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Pseudo Complex Correlation Coefficient: with Application to Correlated Information Sources for NOMA in 5G systems

Kyuhyuk Chung

Professor, Department of Software Science, Dankook University, Korea
khchung@dankook.ac.kr

Abstract

In this paper, the authors propose the pseudo complex correlation coefficient (PCCC) of the two complex random variables (RV), because the four real correlation coefficients (RCC) of the corresponding four real RVs cannot be obtained only from the complex correlation coefficient (CCC) of given two complex RV. Such observation is motivated by the general statement; "The complex jointly-Gaussian random M-vector cannot be completely described by the complex covariance matrix, even though the real Gaussian random 2M-vector can be completely descried by the real covariance matrix. Therefore, in order to describe completely the complex jointly-Gaussian random M-vector, we need an additional matrix, namely the complex pseudo-covariance matrix, along with the complex covariance matrix."

Then, we apply PCCC to correlated information sources (CIS) for non-orthogonal multiple access (NOMA) in 5G system, and investigate impact of the proposed PCCC on the achievable data rate of the stronger channel user in the conventional successive interference cancellation (SIC) NOMA with CIS. It is shown that for the given same CCC, the achievable data rates with the different PCCC are different, because the corresponding RCC are different. We also show that as the absolute value of the same CCC increases, the impact of the different PCCC becomes more significant.

Keywords: *NOMA, Correlation coefficient, Superposition coding, Successive interference cancellation, Power allocation.*

1. Introduction

In most existing works on the advanced smart convergence, the internet of things (IoT) and the fifth generation (5G) and beyond 5G (B5G) mobile communications have been intensively investigated [1]. One of the promising multiple access (MA) in 5G mobile networks is non-orthogonal multiple access (NOMA) [2-4]. A recent work on NOMA showed that mobile networks can be 1000 faster than orthogonal multiple access (OMA) in the fourth generation (4G) mobile communications, such as long term evolution advanced (LTE-A) [5-7]. By sharing the channel resources, such improvements can be achieved [8]. Due to sharing the resources, the user-fairness should be established between the users [9]. Also, underwater visible light communication

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Corresponding Author: khchung@dankook.ac.kr
Tel: +82-32-8005-3237, Fax: +82-504-203-2043
Professor, Department of Software Science, Dankook University, Korea

(VLC) was studied in NOMA [10]. In NOMA, a multi-antenna base station was investigated [11]. The bit-error rate (BER) was considered as the practical performance measure for NOMA [12]. The impact of local oscillator imperfection for NOMA was considered in [13].

Recently, the authors of this paper investigated correlated information sources (CIS) for NOMA networks [14]. However, in [14], CIS were described only by complex correlation coefficients (CCC). Such description is not complete for a given CIS. Therefore, in order to describe CIS completely, we propose in this paper the pseudo-complex correlation coefficients (PCCC), along with CCC.

The remainder of this paper is organized as follows. In Section 2, the system and channel model are described. The preliminaries for the pseudo covariance matrix is presented in Section 3. We propose PCCC formally in Section 4. The numerical results are presented and discussed in Section 5. Finally, the conclusions are presented in Section 6.

The main contributions of this paper is summarized as follows:

- We propose the novel PCCC, in order to describe completely CIS in NOMA.
- We formally derive PCCC, along with CCC, to obtain the four corresponding real correlation coefficients (RCC).
- It is shown by numerically that even though CCC is the same for two cases, the achievable data rate of NOMA with CIS can be different for two cases.
- Furthermore, the absolute values of CCC increases, the role of PCCC becomes more significant.

2. System and Channel Model

In a cellular downlink NOMA transmission system, all the users are assumed to be experiencing block fading, in the narrow band system. For wideband systems, orthogonal frequency division multiplexing (OFDM) can transform a fast fading channel into slow fading ones. A base station and M users are within the cell. The complex channel coefficient between the *m*th user and the base station is denoted by h_m. The channels are sorted as $|h_1| \geq \cdots \geq |h_M|$. The base station sends the superimposed signal $z = \sum_{m=1}^{M} \sqrt{\beta_m P_A} c_m$, where c_m is the message for the *m*th user, β_m is the power allocation coefficient, with $\sum_{m=1}^{M} \beta_m = 1$, and P_A is the average total allocated power. The power of the message c_m for the *m*th user is normalized as unit power, denoted by $\sigma_m^2 = \rho_{m,m} = \mathbb{E}\left[c_m c_m^*\right] = \mathbb{E}\left[\left|c_m\right|^2\right] = 1$, with $\sigma_{m,r}^2 = \rho_{m,r,m,r} = \mathbb{E}\left[\mathrm{Re}\{c_1\}^2\right] = 1/2$ and $\sigma_{m,i}^2 = \rho_{m,i,m,i} = \mathbb{E}\left[\mathrm{Im}\{c_1\}^2\right] = 1/2$, $\forall m,\ 1 \leq m \leq M$. The CCC between the *m*th user and the *n*th user is denoted by $\rho_{m,n} = \mathbb{E}\left[c_m c_n^*\right]$, with RCC $\rho_{m,r,n,r} = \mathbb{E}\left[\mathrm{Re}\{c_m\}\mathrm{Re}\{c_n\}\right]$, $\rho_{m,i,n,i} = \mathbb{E}\left[\mathrm{Im}\{c_m\}\mathrm{Im}\{c_n\}\right]$, $\rho_{m,r,n,i} = \mathbb{E}\left[\mathrm{Re}\{c_m\}\mathrm{Im}\{c_n\}\right]$, and $\rho_{m,i,n,r} = \mathbb{E}\left[\mathrm{Im}\{c_m\}\mathrm{Re}\{c_n\}\right]$, $\forall m,n,\ m \neq n,\ 1 \leq m,n \leq M$. Due to the correlation, the power of the superimposed signal z is larger than P_A. Thus, given the constant total transmitted power P at the base station, P_A is effectively scaled down [14]

$$P_A = \frac{P}{\sum_{i=1}^{M} \sum_{j=1}^{M} \rho_{i,j} \sqrt{\beta_i \beta_j}}. \tag{1}$$

The observation at the *m*th user is given by

$$y_m = h_m z + n_m, \tag{2}$$

where n_m is complex additive white Gaussian noise (AWGN) at the *m*th user, $n_m \sim CN(0,\sigma^2)$.

3. Preliminaries for Pseudo Covariance Matrix

Let C be a random complex *M*-vector for the *M* users' signals of correlated information sources

$$C = \left[\sqrt{P_A\beta_1}c_1, \sqrt{P_A\beta_2}c_2, \cdots, \sqrt{P_A\beta_M}c_M\right]^{\mathrm{T}}, \tag{3}$$

and let the corresponding real random 2*M*-vector D consist of the real and imaginary components of C, taken in this order

$$D = \left[\mathrm{Re}\{\sqrt{P_A\beta_1}c_1\}, \mathrm{Re}\{\sqrt{P_A\beta_2}c_2\}, \cdots, \mathrm{Re}\{\sqrt{P_A\beta_M}c_M\}, \mathrm{Im}\{\sqrt{P_A\beta_1}c_1\}, \mathrm{Im}\{\sqrt{P_A\beta_2}c_2\}, \cdots, \mathrm{Im}\{\sqrt{P_A\beta_M}c_M\}\right]^{\mathrm{T}}. \tag{4}$$

Note that the superimposed signal is given by

$$\begin{aligned} z &= \sqrt{P_A\beta_1}c_1 + \sqrt{P_A\beta_2}c_2 + \cdots + \sqrt{P_A\beta_M}c_M \\ &= \mathrm{Re}\{\sqrt{P_A\beta_1}c_1\} + \mathrm{Re}\{\sqrt{P_A\beta_2}c_2\} + \cdots + \mathrm{Re}\{\sqrt{P_A\beta_M}c_M\} \\ &+ j\left(\mathrm{Im}\{\sqrt{P_A\beta_1}c_1\} + \mathrm{Im}\{c_2\sqrt{P_A\beta_2}\} + \cdots + \mathrm{Im}\{\sqrt{P_A\beta_M}c_M\}\right). \end{aligned} \tag{5}$$

Then, the covariance matrix K_C of the jointly-Gaussian complex random *M*-vector C is given by

$$\mathrm{K}_C = P_A \begin{bmatrix} \beta_1 & \sqrt{\beta_1}\sqrt{\beta_2}\rho_{1,2} & \cdots & \sqrt{\beta_1}\sqrt{\beta_M}\rho_{1,M} \\ \sqrt{\beta_2}\sqrt{\beta_1}\rho_{2,1} & \beta_2 & \cdots & \sqrt{\beta_2}\sqrt{\beta_M}\rho_{2,M} \\ \vdots & & \ddots & \vdots \\ \sqrt{\beta_M}\sqrt{\beta_1}\rho_{M,1} & \sqrt{\beta_M}\sqrt{\beta_2}\rho_{M,2} & \cdots & \beta_M \end{bmatrix}, \tag{6}$$

and the covariance matrix K_D of the corresponding real random 2*M*-vector D is given by

$$\mathrm{K}_D = P_A \left[\begin{array}{c:c} \mathrm{K}_{D,r,r} & \mathrm{K}_{D,r,i} \\ \hdashline \mathrm{K}_{D,i,r} & \mathrm{K}_{D,i,i} \end{array}\right], \tag{7}$$

where

$$\mathrm{K}_{D,r,r}=\begin{bmatrix}\beta_1 0.5 & \sqrt{\beta_1}\sqrt{\beta_2}0.5\rho_{1,r,2,r} & \cdots & \sqrt{\beta_1}\sqrt{\beta_M}0.5\rho_{1,r,M,r}\\ \sqrt{\beta_2}\sqrt{\beta_1}0.5\rho_{2,r,1,r} & \beta_2 0.5 & \cdots & \sqrt{\beta_2}\sqrt{\beta_M}0.5\rho_{2,r,M,r}\\ \vdots & & \ddots & \vdots\\ \sqrt{\beta_M}\sqrt{\beta_1}0.5\rho_{M,r,1,r} & \sqrt{\beta_M}\sqrt{\beta_2}0.5\rho_{M,r,2,r} & \cdots & \beta_M 0.5\end{bmatrix}, \quad (8)$$

$$\mathrm{K}_{D,r,i}=\begin{bmatrix}\beta_1 0.5\rho_{1,r,1,i} & \sqrt{\beta_1}\sqrt{\beta_2}0.5\rho_{1,r,2,i} & \cdots & \sqrt{\beta_1}\sqrt{\beta_M}0.5\rho_{1,r,M,i}\\ \sqrt{\beta_2}\sqrt{\beta_1}0.5\rho_{2,r,1,i} & \beta_2 0.5\rho_{2,r,2,i} & \cdots & \sqrt{\beta_2}\sqrt{\beta_M}0.5\rho_{2,r,M,i}\\ \vdots & & \ddots & \vdots\\ \sqrt{\beta_M}\sqrt{\beta_1}0.5\rho_{M,r,1,i} & \sqrt{\beta_M}\sqrt{\beta_2}0.5\rho_{M,r,2,i} & \cdots & \beta_M 0.5\rho_{M,r,M,i}\end{bmatrix}, \quad (9)$$

$$\mathrm{K}_{D,i,r}=\begin{bmatrix}\beta_1 0.5\rho_{1,i,1,r} & \sqrt{\beta_1}\sqrt{\beta_2}0.5\rho_{1,i,2,r} & \cdots & \sqrt{\beta_1}\sqrt{\beta_M}0.5\rho_{1,i,M,r}\\ \sqrt{\beta_2}\sqrt{\beta_1}0.5\rho_{2,i,1,r} & \beta_2 0.5\rho_{1,i,2,r}\rho_{2,i,2,r} & \cdots & \sqrt{\beta_2}\sqrt{\beta_M}0.5\rho_{2,i,M,r}\\ \vdots & & \ddots & \vdots\\ \sqrt{\beta_M}\sqrt{\beta_1}0.5\rho_{M,i,1,r} & \sqrt{\beta_M}\sqrt{\beta_2}0.5\rho_{M,i,2,r} & \cdots & \beta_M 0.5\rho_{M,i,M,r}\end{bmatrix}, \quad (10)$$

and

$$\mathrm{K}_{D,i,i}=\begin{bmatrix}\beta_1 0.5 & \sqrt{\beta_1}\sqrt{\beta_2}0.5\rho_{1,i,2,i} & \cdots & \sqrt{\beta_1}\sqrt{\beta_M}0.5\rho_{1,i,M,i}\\ \sqrt{\beta_2}\sqrt{\beta_1}0.5\rho_{2,i,1,i} & \beta_2\sigma_{2,i}^2 & \cdots & \sqrt{\beta_2}\sqrt{\beta_M}0.5\rho_{2,i,M,i}\\ \vdots & & \ddots & \vdots\\ \sqrt{\beta_M}\sqrt{\beta_1}0.5\rho_{M,i,1,i} & \sqrt{\beta_M}\sqrt{\beta_2}0.5\rho_{M,i,2,i} & \cdots & \beta_M\sigma_{M,i}^2\end{bmatrix}. \quad (11)$$

where we use the assumption in Section 2, i.e., $\sigma_{m,r}^2=\rho_{m,r,m,r}=\mathbb{E}\left[\mathrm{Re}\{c_1\}^2\right]=1/2$ and $\sigma_{m,i}^2=\rho_{m,i,m,i}=\mathbb{E}\left[\mathrm{Im}\{c_1\}^2\right]=1/2$, $\forall m,\ 1\le m\le M$.

The complex jointly-Gaussian random M-vector C cannot be completely described only by the covariance matrix K_C, even though the real Gaussian random $2M$-vector D can be completely descried only by the covariance matrix K_D. Therefore, in order to describe completely C, we need an additional matrix, namely the pseudo-covariance matrix M_C, which is given by

$$\mathrm{M}_C=P_A\begin{bmatrix}\beta_1\mathbb{E}[c_1c_1] & \sqrt{\beta_1}\sqrt{\beta_2}\mathbb{E}[c_1c_2] & \cdots & \sqrt{\beta_1}\sqrt{\beta_M}\mathbb{E}[c_1c_M]\\ \sqrt{\beta_2}\sqrt{\beta_1}\mathbb{E}[c_2c_1] & \beta_2\mathbb{E}[c_2c_2] & \cdots & \sqrt{\beta_2}\sqrt{\beta_M}\mathbb{E}[c_2c_M]\\ \vdots & & \ddots & \vdots\\ \sqrt{\beta_M}\sqrt{\beta_1}\mathbb{E}[c_Mc_1] & \sqrt{\beta_M}\sqrt{\beta_2}\mathbb{E}[c_Mc_2] & \cdots & \beta_M\mathbb{E}[c_Mc_M]\end{bmatrix}. \quad (12)$$

Then, we can represent K_D, in terms of K_C and M_C,

$$\mathrm{K}_D = \left[\begin{array}{c:c} \frac{1}{2}[\mathrm{Re}\{\mathrm{K}_C\} + \mathrm{Re}\{\mathrm{M}_C\}] & \frac{1}{2}[-\mathrm{Im}\{\mathrm{K}_C\} + \mathrm{Im}\{\mathrm{M}_C\}] \\ \hdashline \frac{1}{2}[\mathrm{Im}\{\mathrm{K}_C\} + \mathrm{Im}\{\mathrm{M}_C\}] & \frac{1}{2}[\mathrm{Re}\{\mathrm{K}_C\} - \mathrm{Re}\{\mathrm{M}_C\}] \end{array}\right]. \tag{13}$$

4. Pseudo Complex Correlation Coefficient

In this section, we start the summary of the previous section:

"The complex jointly-Gaussian random *M*-vector C cannot be completely described by the covariance matrix K_C, even though the real Gaussian random 2*M*-vector D can be completely descried by the covariance matrix K_D. Therefore, in order to describe completely C, we need an additional matrix, namely the pseudo-covariance matrix M_C, along with K_C."

We apply such same logic to CCC as follow:

"The $({}_{2M}C_2 - M)$ corresponding RCC $\{\rho_{m,r,n,r}, \rho_{m,i,n,i}, \rho_{m,r,n,i}, \rho_{m,i,n,r}\}$ cannot be obtained from the ${}_{M}C_2$ CCC $\rho_{m,n}$. Thus, in order to obtain the set of $({}_{2M}C_2 - M)$ RCC, we need an additional set of ${}_{M}C_2$ new complex correlation coefficients, called the "ps"eudo complex correlation coefficients (PCCC), defined by $\psi_{m,n} = \mathbb{E}[c_m c_n]$, along with a set of ${}_{M}C_2$ CCC $\rho_{m,n}$, where $\forall m,n,\ 1 \le m,n \le M$, with $\psi_{m,m} = \mathbb{E}[c_m c_m] = j$. (The Greek letter ψ stands for "ps" in English.)" The existing $\rho_{m,n}$ is represented by, in terms of $\{\rho_{m,r,n,r}, \rho_{m,i,n,i}, \rho_{m,r,n,i}, \rho_{m,i,n,r}\}$,

$$\rho_{m,n} = \frac{1}{2}\rho_{m,r,m,r} + \frac{1}{2}\rho_{m,i,n,i} + j\left(-\frac{1}{2}\rho_{m,r,n,i} + \frac{1}{2}\rho_{m,i,n,r}\right), \tag{14}$$

whereas the new $\psi_{m,n}$ is represented by, in terms of $\{\rho_{m,r,n,r}, \rho_{m,i,n,i}, \rho_{m,r,n,i}, \rho_{m,i,n,r}\}$,

$$\psi_{m,n} = \frac{1}{2}\rho_{m,r,m,r} - \frac{1}{2}\rho_{m,i,n,i} + j\left(+\frac{1}{2}\rho_{m,r,n,i} + \frac{1}{2}\rho_{m,i,n,r}\right). \tag{15}$$

Thus, now the $({}_{2M}C_2 - M)$ corresponding RCC $\{\rho_{m,r,n,r}, \rho_{m,i,n,i}, \rho_{m,r,n,i}, \rho_{m,i,n,r}\}$ can be obtained from the ${}_{M}C_2$ CCC $\rho_{m,n}$, along with the ${}_{M}C_2$ PCCC $\psi_{m,n}$,

$$\begin{aligned} \rho_{m,r,n,r} &= \mathrm{Re}\{\rho_{m,n}\} + \mathrm{Re}\{\psi_{m,n}\}, \\ \rho_{m,i,n,i} &= \mathrm{Re}\{\rho_{m,n}\} - \mathrm{Re}\{\psi_{m,n}\}, \\ \rho_{m,r,n,i} &= -\mathrm{Im}\{\rho_{m,n}\} + \mathrm{Im}\{\psi_{m,n}\}, \\ \rho_{m,i,n,r} &= \mathrm{Im}\{\rho_{m,n}\} + \mathrm{Im}\{\psi_{m,n}\}. \end{aligned} \tag{16}$$

Then, M_C in the equation (12) now can be written compactly by, with the proposed $\psi_{m,n}$,

$$\mathrm{M}_C = P_A \begin{bmatrix} \beta_1 j & \sqrt{\beta_1}\sqrt{\beta_2}\psi_{1,2} & \cdots & \sqrt{\beta_1}\sqrt{\beta_M}\psi_{1,M} \\ \sqrt{\beta_2}\sqrt{\beta_1}\psi_{2,1} & \beta_2 j & \cdots & \sqrt{\beta_2}\sqrt{\beta_M}\psi_{2,M} \\ \vdots & & \ddots & \vdots \\ \sqrt{\beta_M}\sqrt{\beta_1}\psi_{M,1} & \sqrt{\beta_M}\sqrt{\beta_2}\psi_{M,2} & \cdots & \beta_M j \end{bmatrix}. \tag{17}$$

5. Numerical Results and Discussions

The constant total transmitted signal-to-noise power ratio (SNR) is $P/\sigma^2 = 50$, and the channel gain $|h_1|$ is assumed to be $\sqrt{2}$. Without loss of generality, we assume the two-user NOMA with $m=1$ and $n=2$.

We consider two cases with the same CCC $\rho_{m,n}$ of the different PCCCs $\psi_{m,n}$. Such cases imply that $\rho_{m,r,n,r}$ and $\rho_{m,i,n,i}$ are different for each case as follows,

$$\begin{aligned} &\textit{case}\ \#1:\ \rho_{1,2} = \sqrt{1/2 \times 25/100}, \quad \psi_{1,2} = 0, \quad \rho_{1,r,2,r} = \sqrt{1/2 \times 25/100}, \quad \rho_{1,i,2,i} = \sqrt{1/2 \times 25/100}, \\ &\textit{case}\ \#2:\ \rho_{1,2} = \sqrt{1/2 \times 25/100}, \quad \psi_{1,2} = \sqrt{1/2 \times 25/100}, \quad \rho_{1,r,2,r} = 2 \times \sqrt{1/2 \times 25/100}, \quad \rho_{1,i,2,i} = 0, \end{aligned} \tag{18}$$

where we assume that $\rho_{m,r,n,i} = 0$ and $\rho_{m,i,n,r} = 0$, without loss of generality. It should be noted that the case for $\psi_{1,2} = 0$ corresponds the conventional scheme in [14]. Then, the achievable data rate of the stronger channel gain user is given by

$$R_1^{(\mathrm{SIC;\,CIS})} = \frac{1}{2}\log_2\left(\frac{|h_1|^2 \frac{P_A\beta_1}{2}\left(1-|\rho_{1,r,2,r}|^2\right) + \sigma^2/2}{\sigma^2/2}\right) + \frac{1}{2}\log_2\left(\frac{|h_1|^2 \frac{P_A\beta_1}{2}\left(1-|\rho_{1,i,2,i}|^2\right) + \sigma^2/2}{\sigma^2/2}\right). \tag{19}$$

It should be noted that for case #1, $R_1^{(\mathrm{SIC;\,CIS})}$ is simplified as [14]

$$R_1^{(\mathrm{SIC;\,CIS})} = \log_2\left(\frac{|h_1|^2 \frac{P_A\beta_1}{2}\left(1-|\rho_{1,r,2,r}|^2\right) + \sigma^2/2}{\sigma^2/2}\right) = \log_2\left(\frac{|h_1|^2 P_A\beta_1\left(1-|\rho_{1,2}|^2\right) + \sigma^2}{\sigma^2}\right). \tag{20}$$

However, for case #2, such simplification is not possible. In Fig. 1, we depict $R_1^{(\mathrm{SIC;\,CIS})}$, for two cases.

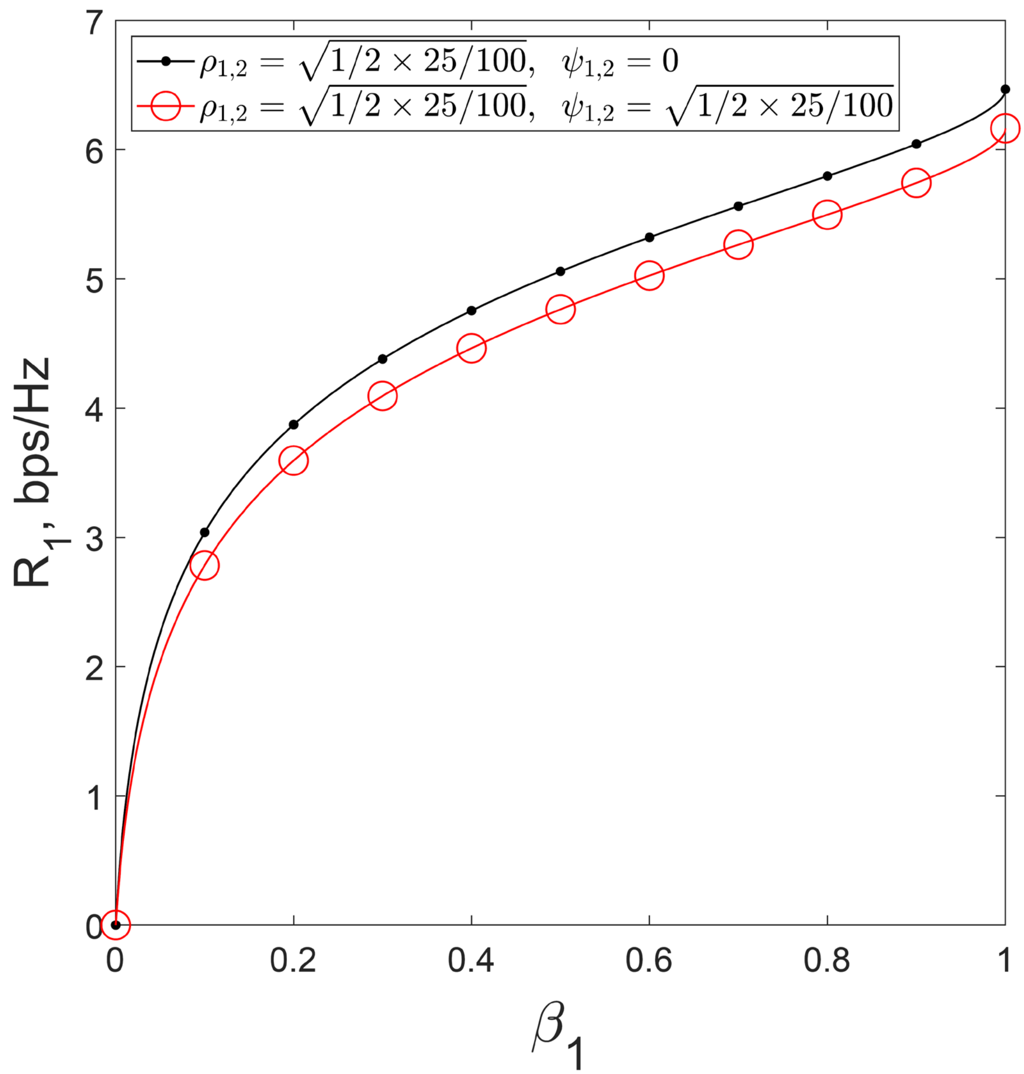

Figure 1. Comparison of $R_1^{(\mathrm{SIC;\,CIS})}$ for two cases.

As shown in Fig. 1, $R_1^{(\mathrm{SIC;\,CIS})}$ for case #2 is smaller than $R_1^{(\mathrm{SIC;\,CIS})}$ for case #1. The main point of this numerical result in Fig. 1 is that even though the CCC $\rho_{m,n}$ is the same for two cases, $R_1^{(\mathrm{SIC;\,CIS})}$ can be different for two cases. Therefore, in order to describe completely CIS in NOMA, we need PCCC $\psi_{m,n}$, along with $\rho_{m,n}$.

Then, in Fig. 2, we investigate the impact of the increased absolute value of CCC $\rho_{m,n}$ on $R_1^{(\mathrm{SIC;\,CIS})}$, when $\rho_{m,n}$, $\psi_{m,n}$, $\rho_{m,r,n,r}$ and $\rho_{m,i,n,i}$ are changed as follows,

$$\begin{aligned} &case\,\#1:\ \rho_{1,2} = \sqrt{1/2 \times 49/100}, \quad \psi_{1,2} = 0, \quad \rho_{1,r,2,r} = \sqrt{1/2 \times 49/100}, \quad \rho_{1,i,2,i} = \sqrt{1/2 \times 49/100}, \\ &case\,\#2:\ \rho_{1,2} = \sqrt{1/2 \times 49/100}, \quad \psi_{1,2} = \sqrt{1/2 \times 49/100}, \quad \rho_{1,r,2,r} = 2 \times \sqrt{1/2 \times 49/100}, \quad \rho_{1,i,2,i} = 0. \end{aligned} \tag{21}$$

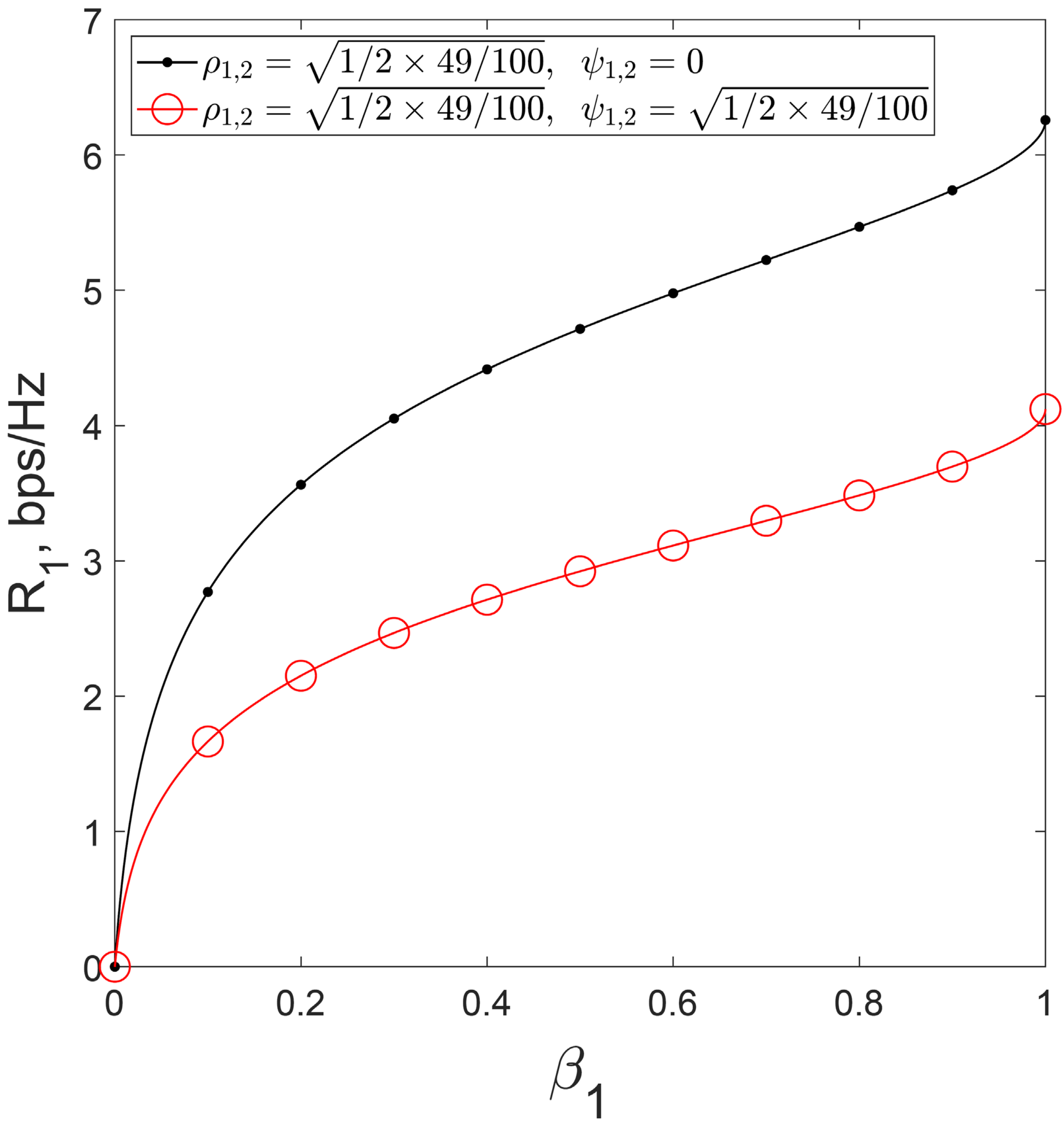

Figure 2. Comparison of $R_1^{(\mathrm{SIC;\,CIS})}$ for two cases.

As shown in Fig. 2, $R_1^{(\mathrm{SIC;\,CIS})}$ for case #2 is much smaller than $R_1^{(\mathrm{SIC;\,CIS})}$ for case #1. The main point of this numerical result in Fig. 2 is that as the absolute value of CCC $\rho_{m,n}$ increases, the role of PCCC $\psi_{m,n}$ becomes more significant.

6. Conclusion

In this paper, we proposed PCCC, in order to describe completely CIS in NOMA. We first observed he general statement: ″The complex jointly-Gaussian random M-vector cannot be completely described by the complex covariance matrix, even though the real Gaussian random $2M$-vector can be completely descried by the real covariance matrix. Therefore, in order to describe completely the complex jointly-Gaussian random M-vector, we need an additional matrix, namely the complex pseudo-covariance matrix, along with the complex covariance matrix.” Based on such general statement, we formally derived PCCC, along with CCC, to obtain the four corresponding RCC.

Then, we investigated the impact of PCCC on the achievable data rate of NOMA with CIS, in 5G networks. It was shown by numerically that even though the CCC $\rho_{m,n}$ is the same for two cases, $R_1^{(\mathrm{SIC;\,CIS})}$ can be different for two cases. Therefore, in order to describe completely CIS in NOMA, we need PCCC $\psi_{m,n}$, along with $\rho_{m,n}$. We also showed that the absolute values of CCC $\rho_{m,n}$ increases, the role of PCCC $\psi_{m,n}$ becomes more significant.

In result, in order to describe CIS completely in NOMA, the information of PCCC $\psi_{m,n}$ should be provided, along with $\rho_{m,n}$. Otherwise, the results might be misleading.

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