs(Microwave Integrated Circuits) and MMICs(Microwave Monolithic Integrated Circuits).

Among planar transmission lines, the microstrip line is one of the most popular types, primarily because it can be fabricated by an easy process at low cost and

with good agreement between prediction and measurement. However, as operating frequencies are raised, the substrate needs to be thinner. As more integration with passive or active elements is required, the substrate dielectric constant also needs to be higher^{[1],[2]}. In this case, high characteristic impedance is not easy to fabricate.

In this paper, ways for realizing the high characteristic impedances in a microstrip transmission line are discussed. As a review, the relationship between widths and characteristic impedances of a microstrip line is explained, showing that characteristic impedance higher than $120\ \Omega$ is not easy to fabricate with a higher relative dielectric constant and a thinner substrate. To solve the problem, an equivalent high impedance transmission-line section is derived using a pair of coupled transmission-line sections.

A pair of coupled transmission-line sections was first introduced in 1922^[3] and significant progress was made during the 1940s and 1950s^{[4],[5]}. In the 1960s and 1970s, numerous papers^{[6],[7]} extended this theory, and its application and development has continued since then^{[8]~[10]}. A pair of coupled transmission-line sections is inherently a four-port circuit and its frequency response is determined depending on the choice of how to terminate two of four ports. Terminating two ports in shorts in a diagonal direction results in a two-port circuit having particular characteristics that include very high characteristic impedance of a transmission-line section available with a 180° phase shift. Due to these particular characteristics, this type has been used for a variety of applications such as ring hybrids^[11], baluns^{[12],[13]} phase shifters, impedance transformers, filters^{[14],[15]} and so on. However, only when the coupling power is more than half (−3 dB coupling power) and its electrical length is fixed, can perfect matching be obtained. However, the coupled-line sections with more than −3 dB coupling power are not easy to realize. To have transmission line characteristics without any restriction of both coupling power and electrical length, an equivalent high impedance transmission-line section is derived, consisting of a pair of coupled-line sections with two shorts and two open stubs. For this type, the conventional design equations of even- and odd-mode impedances need to be modified, based on the fact that the modified design equations have linear combinations with the conventional ones. In addition, to have perfect matching, at least at a design center frequency with any electrical length, two different solutions for the open stubs are computed. Using these two solutions, several transmission-line sections having a characteristic impedance of 160 Ω can be simulated and the simulation results then show that where the available bandwidth is located is dependent on which solution is chosen. As an application of the high impedance transmission-line sections, two types of branch-line hybrids are suggested.

The branch-line hybrids were first introduced in 1947 by W. Munford^[16] and have been used for many applications, including phase shifters, high power amplifiers, measurement systems and so on. In particular, the branch-line hybrids with very weak coupling power may be used in place of directional couplers or feed-forward amplifiers^[10], with the advantage that two outputs of the branch-line hybrids are located in the same side^[17]. In this paper, to realize the very weak coupling power, two types of branch-line hybrids are introduced using power flow behavior. A microstrip branch-line hybrid with −15 dB coupling power is fabricated and measured.

This paper is constructed with five sections. Section

I gives a brief introduction of this paper, including an explanation of why high characteristic impedance is not easy to realize with microstrip technology. Section II shows how difficult it is to fabricate more than 120 Ω characteristic impedances of a microstrip line on a thin substrate with high dielectric constant. To solve the problem, in section III, we explain how to get an equivalent high impedance transmission-line section using a pair of coupled transmission-line sections. As an application of the high impedance transmission-line section, in section IV, two-types of branch-line hybrids are introduced and a microstrip branch-line hybrid with −15 dB coupling power is fabricated and tested. The paper then concludes with section V.

II. Microstrip Transmission Lines

The microstrip is by far the most popular microwave transmission line, especially for MICs and MMICs. The major advantage of microstrip transmission lines is that all active and passive components can be mounted on the top of the board. The cross-sectional view of a microstrip line is depicted in Fig. 1. The microstrip line consists of a conductive strip of width " W " and thickness " t " and a wider ground plane, separated by a dielectric layer (substrate) of thickness " h ". Unlike the stripline, where all fields are contained within a homogeneous dielectric region, the microstrip has some of its field lines in the dielectric region, concentrated between strip conductor and the ground plane, and some other fraction in the air region above the substrate. For this reason, the microstrip line cannot support a pure TEM wave and it is also more difficult to realize high characteristic impedances.

Because part of the field of the microstrip conductor exists in air, the effective dielectric constant ϵ_{eff} is somewhat less than the relative dielectric constant ϵ_r . The effective dielectric constant ϵ_{eff} is also a function of the ratio of width to height W/h (and also the ratio of height to width h/W), as well the relative dielectric constant. Depending on the value of W/h , two separate solutions exist, such as

Fig. 1. Cross-sectional view of microstrip line.

$$\epsilon_{eff} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left[\left(1 + \frac{12h}{W} \right)^{-1/2} + 0.04 \left(1 - \frac{W}{h} \right)^2 \right] \quad \text{for } \frac{W}{h} < 1 \quad (1a)$$

$$\epsilon_{eff} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left(1 + \frac{12h}{W} \right)^{-1/2} \quad \text{for } \frac{W}{h} \geq 1 \quad (1b)$$

The characteristic impedance $Z_0^{[18]}$ is also a function of the ratio of height to width, h/W , of the transmission line, and also has separate solutions depending on the value of W/h

$$Z_0 = \frac{60}{\sqrt{\epsilon_{eff}}} \ln \left(\frac{8h}{W} + \frac{W}{4h} \right), \quad \text{for } \frac{W}{h} < 1 \quad (2a)$$

$$Z_0 = \frac{120\pi}{\sqrt{\epsilon_{eff}}} \left[\frac{W}{h} + 1.393 + 0.677 \ln \left(\frac{W}{h} + 1.444 \right) \right]^{-1} \quad \text{for } \frac{W}{h} \geq 1 \quad (2b)$$

Using (1) and (2), the widths of the microstrip line have been calculated depending on substrates and characteristic impedances. The calculation results are plotted in Fig. 2 and listed in Table 1. For this, four types of substrates are used: ($\epsilon_r=3.5$, $h=0.76$ mm), ($\epsilon_r=3.5$, $h=0.38$ mm), ($\epsilon_r=7$, $h=0.76$ mm) and ($\epsilon_r=7$, $h=0.38$ mm). Fig. 2 shows that a greater value of W can be obtained from a substrate with lower ϵ_r and higher h . Based on Table 1, when $Z_0=120 \Omega$ and $h=0.76$ mm, $W=221 \mu\text{m}$ with $\epsilon_r=3.5$, while $W=62.4 \mu\text{m}$ with $\epsilon_r=7$. When $Z_0=120 \Omega$ and $h=0.38$ mm, $W=96 \mu\text{m}$ with $\epsilon_r=3.5$, while $W=22.4 \mu\text{m}$ with $\epsilon_r=7$. If Z_0 is more than 120Ω , the width becomes too small to fabricate with a normal PCB(Printed Circuit Board) technology. To

Fig. 2. Microstrip width W versus characteristic impedance Z_0 with different substrates.

Table 1. Microstrip widths.

Z_0	100 Ω	120 Ω	160 Ω	200 Ω
$h=0.76$ mm				
$\epsilon_r=3.5$	392 μm	221 μm	61 μm	11 μm
$\epsilon_r=7$	146 μm	62.4 μm	6.8 μm	0.69 μm
$h=0.38$ mm				
$\epsilon_r=3.5$	181 μm	96 μm	20 μm	2.9 μm
$\epsilon_r=7$	62 μm	22.4 μm	1.9 μm	X

realize more than 120Ω characteristic impedances without any problem, one set of coupled transmission-line sections with two shorts may be used.

III. High Impedance Transmission-Line Sections

3-1 Coupled Transmission-Line Sections with Two Shorts

A four-port circuit and a two-port circuit of coupled-line sections are depicted in Fig. 3(a) and (b), respectively. The two-port circuit is obtained from the four-port circuit by terminating ports ③ and ④ in Fig. 3(a) in shorts. Therefore, the admittance parameters of the two-port circuit are derived by applying the short boundary condition to the four-port circuit and are given as

$$Y_{11} = Y_{22} = -j \frac{Y_{0e} + Y_{0o}}{2} \cot \Theta, \quad (3a)$$

$$Y_{12} = Y_{21} = -j \frac{Y_{0o} - Y_{0e}}{2} \csc \Theta \quad (3b)$$

where $Y_{0e} = Z_{0e}^{-1}$ and $Y_{0o} = Z_{0o}^{-1}$.

Equation (3) may be rewritten as:

$$[Y] = -jY_{0e} \cot \Theta \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + j \frac{Y_{0o} - Y_{0e}}{2} \begin{bmatrix} -\cot(\pi + \Theta) & \csc(\pi + \Theta) \\ \csc(\pi + \Theta) & -\cot(\pi + \Theta) \end{bmatrix} \quad (4)$$

In (4), the first term indicates two identical short stubs with a characteristic admittance Y_{0e} , while the second term is a transmission line section with a characteristic admittance $(Y_{0o} - Y_{0e})/2$ and an electrical length of $\pi + \Theta$. Therefore, its equivalent circuit can be derived as in Fig. 3(c). Assuming a real termination impedance of Z_0 and a pure TEM(transverse electromagnetic) propagation, the conventional design equations of even- and odd-mode impedances^{[19],[20]} are:

$$Z_{0e} = Z_0 \sqrt{\frac{1+C}{1-C}} \quad (5a)$$

Fig. 3. Coupled-line sections.

$$Z_{0o} = Z_0 \sqrt{\frac{1-C}{1+C}}, \quad (5b)$$

where C is the coupling coefficient.

In this case, the line characteristic impedance Z_c in Fig. 3(c) is

$$Z_c = \left(\frac{Y_{0o} - Y_{0e}}{2} \right)^{-1} = Z_0 \sqrt{\frac{1}{C^2} - 1} \quad (6)$$

In (6), if the coupling power is close to zero, almost infinite characteristic impedance is obtained and the characteristic impedance of Z_c is dependent on the coupling power of C^2 .

Terminating port ② in Fig. 3(c) in Z_0 results in one port circuit as depicted in Fig. 4, where input impedance Z_{in} is calculated as:

$$Z_{in} = Y_{in}^{-1} = \left[-jY_{0e} \cot \Theta + Y_c \frac{Y_L + jY_c \tan \Theta}{Y_c + jY_L \tan \Theta} \right]^{-1}, \quad (7)$$

where $Y_{0e} = Z_{0e}^{-1}$, $Y_c = Z_c^{-1}$ and $Y_L = -jY_{0e} \cot \Theta + Y_0$ with $Y_0 = Z_0^{-1}$.

To display the dependency on the coupling power, the input impedance Z_{in} in (7) has been calculated as coupling coefficient C and electrical length Θ are varied and the results are plotted in Fig. 5. Real and imaginary parts of input impedances normalized to Z_0 are indicated in Figs. 5(a) and (b), respectively. For the calculation, the mathematical software Matlab. 6 was used. When $\Theta = 90^\circ$ in Fig. 5, the real part of the input

 Fig. 4. One-port circuit with port ② terminated in Z_0 .

impedance normalized to Z_0 is unity with coupling coefficient -3 dB, and more or less than unity with other cases. The imaginary part is zero, regardless of coupling coefficients. A scattering parameter S_{11} is calculated based on $S_{11} = (Z_{in} - Z_0) / (Z_{in} + Z_0)$. Calculation results are plotted in Fig. 6 as electrical length Θ and coupling coefficient C are varied.

Almost perfect matching occurs around $\Theta = 45^\circ$ and 135° with $C = -2$ dB and perfect matching appears

 (a) Real part of input impedance normalized to Z_0

 (b) Imaginary part of input impedance normalized to Z_0

 Fig. 5. Calculation results of input impedance normalized to Z_0 .

Fig. 6. Return loss versus electrical length depending on coupling coefficient.

around $\Theta = 90^\circ$ with $C = -3$ dB. From the results in Figs. 5 and 6, the coupling power should be more than half, so that the coupled-line sections with two shorts in Fig. 3(b) can have transmission-line characteristics (perfect matching). As is well known, the coupled-line sections with more than -3 dB coupling power can not be easily realized in microstrip technology. For the two-port circuit of coupled-line sections shown in Fig. 3(b) to be equivalent to a high impedance transmission-line sections, independently of coupling power and electrical length, two conditions must be satisfied: the first one is that the characteristic impedance Z_c should be always equal to the termination impedance Z_0 in Fig. 3(c), with any coupling power. The second one is that the coupled-line sections with two shorts should have perfect matching at least at a design center frequency, independently of the electrical length Θ in Fig. 3.

3-2 High Impedance Transmission-Line Sections

3-2-1 First Condition

To satisfy the first condition, the conventional even- and odd-mode impedances should be modified and the modified even- and odd-mode impedances, Z_{0e}^m and Z_{0o}^m are then satisfied with the following relation:

$$C = \frac{Z_{0e}^m - Z_{0o}^m}{Z_{0e}^m + Z_{0o}^m} \quad (8)$$

From the coupling coefficient in (8), the modified even- and odd-mode impedances are linear combinations of conventional ones, such as:

$$Z_{0e}^m = Z_{0e} \cdot M, \quad (9a)$$

$$Z_{0o}^m = Z_{0o} \cdot M. \quad (9b)$$

When $\Theta = 90^\circ$ in Fig. 4, only a transmission-line section remains and this transforms Z_{in} into Z_0 . With the modified even- and odd-mode impedances, the characteristic impedance Z_c is:

$$Z_c^{-1} = \frac{Y_{0o}^m - Y_{0e}^m}{2} = Z_0^{-1}, \quad (10)$$

where $Y_{0o}^m = 1/Z_{0o}^m$ and $Y_{0e}^m = 1/Z_{0e}^m$.

Substituting the modified the even- and odd-mode impedances into (5) and (10), M is computed as:

$$M = \frac{C}{\sqrt{1 - C^2}} \quad (11)$$

Thus, the resulting modified even- and odd-mode impedances are:

$$Z_{0e}^m = Z_0 \frac{C}{1 - C}, \quad (12a)$$

$$Z_{0o}^m = Z_0 \frac{C}{1 + C}. \quad (12b)$$

With the modified even- and odd-mode impedance given in (12), the line characteristic impedance Z_c is always Z_0 , independently of coupling power and electrical length. Substituting $(Z_{0e}^m - Z_{0o}^m)/(Z_{0e}^m + Z_{0o}^m)$ into C in (12) gives the characteristic impedance Z_c as:

$$Z_0 = Z_c = \frac{2Z_{0e}^m}{\frac{Z_{0e}^m}{Z_{0o}^m} - 1} \quad (13)$$

If the denominator is closed to zero in (13), the infinite value of the characteristic impedance Z_c can be obtained. Since the odd-mode impedances should be at least $30 \, \Omega$ for easy realization in planar structure, Fig. 7 shows the relation between Z_c and Z_{0e}^m in the case that the odd-mode impedances are more than $30 \, \Omega$. If a characteristic impedance of $150 \, \Omega$ is desired to be realized,

Table 2. Modified even- and odd-mode impedances for high characteristic impedances.

$Z_c = Z_0$	(Z_{0e}^m, Z_{0o}^m)
$Z_c = 170 \, \Omega$	(46.3, 30), (75.6, 40), (121.4, 50), (204, 60)
$Z_c = 160 \, \Omega$	(48, 30), (80, 40), (133.4, 50), (240, 60)
$Z_c = 150 \, \Omega$	(50, 30), (85.7, 40), (150, 50), (300, 60)
$Z_c = 120 \, \Omega$	(60, 30), (120, 40), (300, 50), (*, 60), *: no solution.
$Z_c = 110 \, \Omega$	(66, 30), (147, 40), (550, 50), (*, 60), *: no solution.

then several points, which a parallel line of 150Ω intersects, are the solutions. In this way, the modified even- and odd-mode impedances that correspond to each characteristic impedance can be calculated. These calculation results are presented in Table 2.

Fig. 8 shows the normalized modified even- and odd-mode impedances versus coupling coefficient in dB scale. In Table 2, when $Z_0=160 \Omega$, one of solutions is (80, 40) and the modified even-mode impedance normalized to the characteristic impedance is 0.5. In this case, the coupling coefficient is -9.54 dB, indicated as dotted lines in Fig. 8. In this way, coupling power, which plays an important role in the fabrication, can be determined from Fig. 8.

3-2-2 Second Condition

The second condition requires that the coupled-line sections with two shorts should be perfectly matched at least at a design center frequency, independently of the electrical length Θ in Fig. 3(b). To satisfy this, two

Fig. 7. Relation between Z_c , Z_0^m and Z_0^m

Fig. 8. Relation between coupling coefficient and normalized modified even- and odd-mode impedances.

identical open stubs are added to both ports, as depicted in Fig. 9 (a), where Y_{op} and Θ_{op} are the characteristic admittance and electrical length of the open stub, respectively. The circuit in Fig. 9(a) then becomes to be equivalent to a high impedance transmission-line section and symmetrical. Therefore, even- and odd-mode excitations may be used for the analyses and those in Fig. 9(b) and (c) are even- and odd-mode equivalent circuits. The even- and odd-mode input admittances in Fig. 9(b) and (c) are:

$$Y_{even} = j(Y_{op} \tan \Theta_{op} - Y_{0e}^m \cot \Theta) - jY_c \cot \frac{\Theta}{2} \quad (14a)$$

$$Y_{odd} = j(Y_{op} \tan \Theta_{op} - Y_{0e}^m \cot \Theta) + jY_c \tan \frac{\Theta}{2}, \quad (14b)$$

where $Y_c = Z_c^{-1}$, $Y_{op} = Z_{op}^{-1}$ and $Y_{0e}^m = (Z_{0e}^m)^{-1}$.

Thus, the reflection coefficient $S_{11}=S_{22}$ may be written in terms of eigenvalues^[21], such as:

$$S = \frac{s_1 + s_2}{2} = \frac{Y_0^2 - (Y_{even}Y_{odd})}{(Y_0 + Y_{even})(Y_0 + Y_{odd})}, \quad (15)$$

(a) Equivalent high impedance transmission-line section

(b) Even-mode equivalent circuit

(c) Odd-mode equivalent circuit

Fig. 9. High impedance transmission-line section.

where $s_1 = (Y_0 - Y_{even}) / (Y_0 + Y_{even})$ and $s_2 = (Y_0 - Y_{odd}) / (Y_0 + Y_{odd})$

For the circuit in Fig. 9(a) to be perfectly matched, the numerator in (15) should be zero, which results in:

$$\begin{aligned} & (Y_{op} \tan \Theta_{op} - Y_{0e}^m \cot \Theta) \cdot \\ & \left(Y_{op} \tan \Theta_{op} - Y_{0e}^m \cot \Theta + Y_0 \tan \frac{\Theta}{2} - Y_0 \cot \frac{\Theta}{2} \right) = 0 \end{aligned} \quad (16)$$

From (16), two solutions for the open stubs are derived as:

$$\text{Sol. I ; } Y_{op} \tan \Theta_{op} = Y_{0e}^m \cot \Theta, \quad (17a)$$

$$\text{Sol. II ; } Y_{op} \tan \Theta_{op} = Y_{0e}^m \cot \Theta, \quad (17b)$$

Several high impedance transmission line sections have been simulated at a center frequency of 1 GHz and the results are plotted in Fig. 10, where insertion and return losses are in Fig. 10 (a) and (b), respectively. For the simulation, characteristic impedance Z_c and termination impedance Z_0 were fixed at $160 \, \Omega$ ($Z_{0e}^m = 102.8 \, \Omega$, $Z_{0o}^m = 45 \, \Omega$), and their electrical lengths Θ were varied from 90° to 60° at equal interval of 10° . For this simulation, Sol. I in (17a) is adopted for the open stubs and Table 3 gives each bandwidth. Here, the bandwidth is defined as a frequency range where insertion loss is more than -3 dB and return loss less than -15 dB.

Based on Fig. 10 and Table 3, the bandwidths of different high impedance transmission-line sections are similar to each other and perfect matching can be achieved at a design center frequency of 1 GHz, independently of the electrical length Θ . In addition to the operating center frequency, almost perfect matching occurs at higher than 1 GHz and the additional perfect matching point is dependent on the electrical length Θ . With Sol. II, the same simulation process has been carried out and only return loss frequency response is plotted in Fig. 11. Comparing the two types of return losses in Figs. 10(b) and 11, additional perfect matching with Sol. II in (17b) appears at lower than the operating center frequency of 1 GHz. Using this concept, high impedance transmission-line sections can be designed with wider bandwidths.

3-2-3 Measurements

To verify the high impedance transmission-line section, an impedance transformer to transform $1 \, \text{k}\Omega$ into $100 \, \Omega$ was designed at a center frequency of 2 GHz and fabricated on a substrate ($\epsilon_r = 3.5$, $h = 0.76 \, \text{mm}$) with microstrip technology. In this case, the characteristic impedance of the impedance transformer is 316.2

(a) Insertion loss for Sol. I

(b) Return loss for Sol. I

Fig. 10. Simulation results of high impedance transmission-line section with $160 \, \Omega$ characteristic impedance.

Table 3. Bandwidths of high impedance transmission-line sections with $160 \, \Omega$ characteristic impedance.

	$\Theta = 90^\circ$	$\Theta = 80^\circ$	$\Theta = 70^\circ$	$\Theta = 60^\circ$
Insertion loss more than -3 dB				
GHz	0.63 ~ 1.37	0.66 ~ 1.43	0.7 ~ 1.47	0.74 ~ 1.47
BW	(0.74)	(0.77)	(0.77)	(0.73)
Return loss less than -15 dB				
GHz	0.84 ~ 1.16	0.87 ~ 1.2	0.9 ~ 1.25	0.93 ~ 1.28
BW	(0.32)	(0.33)	(0.35)	(0.35)

Ω and the electrical length Θ is 90° , which requires transformation of a real impedance of $1 \, \text{k}\Omega$ into another real impedance of $100 \, \Omega$.

To realize the characteristic impedance of $316.2 \, \Omega$, there are many solutions that depend on coupling power, as written in Table 4. The characteristic impedance of

Fig. 11. Simulation results of return loss frequency response for Sol. II.

(a) Circuit schematic

(b) Layout

Fig. 12. Measurement circuit.

Table 4. Modified even- and odd-mode impedances to have a line characteristic impedance of 316.2 Ω.

C	-9 dB	-11 dB	-13 dB	-15 dB
Z_{0e}^m	173.9 Ω	124.1 Ω	91.2 Ω	68.39 Ω
Z_{0o}^m	82.8 Ω	69.5 Ω	57.8 Ω	47.7 Ω

316.2 Ω can be realized without any significant problem; generally, when its even-mode impedances are less than 120 Ω. Since the modified even- and odd-mode impedances were derived under the assumption of pure TEM propagation, the average value of even- and odd-mode phase velocities may be used [22]. In addition,

Table 5. Fabrication data for an impedance transformer.

Substrate($h=0.76$ mm and $\epsilon_r=3.4$)	
$Z_{0e}^m=91.2$ Ω $Z_{0o}^m=57.8$ Ω	$W=0.76$ mm, $s=0.49$ mm, $l=23.9$ mm
$Z_{IPT}=70.71$ Ω	$W=0.895$ mm $l=23.6$ mm
Feeding line: 50 Ω	$W=1.678$ mm

tion, fabrication data of microstrip coupled-line sections are always available with commercial circuit simulation tools like ADS(Advanced Design System), if the modified even- and odd-mode impedances are given. In our case, the coupling power of -13 dB was chosen so that the gap of the microstrip coupled transmission-line sections is about 0.5 mm.

Since the high impedance transmission-line section transforms 1 kΩ into 100 Ω, an additional 90° impedance transformer is needed for the measurements. Fig. 12 gives a detailed circuit schematic and layout for the measurements. Its fabrication data are given in Table 5, where Z_{IPT} is the characteristic impedance of an additional impedance transformer to transform 100 Ω into 50 Ω.

Fig. 13 compares the measured frequency response with a simulated one, where return loss frequency responses are plotted. As demonstrated in Figs. 10 and 11, only one solution exists for the open stub in Fig. 9(a) when the electrical length is 90°. That is, perfect matching can be achieved only at a design center frequency of 2 GHz. The measured matching point is somewhat shifted to a higher center frequency, but the measured result can be accepted, as it has good agreement with the simulated one.

Fig. 13. Results measured and simulated are compared.

IV. Branch-Line Hybrids

As an application of high impedance transmission-line sections, branch-line hybrids with very weak coupling power will be discussed. Fig. 14 shows branch-line hybrids with arbitrary termination impedances R_a , R_b , R_c , and R_d . When power is fed into port ①, two types of branch-line hybrids are available, depending on which port is isolated; port ④ is isolated in Fig. 14(a) and port ② is isolated in Fig. 14(b). For the case that port ④ is isolated in Fig. 14(a), when unity power is fed into port ①, $\tilde{a}$ and $\tilde{i}_a$ waves are generated at the port ① junction. The $\tilde{a}$ wave is divided into $\tilde{b}$ and $\tilde{b}_{ic}$ waves at the port ② junction and only the $\tilde{b}$ wave is transmitted into port ② load R_b . The $\tilde{b}_{ic}$ wave is also divided into $\tilde{c}$ and $\tilde{i}_c$ waves at the port ③ junction and the $\tilde{c}$ wave is delivered to the port ③ load R_c . If it is desired that the branch-line hybrid be perfectly isolated, the two waves at port ④ $\tilde{i}_a$ and $\tilde{i}_c$ should be cancelled and therefore should be 180° out of phase. Thus,

$$\Theta_1 + \Theta_2 + \Theta_3 - \Theta_4 = \pm\pi \quad (18)$$

Fig. 14. Branch-line hybrids with arbitrary termination impedances R_a , R_b , R_c , and R_d .

Fig. 15. Two types of branch-line hybrids.

In addition, if the branch-line hybrid is to be designed for the two waves at ports ② and ③ to have a $\pm 90^\circ$ phase difference, then Θ_2 should be $\pm 90^\circ$. Therefore, two types of branch-line hybrids are available, as described in Fig. 15. One is the case where all of the transmission-line sections are 90° long, as shown in Fig. 15(a). The other case is where two transmission-line sections are 90° long and two others are 270° long, as shown in Fig. 15(b).

If it is assumed that the branch-line hybrid is perfectly isolated, then the sum of $\tilde{i}_a$ and $\tilde{i}_c$ is zero. Thus, one possible case is that the two characteristic impedances Z_3 and Z_4 may be considered as infinite, which results in $|\tilde{a}|=1$ and $|\tilde{b}_{ic}|=|\tilde{c}|$. If the power delivered to port ③ is defined as coupling power C_b^2 , then the power transmitted to port ② is $1 - C_b^2$. For the unity power at port ① to be divided in this way, an input impedance looking into the transmission-line section with Z_2 and Θ_2 in Fig. 14 (a) is $R_b(1 - C_b^2)/C_b^2$, due to the assumed infinite impedances of Z_3 and Z_4 . Therefore, the equivalent circuits for Z_1 - and Z_2 -transmission-line sections in Fig. 14(a) are derived to yield those in Fig. 16, where the transmission-line section with Z_2 transforms a real impedance of $R_b(1 - C_b^2)/C_b^2$ into another real impedance of R_c , and that with Z_1 does the same. Therefore, the design equations for perfect matching at port ① are:

$$Z_1 = \sqrt{1 - C_b^2} \sqrt{R_a R_b} \quad (19a)$$

$$Z_2 = \sqrt{\frac{1 - C_b^2}{C_b^2}} \sqrt{R_b R_c} \quad (19b)$$

The branch-line hybrids are passive and therefore should be satisfied with a unitary property. When power is fed into ports ① and ③, the unitary property is expressed as:

(a) Equivalent circuit for a transmission-line section for Z_1 (b) Equivalent circuit for a transmission-line section for Z_2

Fig. 16. Equivalent circuits with power fed into port ①.

$$|S_{11}|^2 + |S_{21}|^2 + |S_{31}|^2 + |S_{41}|^2 = 1 \quad (20a)$$

$$|S_{13}|^2 + |S_{23}|^2 + |S_{33}|^2 + |S_{43}|^2 = 1 \quad (20b)$$

Since $|S_{21}|^2 = 1 - C_b^2$ and $|S_{31}|^2 = C_b^2$ from the above assumption, $|S_{13}|^2 = C_b^2$ and $|S_{43}|^2 = 1 - C_b^2$. Therefore, when the power is excited at port ③, the same process explained above may be applied and the remaining characteristic impedances are derived as:

$$Z_3 = \sqrt{1 - C_b^2} \sqrt{R_c R_d} \quad (21a)$$

$$Z_4 = \sqrt{\frac{1 - C_b^2}{C_b^2}} \sqrt{R_a R_d} \quad (21b)$$

In (19) and (21), if the coupling power C_b^2 is -3 dB; that is, $C_b^2 = 1/2$ and all the termination impedances are equally Z_0 , then $Z_1 = Z_3 = Z_0/\sqrt{2}$ and $Z_2 = Z_4 = Z_0$, which are well-known design equations of 3-dB branch-line hybrids^{[23],[24]}. The branch-line hybrids have been used as impedance transformers^{[25]~[27]}. If $R_a = R_d = Z_{01}$, $R_b = R_c = Z_{02}$, and $|S_{31}|/|S_{21}| = C_b/\sqrt{1 - C_b^2} = k$, then the design equations in (19) and (21) are the same as those given in [25], where Z_{01} and Z_{02} are input and output impedances. If $|S_{31}|/|S_{21}| = C_b/\sqrt{1 - C_b^2} = d_2/d_1$, then the design equations in (19) and (21) are the same as those in [17].

In the case that port ② is isolated in Fig. 14(b), in a similar way, design equations for the transmission-line sections are derived as:

$$Z_1 = \sqrt{\frac{1 - C_b^2}{C_b^2}} \sqrt{R_a R_b} \quad (22a)$$

$$Z_2 = \sqrt{1 - C_b^2} \sqrt{R_b R_c} \quad (22b)$$

Table 6. Characteristic impedances with all the termination impedances fixed at 50Ω and coupling power varied.

C_b	-5 dB	-10 dB	-15 dB
$Z_1 = Z_3$	41.3Ω	47.4Ω	49.2Ω
$Z_2 = Z_{34}$	73.5Ω	150Ω	276.7Ω

$$Z_3 = \sqrt{\frac{1 - C_b^2}{C_b^2}} \sqrt{R_c R_d} \quad (22c)$$

$$Z_4 = \sqrt{1 - C_b^2} \sqrt{R_d R_a} \quad (22d)$$

If it is assumed that the branch-line hybrid is terminated in equal impedance 50Ω , this results in a symmetric branch-line hybrid with $Z_1 = Z_3$ and $Z_2 = Z_4$. Characteristic impedances of the branch-line hybrid in Fig. 14(a) are calculated depending on coupling power and are written in Table 6, where Z_2 is already 150Ω with -10 dB coupling power and 276.7Ω with -15 dB coupling power.

A branch-line hybrid with coupling power of -10 dB has been simulated at a center frequency of 1 GHz. For this, two identical transmission-line sections having a characteristic impedance of 150Ω are needed and Table 2 gives several sets of modified even- and odd-mode impedances for the characteristic impedance of 150Ω .

When $\Theta = 90^\circ$ in Fig. 9(a), the resulting phase difference of the high impedance transmission-line section is 270° and the branch-line hybrid in Fig. 15(b) is used for this simulation. Therefore the phase difference between two output ports is 270° or -90° . Simulated scattering parameters produced by port ① are plotted in Fig. 17, where solid lines, dotted lines, solid lines with symbols, and dotted lines with symbols indicate those with $(Z_{0e}^m, Z_{0o}^m) = (300 \Omega, 60 \Omega)$, $(150 \Omega, 50 \Omega)$, $(85.7 \Omega, 40 \Omega)$ and $(50 \Omega, 30 \Omega)$, respectively. Referring to Fig. 8, where the relationship between modified even- and odd-mode impedances and coupling coefficient is drawn, the higher even- or odd-mode impedances have higher coupling coefficients. Therefore, the solid lines have the highest coupling power of C^2 , whereas the dotted lines with symbols have lowest. Since the branch-line hybrid was designed with -10 dB coupling power, or, $C_b^2 = -10$ dB, $|S_{31}|$ is -10 dB and $|S_{21}| = -0.457$ dB. The simulated branch-line hybrids are all perfectly matched and isolated at a design center frequency of 1 GHz. The bandwidths are proportional to the even-mode impedance, but all of the cases are acceptable, considering that a typical conventional branch-line hybrid has about a 10% bandwidth.

Fig. 17. Simulation frequency responses contributed by port ①.

4-1 Measurements

A microstrip branch-line hybrid with all the termination impedances $50\ \Omega$ and coupling power $-15\ \text{dB}$ was designed at a center frequency of $2\ \text{GHz}$ and fabricated on a substrate ($\epsilon_r=3.5$, $h=0.76\ \text{mm}$). Since all the termination impedances are equally $50\ \Omega$, the characteristic impedances are $49\ \Omega$ and $276.7\ \Omega$, as given in Table 6. In order to realize the high characteristic impedance of $276.7\ \Omega$, the high impedance transmission-line section shown in Fig. 9(a) can be used. Table 7 gives several sets of modified even- and odd-mode impedances to produce the characteristic impedance of $276.7\ \Omega$ and $Z_{0e}^m = 57\ \Omega$ and $Z_{0o}^m = 40.4\ \Omega$ were chosen in our case. The fabricated microstrip branch-line hybrid is illustrated in Fig. 18 and its fabrication data are summarized in Table 8. Measured and simulated frequency responses are compared in Fig. 19, where solid and dotted lines are measured and simulated frequency responses, respectively. The measured S_{21} and S_{31} are very close to the simulated ones, and matching and isolation are almost perfect, even though the measured center frequency is slightly shifted.

 Table 7. Several sets of modified even- and odd-mode impedances to have $276.7\ \Omega$ characteristic impedance.

C	-9 dB		-15 dB	-15.3 dB	-16 dB
Z_{0e}^m	$152.3\ \Omega$		$59.9\ \Omega$	$57\ \Omega$	$52\ \Omega$
Z_{0o}^m	$72.5\ \Omega$		$41.8\ \Omega$	$40.4\ \Omega$	$37.8\ \Omega$

 Table 8. Fabrication data for a branch-line hybrid with $-15\ \text{dB}$ coupling power.

Substrate($h=0.76\ \text{mm}$ and $\epsilon_r=3.4$)	
$Z_{0e}^m=57\ \Omega$ $Z_{0o}^m=40.4\ \Omega$	$W=1.69\ \text{mm}$, $s=0.5\ \text{mm}$, $l=22.879\ \text{mm}$
$Z_1=Z_3=49.2\ \Omega$	$W=1.72\ \text{mm}$, $l=22.66\ \text{mm}$
Feeding line: $50\ \Omega$	$W=1.678\ \text{mm}$

 Fig. 18. A fabricated microstrip branch-line hybrid from referenes^[28].

Fig. 19. Comparison of measured and simulated results.

V. Conclusion

As more integration with active and passive elements is required, a substrates' dielectric constant should increase. As operating frequency is raised, the substrate should be thinner. Under these conditions, realizable characteristic impedances of microstrip line are restricted.

In this paper, in order to solve this problem, an equivalent transmission-line section was suggested for high characteristic impedances, consisting of a pair of coupled-line sections and two identical open stubs. The high impedance transmission-line sections were perfectly matched at a design center frequency, independently of coupling power and electrical length of the coupled-line sections. Additional matching points were determined depending on which solution for the open stubs was chosen.

With the high impedance transmission-line sections, various potential applications will become a reality. Wider bandwidths of high impedance transmission-line sections will be possible if the two types of solutions for open stubs are combined properly.

References

- [1] H. A. Wheeler, "Transmission-line properties of a strip on a dielectric sheet on a plane", *IEEE Trans. Microwave Theory Tech.*, vol. 25, no. 8, pp. 631-647, Aug. 1977.
- [2] H. A. Wheeler, "Transmission-line properties of parallel strips separated by a dielectric sheet", *IEEE Trans. Microwave Theory Tech.*, vol. 3, no. 3, pp. 172-185, Mar. 1965.
- [3] S. B. Cohn, R. Levy, "History of microwave passive components with particular attention to directional couplers", *IEEE Trans. Microwave Theory Tech.*, vol. MTT-32, pp. 1046-1054, Sep. 1984.
- [4] E. M. T. Hones, J. T. Bolljahn, "Coupled-strip- transmission-line filters and directional couplers", *IRE Trans. Microwave Theory Tech.*, vol. MTT-4, pp. 75-81, Apr. 1956.
- [5] S. B. Cohn, "Parallel-coupled transmission line resonators", *IRE Trans. Microwave Theory Tech.*, vol. MTT-6, pp. 223-231, Apr. 1958.
- [6] G. L. Matthaei, "Interdigital bandpass filters", *IRE Trans. Microwave Theory Tech.*, vol. MTT-10, pp. 479-491, Nov. 1962.
- [7] R. Levy, "General synthesis of asymmetric multi-element coupled transmission line directional couplers", *IEEE Trans. Microwave Theory Tech.*, vol. MTT-11, pp. 227-231, Jul. 1963.
- [8] V. J. Tripathi, "Asymmetric coupled transmission lines in an inhomogeneous medium", *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 734-739, Sep. 1975.
- [9] H.-R. Ahn, B. Kim, "Transmission-line directional couplers for impedance transforming", *IEEE Microwave and Wireless Components Letters*, pp. 537-539, Oct. 2006.
- [10] H.-R. Ahn, B. Kim, "Toward integrated circuit size reduction", *IEEE Microwave Magazine*, pp. 65-75, Feb. 2008.
- [11] S. March, "A wideband stripline hybrid ring", *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 361-362, Jun. 1968.
- [12] L. K. Yeung, K. L. Wu, "A dual-band coupled-line balun filter", *IEEE Trans. Microwave Theory Tech.*, vol. 55, no. 11, pp. 2406-2411, Nov. 2007.
- [13] X. Gao, L. K. Yeung, and K. L. Wu, "A dual-band balun using partially coupled stepped-impedance coupled-line resonators", *IEEE Trans. Microwave Theory Tech.*, vol. 55, no. 11, pp. 1455-1460, Jun. 2008.
- [14] R. Phromlounsri, M. Chongcheawchamnan, and I. D. Robertson, "Inductively compensated parallel coupled microstrip lines and their applications", *IEEE Trans. Microwave Theory Tech.*, vol. 54, no. 9, pp. 3571-3582, Sep. 2006.
- [15] H. Hayasho, T. Nakagawa, and K. Araki, "A miniaturized MMIC analog phase shifter using two quarter-wave-length transmission lines", *IEEE Trans. Microwave Theory Tech.*, vol. 50, no. 1, pp. 150-154, Jan. 2002.
- [16] W. W. Munford, "Directional couplers", *Proc. IRE.*, vol. 35, pp. 159-165, Feb. 1947.
- [17] H.-R. Ahn, I. Wolff, "Asymmetric four-port and branch-line hybrids", *IEEE Trans. Microwave Theory Tech.*, vol. 48, pp. 1585-1588, Sep. 2000.
- [18] D. M. Pozar, *Microwave Engineering*, Addison-Wesley, p. 185, Jun. 1990.
- [19] H.-R. Ahn, *Asymmetric Passive Components in Microwave Integrated Circuits*, John Wiley & Sons INC., p. 65, Aug. 2006.
- [20] R. Mongia, I. Bahl, and P. Bhartia, *RF and Microwave Coupled-Line Circuits*, Boston, Artech House INC., p. 190, 1999.
- [21] J. Helszajn, *Passive and Active Microwave Circuits*, John Wiley & Sons INC., p. 7, 1978.
- [22] D. Kajfez, B. S. Vidula, "Design equations for symmetric microstrip DC blocks", *IEEE MTT*, vol. MTT-28, no. 9, pp. 974-981, Sep. 1980.
- [23] R. G. Brown, R. A. Sharpe, W. L. Hughes, and R. E. Post, *Lines, Waves and Antennas*, Wiley, pp. 124-128, 1973.
- [24] J. A. G. Malherbe, *Microwave Transmission Line Couplers*, Artech, p. 23, 1988.
- [25] R. K. Gupta, S. E. Anderson, and W. Getsinger, "Impedance-transforming 3-dB hybrids", *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, pp. 1303-1307, Dec. 1987.
- [26] S. Kummar, C. Tannous, and T. Danshin, "A multisection broadband impedance transforming branch-

line hybrid", *IEEE Trans. Microwave Theory Tech.*, vol. 43, pp. 2517-2523, Nov. 1995.

- [27] L. F. Lind, "Synthesis of asymmetrical branch-guided directional coupler-impedance transformers", *IEEE Trans. Microwave Theory Tech.*, vol. MTT-17, pp. 45-48, Jan. 1969.

- [28] H. R. Ahn, J. Kim, and B. Kim, "Branch-line hybrids with -15 dB coupling power", in *APMC 2008 CD, B1-3 paper*, Hong Kong, Dec. 2008.

Hee-Ran Ahn

received the B.S., M.S., and Ph.D. degrees in electronic engineering from Sogang University, Seoul, Korea, in 1988, 1990 and 1994, respectively. From 1991 to 1995, she was Part-Time Lecturer at Sogang University, Seoul, Korea and Post-Doctorial Fellow from 1996 to 1997 at Duisburg-Essen University, Duisburg, Germany. From February 1997 to 2002, she was with the Department of Electrical Engineering at Duisburg-Essen University, Duisburg, Germany where she worked for the Habilitation dealing with asymmetric passive components in microwave circuits. She is currently with Dept. of Electronics and Electrical Engineering at POSTECH(Pohang University of Science and Technology), Pohang, Korea. From March 2003 to February 2005, she was with Division of Electrical Engineering, Dept. of EECS(Electrical Engineering and Computer Science) at KAIST(Korea Advanced Institute of Science and Technology), Daejeon, Korea as a Visiting professor. Her interests include high frequency and microwave circuit design and biomedical application using microwave theory and techniques. She authored a book entitled, "Asymmetric passive component in microwave integrated circuits", Wiley, 2006.

Bumman Kim

received the Ph.D. degree in electrical engineering from Carnegie Mellon University, Pittsburgh, PA, in 1979. From 1978 to 1981, he was engaged in fiber-optic network component research with GTE Laboratories Inc. In 1981, he joined the Central Research Laboratories, Texas Instruments Incorporated, where he was involved in development of GaAs power field-effect transistors (FETs) and monolithic microwave integrated circuits(MMICs). He has developed a large-signal model of a power FET, dual-gate FETs for gain control, high-power distributed amplifiers, and various millimeter-wave MMICs. In 1989, he joined the Pohang University of Science and Technology (POSTECH), Pohang, Gyungbuk, Korea, where he is a Namko Professor with the Department of Electrical Engineering, and Director of the Microwave Application Research Center, where he is involved in device and circuit technology for RF integrated circuits(RFICs). He was a Visiting Professor of electrical engineering with the California Institute of Technology, Pasadena, in 2001. He has authored over 200 technical papers. Dr. Kim is a member of the Korean Academy of Science and Technology and the Academy of Engineering of Korea. He was an associate editor for the Ieee Transactions On Microwave Theory And Techniques and a Distinguished Lecturer of the IEEE Microwave Theory and Techniques Society(IEEE MTT-S).