

# Conceptual understanding of ubiquitous superconductivity

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(Received 15 November 2020; revised or reviewed 11 December 2020; accepted 12 December 2020)

## Abstract

Since the discovery of superconductivity, the unique and mysterious phenomenon has been observed in various metallic material systems. Now days, the superconductivity becomes ubiquitous because almost every metallic material system shows the superconductivity when it is cooled down enough. This ubiquity of the superconductivity is associated with the fermionic nature and itinerancy of electrons in metallic materials. Because fermions are governed by the Pauli's exclusion principle the total energy of fermions is much larger than that of bosons. Therefore, fermionic itinerant electrons are fundamentally instable. Itinerant electrons are able to find "a way" to lead them to their lowest possible energy state through an available bosonization (or pairing) process and Bose-Einstein condensation. Therefore, the lowest possible energy state of itinerant electrons will be a superconducting state, which is "their ultimate destination". This may explain the reason why the superconductivity is ubiquitous.

*Keywords:* superconductivity, itinerant electrons, cooper pairs, electron-phonon interaction, bosonization, retarded interactions

## 1. INTRODUCTION

The superconductivity was first discovered in pure Hg by Heike Kamerlingh Onnes in 1911 [1]. At that time, this new and extraordinary phenomenon was a real surprise. The superconductivity has two distinctive characteristics: perfect conductivity and perfect diamagnetism [2, 3]. However, since the first discovery of superconductivity the unique phenomenon has been observed in most metals and alloys when they were cooled down enough. Up to now, superconductivity has been observed in a number of metallic material systems including pure elements, alloys, heavy fermion systems, MgB<sub>2</sub>, doped copper-oxides, organic compounds, iron-pnictides, iron-chalcogenides, twisted bilayer graphene, topological materials, transition metal dichalcogenides, hydrides under high-pressure, and so on and on [4]. In current days, the superconductivity is a ubiquitous phenomenon which appears in various metallic material systems [5] as we mentioned above. In loosely speaking, almost every metallic material system shows the superconductivity when it is cooled down enough. This may indicate that the superconductivity is a (global) stable equilibrium state in the energy landscape of the metallic materials. In other words, the ultimate destination of itinerant electrons in a metallic material system is the superconducting state.

## 2. ABILITY OF ITINERANT ELECTRONS TO FIND THE LOWEST POSSIBLE ENERGY STATE

Here we are dealing with itinerant electrons. Itinerant electrons may have the ability to find *the way*, which leads

them to the lowest possible energy state. For example, if one introduces electrons on a neutral metal sphere the electrons will move around and eventually settle down to the lowest possible energy state. As shown in Fig.1, when one electron is added in the neutral metal sphere the electron can be anywhere in the sphere because the sphere is equipotential. When another electron is added in the sphere the two electrons will be located on the surface in order to be apart at the farthest distance from each other for getting the lowest electric potential energy. When another is added the three electrons will find their lowest energy states, and so on. This system is a kind of *a natural computer*. A simple analogy of this phenomenon can be that liquid water always flows to the lowest possible place in the geological landscape to minimize the gravitational potential energy resulting in seas or lakes. Here we deal with a charged conductor, which contains intrinsic itinerant electrons, bound electrons, and additionally introduced electrons. We discuss about the behavior of the additionally introduced electrons, which can freely move around in the conducting (or equipotential) sphere. Before the introduction of additional electrons, the neutral metal sphere is in its own equilibrium state. We note that the bound electrons in a material system are governed by the Pauli's exclusion principle but do not have the degrees of freedom of itinerancy. Metallic materials contain the intrinsic itinerant electrons or have finite density of states at the Fermi level. The itinerant electrons will find the route to lead the material system to the lowest possible energy state in the energy landscape. In the following sections, we focus on the intrinsic itinerant electrons in metallic systems and discuss on why they are unstable in a quantum mechanical point of view and how they find the lowest possible energy state, which is the superconducting state, using their ability

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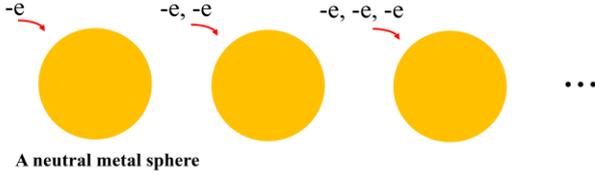


Fig. 1. Introduction electrons to a neutral metal sphere by adding one by one. The introduced electrons will move around and find the most stable state, where the total energy of the system is lowest.

discussed in this section. Therefore, existence of intrinsic itinerant electrons (or metallic material systems) is a minimal requirement for superconductivity.

### 3. UBIQUITY OF SUPERCONDUCTIVITY: FERMIONS AND BOSONS

The superconductivity occurs when two itinerant electrons are paired to acquire the bosonic character and the paired electrons are condensed by the Bose-Einstein condensation [6]. Since electrons are fermions one electron occupies its own quantum state, which is known as the Pauli's exclusion principle [7]. In other words, more than two electrons cannot simultaneously occupy a same quantum state. In contrast, bosons can simultaneously take a same quantum state. Due to these completely different properties (or constraints) between two classes of quantum particles (fermions and bosons) they show fundamentally different occupation statistics, which have been known as the Fermi-Dirac distribution [8] for fermions and Bose-Einstein distribution [9] for bosons. Consequently, they have completely different ground state energies; the total energy of bosons in the ground state is much smaller than that of fermions for the same number of particles (see Fig. 2). Therefore, in a sense, fermions are *fundamentally excited (or unstable)*; fermions may have innate tendency to find a bosonic ground energy state.

However, of course, the exclusion principle, which is the unavoidable constraint, does not allow electrons to be relaxed to the bosonic ground energy state. If there exists effective attractive interaction to make itinerant electrons to be paired, then the electrons will take the opportunity, discard their fermionic character, and acquire bosonic one. Free electrons in vacuum cannot have such an opportunity because they are under only the repulsive Coulomb interaction. However, itinerant electrons in a material system can be under additional interactions caused by their environment (or background) in the material system. The electrons under such additional interactions are called *quasiparticles*, which can be described by the quantum many-body (or self-energy) formalism [10, 11]. If the resulting interaction is effectively attractive the itinerant electrons may take the advantage to lower their energy by being paired and acquiring the bosonic character. Eventually, the bosonized (or paired) electrons will be condensed into a single macroscopic ground state through the Bose-Einstein condensation [6], which is the bosonic ground energy state. We note that the Bose-Einstein

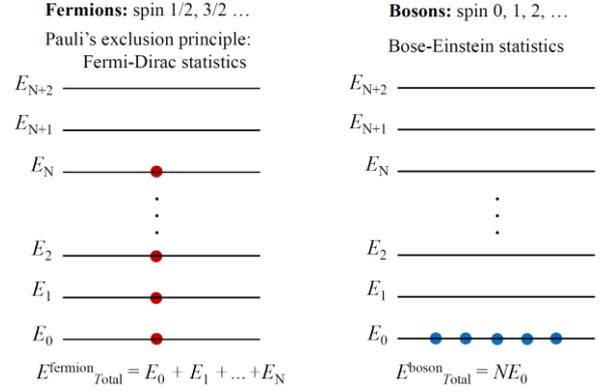


Fig 2. Total energies of two  $N$ -particle systems with fermions and bosons. In the ground state, the total energy of  $N$  fermions is  $E^{\text{fermion}}_{\text{Total}} = E_0 + E_1 + \dots + E_N$  while that of  $N$  bosons is  $E^{\text{boson}}_{\text{Total}} = NE_0$ . In general,  $E^{\text{fermion}}_{\text{Total}} \gg E^{\text{boson}}_{\text{Total}}$ .

condensation is one of the two (bosonization and condensation) processes from itinerant electrons to the superconducting state. The total energy of the paired-and-condensed electrons (or effective bosons) must be lower than that of unpaired electrons (Fig. 2). Therefore, the paired-and-condensed state should be more stable than the unpaired (normal) state. Itinerant electrons in any metallic material systems will follow the innate tendency to find the way, which leads them to the bosonic ground energy state. Since paired-and-condensed electrons have twice the fundamental charge (for each pair) and zero resistivity (or superfluidity), which comes from the Bose-Einstein condensation, they show superconductivity. The Meissner effect appears due to the superfluidity [12]. This new ground state formed by the paired-and-condensed electrons is called the superconducting state [1, 3]. The ubiquity of superconductivity is conceptually understood with this point of view.

### 4. SUPERCONDUCTING MECHANISM

Furthermore, figuring out the microscopic mechanism how the itinerant electrons are paired is a very important issue in the superconductivity society. The microscopic pairing mechanism may be intimately associated with the material system itself; the microscopic pairing mechanism may be different from system to system. However, some of them may share a same pairing mechanism. The microscopic origin of the pairing mechanism in a superconducting system is one of the most important research topics in contemporary condensed matter physics [13-17]. Strong instantaneous Coulomb repulsive interactions always exist between itinerant electrons in any metallic material systems. In general, the repulsive interaction prevents electrons from getting close to form pairs. For formation of electron pairs, attractive interactions between electrons should overcome the repulsive Coulomb interaction; the resulting interaction must be effectively attractive. Therefore, to understand the superconductivity in a system, one has to find out what

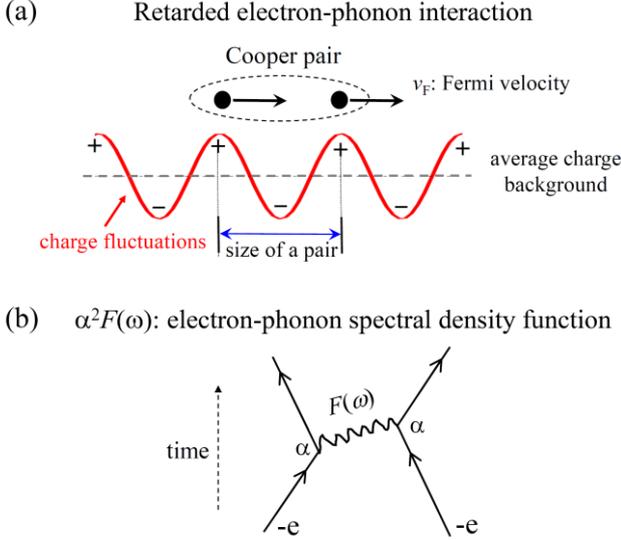


Fig. 3. (a) Retarded electron-phonon interaction through charge fluctuations produced by phonons. The charge fluctuation is oscillating with an average phonon frequency. The distance between electrons in a Cooper pair is directly related to the spatial (or temporal) period of the charge fluctuations above the average charge background. The spatial and temporal periods are related to each other by the Fermi velocity. (b) The electron-phonon spectral density function, which shows interacting electrons by exchanging the force-mediated phonons through a retarded interaction.

interactions cause the effective attraction between itinerant electrons in the system of interest.

For example, for the conventional superconductors, the Bardeen-Cooper-Schrieffer (BCS) theory [18] describes that the electron-phonon (or lattice vibrational quantum) interaction plays the role of the attractive force, which is known as a retarded interaction. The retarded attractive interaction may be more efficient to form electron-electron pairs compared with the instantaneous attractive one.

Instantaneous attractive interactions should be larger than the Coulomb repulsive interaction for resulting in an effectively attractive force at every moment while the retarded attractive interaction does not have to be dominant to the Coulomb repulsive one for the effective attraction at every moment. Experimentally, observation of isotope effects on the superconducting transition temperature is a smoking gun evidence for the phonon-mediated mechanism [19]. The distance between electrons in a Cooper pair is significantly (up to several hundred times) large compared with the average distance between itinerant electrons in conventional superconducting systems; the distance between electrons in the pair is roughly in the order of the coherence length of the superconductivity.

Therefore, the coherency between two electrons in a pair is essentially important for the pairing formation; the coherency is intimately related to both motion of electrons and lattice vibration. In general, the distance is inversely proportional to the strength of superconductivity or the superconducting gap [3]. In other words, the distance can be a measure of the strength of superconductivity; the shorter distance results in the stronger superconductivity.

The shorter distance requires the stronger attractive interaction since the attractive interaction is against the Coulomb repulsive one, which follows the inverse square law of force; electrons in a pair with the smaller size experience the stronger Coulomb repulsive force (or the larger Coulomb potential energy). The phonon frequency is closely associated with the size of the pair; the higher phonon frequency results in the smaller size of electron pairs and the larger retarded attractive interaction. In Fig. 3a we also display a schematic diagram which shows formation of a Cooper pair by the charge fluctuation with an average frequency above the average charge background. The Fermi velocity is another important parameter for determining the size of the Cooper pair; the fast Fermi velocity allows the large Cooper pair [3]. In Fig. 3b we show the Feynman diagram of the electron-phonon spectral density function ( $\alpha^2 F(\omega)$ ), which is known as a glue function. Here  $\alpha$  is the coupling constant between electron and phonon and  $F(\omega)$  is the phonon spectrum. One can get the superconducting transition temperature from the glue function using the McMillan formula [20]. Here, all (repulsive Coulomb and attractive retarded) interactions are associated with the electric interaction because *only the charges* of electron and lattice ions are involved in the interactions.

An electron also has a *spin*, which is the intrinsic magnetic dipole moment of the electron. For example, copper oxide high-temperature superconductors (cuprates) and iron arsenide superconductors (Fe-pnictides) are governed by antiferromagnetic fluctuations because their undoped parent compounds are antiferromagnetic [21, 22]. Therefore, spin-1 antiferromagnetic fluctuations of these material systems may contribute to the formation of electron pairs through a magnetic interaction [23], but the microscopic origin of the magnetic interaction has not been clearly figured out yet. The spin fluctuations may play a similar role as the charge fluctuations in the conventional BCS superconductors. The magnetic interaction also can be a retarded attractive force because the higher average frequency of the electron-boson spectral density (or glue) function [24] shows the higher superconducting transition temperature (or stronger superconductivity) [25]. Therefore, the average frequency of the spin fluctuations can be used to determine the size of the Cooper pair as the phonon frequency for the conventional BCS superconductors. Furthermore, there has been a more radical proposal that even a repulsive interaction between electrons other than the Coulomb interaction can contribute to pairings of electrons [26]. So far, a lot of experimental and theoretical studies have been intensively performed to find out the superconducting mechanisms (microscopic origins of the pairing interactions) of experimentally discovered various superconductors including cuprates and Fe-based high-temperature superconductors [21, 22, 24]. Up to now, the conundrums for pairing mechanism of various superconducting materials including cuprates and Fe-pnictides have not been solved yet.

## 5. CONCLUSIONS

We describe a conceptual explanation for the ubiquity of the superconductivity. The Pauli's exclusion principle enormously increases the ground state energy of a fermionic system compared with a bosonic one. Itinerant electrons in metallic material systems can overcome the Pauli's exclusion principle through pairing (or bosonization) process and the paired electrons are condensed into a more stable superconducting state.

Therefore, the ultimate destination of all itinerant electrons can be the superconducting state because the electrons in metallic materials have *the ability* to find the possible bosonic ground state by taking advantage of possible electric or magnetic processes and even more exotic processes. We note that simple metals (alkali and alkaline metals) and noble metals do not show superconductivity at the lowest temperature to get in experimental laboratories.

We speculate that these good metals may not have the possible process to achieve an effective attractive force between itinerant electrons. Up to now, a lot of superconducting materials have been discovered but microscopic pairing mechanisms of most of them are not known yet. In these systems, itinerant electrons should know the secret of the Cooper pair formation because they are involved in the processes to lead them to the superconducting ground state. Careful experimental studies on itinerant electrons help to uncover the secret of the microscopic mechanism for the pair formation, i.e., provide the smoking gun evidence because the itinerant electrons carry the information on the attractive force. Various spectroscopic techniques including infrared/optical spectroscopy and angle-resolved photoemission spectroscopy can be used for direct measurements of the interaction spectrum, such as the electron-boson spectral density (or glue) function [24]. Particularly, measurements of the (optical and quasiparticle) self-energies of the dressed itinerant electrons (or quasiparticles) may provide the crucial information on the pair formation [15, 16, 24]. We hope that, in near future, researchers in condensed matter physics find out the microscopic origins of the superconducting mechanisms for all currently existing and would-be-discovered superconducting materials. We expect that this paper gives a rough idea to the researchers for understanding the ubiquity of the superconductivity.

## ACKNOWLEDGMENT

This work has been supported by the National Research Foundation of Korea (NRFK Grant No. 2019R1A6A1007307912).

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