Numerical investigation on vortex-induced vibration response characteristics for flexible risers under sheared-oscillatory flows

Hongxiang Xue, Yuchao Yuan, Wenyong Tang

Abstract
Surge motion of top-end platform induced by periodic wave makes marine flexible riser encounter equivalent sheared-oscillatory flow, under which the Vortex-induced Vibration (VIV) response will be more complicated than pure sheared flow or oscillatory flow cases. Based on a time domain force-decomposition model, the VIV response characteristics under sheared-oscillatory flows are investigated numerically in this paper. Firstly, the adopted numerical model is validated well against laboratory experiments under sheared flow and oscillatory flow. Then, 20 sheared-oscillatory flow cases with different oscillation periods and top maximum current velocities are designed and simulated. Under long and short oscillation period cases, the structural response presents several similar features owing to the instantaneous sheared flow profile at each moment, but it also has some different patterns because of the differently varying flow field. Finally, the effects and essential mechanism of oscillation period and top maximum current velocity on VIV response are discussed systematically.

1. Introduction

Vortex-induced Vibration (VIV) for flexible risers is always a challenging research issue in the field of fluid-structure interaction over the past decades. The understanding of VIV under steady flow conditions has been relatively mature with the help of laboratory model tests and numerical simulations. More relevant researches are comprehensively introduced in the review works by Bearman (2011). However, the encountered flow field of flexible risers in the actual marine environment is usually much more complicated than most existing experimental and numerical investigations, especially when the motions of top-end platform are considered.

Fig. 1 is a sketch of platform-TTR-seabed system, where TTR denotes top tensioned riser. TTR is connected with the floating platform at top and its bottom is fixed to the wellhead on seabed. Withstanding the impacts of periodic wave, the top-end platform will experience concomitant motions like surge or heave, driving the dynamic response of submarine riser to become very difficult to be predicted. In fact, taking platform motion effects into consideration is exactly a popular hotspot in the VIV research field recently. Different kinds of platform motion effects should be distinguished in essence. Heave motion introduces a fluctuating axial tension of riser, which leads a periodically varying structural property. Thus, the consequent effect is a parametric excitation. Such VIV cases were investigated by Franzini et al. (2016). Whereas, when horizontal motions i.e. surge of platform are considered, the oscillation movement of top-end will propagate along riser length. Because the riser’s bottom is fixed, the forced oscillation amplitude of different riser sections turns smaller from top to bottom. As a result, transverse motion of platform eventually makes the submarine riser under an equivalent sheared-oscillatory flow field and triggers riser’s VIV significantly. There are several researches concerning VIV under oscillatory flows by model tests (Fu et al., 2014) and numerical simulations (Zhao et al., 2010, Zhao et al. (2013) and Deng et al. (2014)) also carried out relevant Computational Fluid Dynamics (CFD) simulations for combined uniform and oscillatory flow cases.

However, in most existing investigations on VIV under unsteady flow cases, the instantaneous encountered current velocity along the riser is always equal, thus VIV can only excite single mode at a certain moment, which simplifies the actual marine situation excessively. For non-uniform flow profiles, since the current velocity along the riser is unequal, the VIV dominant frequency of
different riser sections may also be different, making the dynamic response present multi-frequency component superposition. Therefore, compared with sheared or oscillatory flow, the sheared-oscillatory flow described in Fig. 1 is much closer to the realistic marine environment and more complicated inevitably. Relevant researches concerning VIV for flexible risers under sheared-oscillatory flow are still quite few.

The temporary industry standard of VIV prediction when designing flexible risers for engineering application is based on a frequency domain approach e.g. VIVANA (Larsen et al., 2009) and SHEAR7 (Vandiver et al., 2005). The most significant advantage of using a frequency domain prediction tool is low demand on computational resources, allowing fast parametric analyses. However, frequency domain models must assume a linear behaviors system and cannot take non-linear aspects such as unsteady flow profile, time-varying structural stiffness and riser-seabed interaction into account. For systems where nonlinearities are important, a time domain procedure must be used for realistic predictions of riser’s VIV response. Based on the traditional frequency domain prediction tools, this paper improves the frequency domain force-decomposition model, expanding it to time domain to deal with unsteady flow cases. Force-decomposition models, which rely on measurements of hydrodynamic forces acting on the structure from experiment, have been widely used to predict VIV for flexible risers under steady flows (e.g. uniform, sheared or stepped flow) in time domain (Ma et al., 2012; Wang et al., 2013), without extravagant computational resources or too many numerical model controlled parameters. The compatibility of force-decomposition model for oscillatory flow cases has been verified by Thorsen et al. (2016) and Yuan et al. (2018), but the existing researches based on force-decomposition model still mainly focus on steady flow or simple pure oscillatory flow cases. Therefore, such a VIV prediction approach is in its infancy as far as complicated unsteady flow cases are concerned.

The main contribution of this paper is proposing an alternative time domain force-decomposition model, not only making it available for more extensive conditions but also improving prediction accuracy and enriching data post-processing, compared with traditional frequency domain analysis tools. More importantly, another novel attempt is using the proposed numerical model to investigate the VIV response characteristics under sheared-oscillatory flows, which is helpful to investigate the riser’s dynamic response in the actual marine environment. This paper is structured as follows. Section 2 is the description of adopted numerical model and analysis methodology. In Section 3, the advantages of adopted time domain model over traditional frequency domain model are verified by comparing with test measurements qualitatively and quantitatively. In Section 4, the availability of the numerical model for oscillatory flow cases is validated well against published experimental results. Section 5 simulates 20 designed cases and investigates the VIV response characteristics under sheared-oscillatory flows. In Section 6, the effects of oscillation period and top maximum current velocity of sheared-oscillatory flow on VIV response are discussed in detail, and some reasonable explanations on relevant mechanical essence are given. Finally, the conclusions are drawn in Section 7.

2. Numerical model and analysis methodology

Since VIV under unsteady flows is still a very challenging issue and in-line VIV is usually much more complicated than cross-flow one, the relevant experimental and numerical researches at present mainly focuses on cross-flow VIV. In this paper, only cross-flow VIV is included as well.

A large aspect ratio of length to diameter is a universal feature of deepwater flexible risers. Thus, the riser model can be considered as a flexural elastic structure satisfying the Euler-Bernoulli beam hypothesis, the governing differential equation for a riser in cross-flow direction could be expressed as Eq. (1). Cartesian coordinate system is utilized, where x-axis is parallel with current velocity, y-axis is perpendicular to incoming flow direction and z-axis is riser’s axial direction.

\[
\left( m + \frac{\pi}{4} C_a \rho_l D^2 \right) \frac{\partial^2 y}{\partial t^2} + \left( c_t (f_r, t) + c_f (A^*, f_r, t) \right) \frac{\partial y}{\partial t} + E I \frac{\partial^4 y}{\partial z^4} - \frac{\partial}{\partial z} \left( T_a(z, t) \frac{\partial y}{\partial z} \right) = F_y(A^*, f_r, t)
\]

where \( m \) is the mass per unit length of the riser, \( C_a \) is the added mass coefficient, generally assumed as a constant e.g. 1.0, \( \rho_l \) is the fluid density, \( c_t \) and \( c_f \) are the structural and hydrodynamic damping coefficients, \( A^* \) is the non-dimensional response.
amplitude to riser’s diameter $D$, $f_r$ is the non-dimensional frequency equal to $fD/V$, $f$ is the response frequency, $E$ is the elastic modulus, $I$ is the moment of inertia, $T_a$ is the effective axial tension, $F_y$ is the VIV excitation force.

VIV excitation force $F_y$ is in phase with riser’s velocity and depends on its response amplitude, vibration frequency, current velocity and direction. Assuming that the excitation force acting on riser segment follows sinusoidal rule in one period, it could be expressed as Eq. (2):

$$F_y = \frac{1}{2}C_V(A^*, f_r, t) \rho D V(V(t))|V(t)| \cos 2 \pi ft$$

(2)

where $C_V$ is the excitation force coefficient and $V$ is the instantaneous current velocity.

2.1. Excitation force model

To obtain the excitation force coefficient $C_V$, a function of non-dimensional amplitude $A^*$ and frequency $f_r$ based on forced vibration experimental data is proposed. Gopalkrishnan (1993) carried out a series of cylinder forced vibration test in MIT towing tank, and gave the contour of VIV excitation force coefficient. This database has been well verified and used by several mature frequency domain software like SHEAR7 and VIVANA, but mainly limited to steady flow or oscillatory flow cases before this paper.

Fig. 2 is the database of $C_V$, adapted from Larsen et al. (2009), a) is the curve of $C_V$ as a function of $A^*$ for a fixed response frequency, and red dotted lines mark three key points controlled by four specific parameters. b) is the controlled parameters to define $C_V$-$A^*$ curves, as a function of non-dimensional response frequency $f_r$. VIV excitation center is approximately $f_r = 0.17$, where corresponds to the largest excitation coefficient according to Fig. 2(b)). The Strouhal number of this series of forced vibration experiments is 0.193.

2.2. Damping model

The total damping considered in adopted VIV simulation method consists of structural damping and hydrodynamic damping. The structural damping coefficient $c_s$ is typically expressed in Eq. (3):

$$c_s = 4\pi mf_2$$

(3)

where $f_2$ is the structural damping ratio.

Hydrodynamic damping is an active research subject in the field of VIV. When $C_V$ is negative, hydrodynamic damping force will work and the energy will be transferred from the riser back to fluid. The hydrodynamic damping coefficient can be calculated in Eq. (4).

$$c_f = \frac{C_V(A^*, f_r, t) \rho D V(t)|V(t)|D}{4\pi A^* f_r}$$

(4)

If $f_r$ is outside the experimental data range, an empirical damping model by Venugopal (1996) is used to simulate VIV hydrodynamic damping. Venugopal (1996) synthesized a broad variety of experimental evidence and recommended the following empirical model made up of high and low non-dimensional frequency regions.

Damping in high non-dimensional frequency region is expressed in Eq. (5):

$$c_f = C_{sf} \rho \sqrt{D} V(t) + C_{sw}$$

(5)

where $C_{sf}$ is an empirical coefficient, $C_{sw}$ is the still water damping given by Eq. (6):

$$c_{sw} = \pi^2 \rho D^2 D \left[ 2 \sqrt{\frac{1}{\pi D^2 f^2 V}} \right] + C_{sw}(A^*)^2$$

(6)

where $r$ is the kinematic viscosity of the fluid, $C_{sw}$ is another empirical coefficient.

Damping in low non-dimensional frequency region is expressed in Eq. (7):

$$c_f = \frac{C_{sf} \rho V(t)^2}{2\pi f}$$

(7)

where $C_{sf}$ is also an empirical coefficient.

The empirical coefficients $C_{sf}$, $C_{sw}$ and $C_f$ are recommended taking 0.18, 0.25 and 0.2.

2.3. Lock-in judgment criterion

Real-time response amplitude and frequency of the riser are the most crucial factors to obtain the hydrodynamic forces. So at each time step, the displacement ($y$) and velocity ($V$) of nodes and time ($t$) should be extracted to calculate the real-time response amplitude and frequency for every element of the riser. The response amplitude $A_{res}$ and frequency $f_{res}$ are calculated by Eq. (8).

$$A_{res} = \frac{|y_b - y_a|}{2f_{res} = 1 / [2 \times (t_b - t_a)]}$$

(8)

where $y_b, y_a, t_b, t_a$ are the displacement and time corresponding to the adjacent two time points $a$ and $b$ with local response velocity.
\( v = 0 \).

The non-dimensional range of \( 0.125, 0.25 \) is determined as the lock-in region according to the recent experimental research by Zheng (2014), which should be corrected for difference of Strouhal number between actual and test circumstance by Eq. (9).

\[
\left( \frac{f_i}{S_f} \right)_{\text{test}} = \left( \frac{f_i}{S_f} \right)_{\text{actual}}
\]

If \( f_i \) locates within the excitation bandwidth, lock-in occurs. The riser element will be synchronized onto the structural natural frequency closest to \( f_i = 0.17, 0.17 \) here also needs to be corrected by Eq. (9). The lock-in natural frequency is regarded as the VIV dominant frequency to calculate excitation force coefficients by Fig. 2. In the adopted numerical model, the second-order digital control loop in Grant et al. (2000) is adopted to guarantee that the excitation force is in phase with the local velocity of the cylinder. The control loop is a regulator system, consisting of a phase error detector and a second-order tracker. The phase difference between the local velocity of the cylinder and the excitation force at lock-in appear as a step function to the regulator system. The phase difference will be gradually adjusted down to zero over a period of time. Then the phase-locked loops (PLL) remains active to keep the phase difference equal to zero.

If \( f_i \) is outside the excitation bandwidth, the dominant frequency is regarded equal to response frequency, the negative \( C_V \) is calculated based on Fig. 2, and then the hydrodynamic damping coefficient can be obtained by Eq. (4). Outside the experimental data, Eqs. (5)–(7) will be adopted.

### 2.4. Structural response solution

The adopted time domain numerical model in this paper is based on direct-integration dynamic analysis. Eq. (1) is solved with the Hilber-Hughes-Taylor (HHT) method, which is a one-step implicit method used successfully in the field of classical mechanical simulation (Hilber et al., 1977). Eq. (10) shows how the positions and velocities update at every time step and the time-discrete momentum equation is expressed as Eq. (11). When \( a \in [-1/3, 0], \beta = (1 - \alpha)2/4 \) and \( \gamma = (1 - 2\alpha)/2 \), HHT method is unconditionally stable. Automatic convergence checks are applied to the force residuals with standard parameters \( a = -0.05 \). The stable time increment is selected as 0.002/\( f_{max} \) where \( f_{max} \) is the maximum expected fundamental frequency. It means there is at least 500 steps during each vibration period of the riser, which has been found sufficient (i.e. reducing the time increment gives no change in the predicted results).

\[
y_{f_t + \Delta t} = y_{f_t} + \Delta t \left( y_{f_t} + \frac{\Delta t^2}{2} \left( f_{f_t} + 2f_{f_t + \Delta t} \right) \right)
\]

\[
y_{f_t + \Delta t} = y_{f_t} + \Delta t \left( y_{f_t} + \frac{\Delta t^2}{2} \left( f_{f_t} + 2f_{f_t + \Delta t} \right) \right)
\]

\[
R_{f_t + \Delta t} = F_{ext_{f_t + \Delta t}} - G_{y_{f_t + \Delta t}} - F_{int_{f_t + \Delta t}}(y_{f_t + \Delta t})
\]

\[
y_{f_t + \Delta t} = (1 - \alpha) y_{f_t} + \alpha y_{f_t + \Delta t}
\]

\[
y_{f_t + \Delta t} = (1 - \alpha) y_{f_t} + \alpha y_{f_t + \Delta t}
\]

where \( F_{ext} \) and \( F_{int} \) are the external and internal forces in the whole system, \( R \) is the force residuals.

Eq. (1) is discretized in space by using finite element method (FEM). The structural kinetic equation can be written as Eq. (12):

\[
(M_s + M_b)y + (C_s + C_f(t))y + (K_s + K_b)y = F_{exc}(t)
\]

where \( M_s \) and \( M_b \) are the structural and added mass matrices, \( C_s \) and \( C_f \) are the structural and hydrodynamic damping matrices, \( K_s \) is the additional lateral stiffness matrix provided by effective axial tension and \( K_b \) is the bending stiffness matrix determined by inherent properties of riser materials, \( F_{exc} \) is the VIV excitation force vector and \( y \) is a vector containing nodal translations and rotations.

Associated with Eq. (11), the force residuals vector \( R_{f_t + \Delta t} \) in HHT method for the dynamic analysis can be expressed in Eq. (13):

\[
R_{f_t + \Delta t} = -(M_s + M_b)y_{f_t + \Delta t} + (1 + \alpha)G_{y_{f_t + \Delta t}} - \alpha G_r
\]

where \( G \) is an internal state vector which can be expressed as \( G = F_{exc} - (C_s + C_f)y \).

To better demonstrate the adopted approach, the time domain analysis flow-chart is shown in Fig. 3. During each time step, hydrodynamic forces will be updated before the structural response calculation.

### 3. Time domain model vs frequency domain model

The adopted hydrodynamic force model is to some extent adapted from the traditional frequency domain prediction tools, but the lock-in judgement criterion is modified evidently for VIV time domain analysis. Quite different from the frequency domain approaches, the present time domain numerical model no longer needs to assume the energy allocation within excitation overlap regions and a specific energy cutoff value, which only make sense in frequency domain prediction tools. Another difference is the update of lock-in region from \( [0.125, 0.2] \) to \( [0.125, 0.25] \) according to the recent forced vibration experimental researches by Zheng (2014). To evaluate the prediction performance of different numerical models, this section selects 13 experimental sheared flow cases of a large-scale riser model published by Lie et al. (2006), and simulates these cases with time domain and frequency domain models respectively.

#### 3.1. Description of sheared flow experiments

Large-scale model experiments of a tensioned steel riser in well-defined sheared current was performed at Hanøytangen outside Bergen on the west coast of Norway in 1997. Detailed description can be found in Lie et al. (2006). The test site was a 180 m long floating quay at 97 m depth. The length of the riser model is 90 m and the diameter is 3 cm. The large-scale model riser was attached to a floating vessel at top-end and kept with constant axial tension by a buoyancy can at bottom end. By moving the top vessel at a constant speed, the subjected current of the riser is equivalent to a linearly sheared flow. The riser was furnished with transducers for measuring bending moment every 3 m down the riser, VIV-induced moments were measured at 29 positions. In addition, inclinometers and tension transducers were installed at both ends. The transducers were sampled at a rate of 120 Hz. The necessary physical parameters of the riser model are listed in Table 1. The Strouhal number is approximately equal to 0.19 according to Lie et al. (2006).

#### 3.2. Validation against experiments for 13 simulated cases

Several researches usually used Eq. (14) to calculate the natural frequencies for a tensioned flexible riser.

\[
f_{ni} = \frac{i}{2\pi} \sqrt{\frac{T_{end}}{m_i}} \pm \frac{\beta p^2}{I_2} \cdot \frac{EI}{m_i}
\]

where \( L \) is the riser’s length, \( T_{end} \) is the axial pretension exposed at one of the riser’s ends, \( m_i \) is the total mass of the whole riser \( m_t = \ldots \)
In fact, Eq. (14) is strictly true only for the cases where axial tension is distributed uniformly along riser’s axial direction. In the actual marine environment, flexible risers in service are always with vertical or catenary configuration, and structural self-gravity makes riser’s tension unequal at different axial locations. Fig. 4 gives the calculated natural frequencies of the 90 m riser model by FEM and Eq. (14). The calculated results by FEM are obviously more consistent with test data than Eq. (14), especially for the high modes. Therefore, this paper carries out modal analysis with FEM to obtain structural natural frequencies before VIV simulation.

Predicted results of time domain and frequency domain models are compared comprehensively in Fig. 5 and Fig. 6. The time domain results with lock-in regions of [0.125, 0.25] and [0.125, 0.2] are both presented. The adopted frequency domain analysis procedure is compiled in reference from Vandiver et al. (2005), and the energy cutoff value is set as 0.7. Fig. 5 collects the predicted average RMS displacement along the 90 m riser under different sheared flow cases. Left is qualitative comparison and right is quantitative analysis where the prediction error is defined in Eq. (15).

\[
\text{prediction error} = \frac{|\text{predicted value} - \text{test data}|}{\text{test data}} \quad (15)
\]

Fig. 5(a) shows that the numerical results are all reasonably in accordance with test data. The average RMS non-dimensional displacement generally keeps between 0.20 and 0.30 with the variation of current velocity, which proves the well-known self-limited feature of VIV. Results of time domain model with lock-in region of [0.125, 0.25] are basically closest to the experimental measurements. While results of time domain model with lock-in region of [0.125, 0.2] and frequency domain model are relatively smaller. According to Fig. 5(b), compared with the other two, time domain model with [0.125, 0.25] has the smallest prediction error except when top current velocity \( V_{\text{top}} \) = 0.44 m/s and 0.74 m/s. As far as the maximum prediction error is concerned, it is 6.4% for time domain model with [0.125, 0.25], also smaller than time domain model with [0.125, 0.2] (18.5%) and frequency domain model (23.9%). The average prediction error of 13 simulated cases for different numerical models is given as well. The minimum value 2.7% corresponds to time domain model with [0.125, 0.25]. With the same lock-in region of [0.125, 0.2], the results of time domain and frequency domain models are approximate, and the value of time domain model (8.5%) is still slightly smaller than frequency domain model (9.0%).

The predicted peak frequencies at the 90 m riser’s midpoint under different sheared flow cases are shown in Fig. 6, left and right are qualitative comparison and quantitative analysis respectively. Since the lock-in region selection has little influence on peak

\[ m_s \] and \( m_b \) are the structural and added mass with \( C_b = 1 \).

Table 1

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Sheared flow case</th>
<th>Oscillatory flow case</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (m)</td>
<td>90</td>
<td>4</td>
</tr>
<tr>
<td>Diameter (m)</td>
<td>0.030</td>
<td>0.024</td>
</tr>
<tr>
<td>Young’s modulus (Pa)</td>
<td>( 2.1 \times 10^8 )</td>
<td>( 6.45 \times 10^8 )</td>
</tr>
<tr>
<td>Bending stiffness (N.m²)</td>
<td>3637.25</td>
<td>10.50</td>
</tr>
<tr>
<td>Mass ratio</td>
<td>3.13</td>
<td>1.53</td>
</tr>
<tr>
<td>Structural damping ratio</td>
<td>0.003</td>
<td>0.015</td>
</tr>
<tr>
<td>Pretension (N)</td>
<td>3700</td>
<td>500</td>
</tr>
</tbody>
</table>

Fig. 3. Flow-chart of VIV analysis.

Fig. 4. Calculated natural frequencies of the 90 m riser model by FEM and Eq. (14).
response frequency for time domain models, here only gives results of time domain model with [0.125, 0.25]. In general, predicted peak frequency and its variation trend with current velocity are both consistent with test data. Peak response frequency of VIV is basically proportional to top current velocity. The difference between numerical results of time domain and frequency domain models is not obvious under most cases, but time domain value is visibly smaller and closer to the experimental measurements when $V_{top} = 0.73$ m/s, $0.74$ m/s and $0.83$ m/s. The prediction error of $f_{peak}$ is also calculated by Eq. (15), time domain value is generally smaller except when $V_{top} = 0.54$ m/s and $0.95$ m/s. The biggest error (around 40%) at $V_{top} = 0.95$ m/s is speculated owing to the randomness or uncertainty of the test measurements, since the test data seems to somehow violate against the well-recognized regularity (i.e. approximately proportional with top current velocity). Associated with the average prediction error of 13 simulated cases, time domain model (6.7%) still has better prediction accuracy than frequency domain model (10.2%).

3.3. Further evaluation for a representative case with $V_{top} = 0.54$ m/s

To evaluate the prediction performance of different numerical models furtherly, a representative case with $V_{top} = 0.54$ m/s is selected for more detailed comparisons. Predicted envelopes of RMS VIV displacement and curvature along the 90 m riser by time domain and frequency domain models are shown in Fig. 7. From Fig. 7(a), the predicted RMS displacement envelopes all present 11th modal vibration shape, consistent with test data. Time domain results evidently have better prediction accuracy than the frequency domain one, especially for the extrema of the inflexion points. Compared with lock-in region of [0.125, 0.25], [0.125, 0.2] underestimates RMS VIV displacement for the region $z/L$ of 0.1–0.35 and overestimates it for the region $z/L$ of 0.7–0.9. For RMS curvature, frequency domain model shows relatively unsatisfactory envelop. Within the region $z/L$ of 0.2–0.9, the predicted value is much smaller than test data. Time domain results with different lock-in regions both present good agreement with experimental measurements. But the prediction error of [0.125, 0.2] is still more visible for the region $z/L$ of 0.4–0.7, and the predicted RMS curvatures of [0.125, 0.25] are larger and more consistent with test data.

More abundant data available for post-processing and analysis is another significant advantage of time domain methods over frequency domain ones, e.g. time histories of response displacement or frequency at a specific axial location are easy to be obtained. Fig.8~Fig.9 compare the predicted displacement time histories and amplitude spectra at the 90 m riser’s midpoint with experimental measurements. The results with two different lock-in regions are both included. The predicted and test time histories of displacement have reasonable similarities with an obvious feature of multi-frequency component superposition, quite different from uniform flow cases. The predicted maximum displacement with [0.125, 0.2] is 0.024 m, evidently larger than [0.125, 0.25] as well as
test data. The amplitude spectra of time domain models with different lock-in regions have the identical peak frequency but different excited bandwidth. Compared to the result with [0.125, 0.2], the predicted response spectrum with [0.125, 0.25] has weaker response energy at peak frequency and broader excited bandwidth (from [2 Hz, 3 Hz] to [1 Hz, 3 Hz]), both of which are more consistent with test data.

Thus, the adopted time domain force-decomposition model is not just a simple copy of the traditional frequency domain prediction tools. By expanding the original hydrodynamic force model to time domain and adopting the improved time domain lock-in judgement criterion, the adopted numerical model shows better performance than traditional frequency domain analysis procedure. Moreover, the updated lock-in region of [0.125, 0.25] in the adopted numerical model is proved more reasonable for VIV predictions.

4. Validation against experiment under oscillatory flows

To the knowledge of the authors, there seems no relevant experimental research on VIV for TTRs under non-uniform unsteady flow so far. This paper validates the adopted VIV model against the published experiments of a 4 m riser model by Fu et al. (2014) under pure oscillatory flow.

4.1. Description of oscillatory flow experiments

The laboratory experiments were conducted in the ocean basin at Shanghai Jiao Tong University. The whole experimental setup was installed under the bottom of the carriage which mainly contains two horizontal tracks. The test riser model was towed in still water with a harmonic in-line motion at various combinations of amplitude and period such that the riser was subjected to different oscillatory flows. The main physical parameters of the riser are listed in Column 3 of Table 1. When fluid oscillates periodically in time domain, the motion equation can be described in Eq. (16).

\[
A(t) = A_m \sin \left( \frac{2\pi}{T} t \right)
\]

\[
V(t) = A_m \frac{2\pi}{T} \cos \left( \frac{2\pi}{T} t \right) = V_m \cos \left( \frac{2\pi}{T} t \right)
\]

(16)

where \(A_m\) and \(V_m\) are the maximum oscillation amplitude and velocity, \(T\) is the oscillation period. Keulegan-Carpenter (KC) number is another key parameter to describe a certain oscillatory flow, which can be expressed as Eq. (17).

\[
KC = \frac{V_m T D}{2\pi A_m}
\]

(17)

Reduced velocity is one of the most important parameters of VIV under steady flows, it can be written as \(V_{m,n} = V_m / f_n D\), where \(f_n\) is the \(n\)th structural natural frequency in still water by assuming \(C_a = 1.0\). The maximum reduced velocity was defined as Eq. (16) for oscillatory flow cases in Fu et al. (2014).

\[
V_{r,1}^{in} = \frac{V_m}{f_1 D} = \frac{2\pi A_m}{f_1 T_1} = \frac{KC}{T_1}
\]

(18)

A specific oscillatory flow case was characterized by a
combination of KC number and maximum reduced velocity \( V^\text{m}_1 \). A large number of cases were classified by Fu et al. (2014) into two categories i.e. high and small KC cases, to discuss respective VIV features. For the sake of conciseness, two representative cases (i.e. Case 1 and Case 2 in Table 2) are simulated in this section to evaluate performance of the adopted numerical model for high and small KC cases. The corresponding Strouhal number is taken to be 0.2, which is known to be approximately true in the subcritical flow regime. The continuous wavelet transform (CWT) technique is used to obtain VIV time-frequency distribution under oscillatory flows, and Morlet wavelet is selected as the mother wavelet, which is coincident with Fu et al. (2014). Eq. (19) is the continuous wavelet transform equation (Grossmann et al., 1984).

\[
WT_f(a, \tau) = \left( f(t), \psi_{a, \tau}(t) \right) = \|a\|^{-1/2} \int_{-\infty}^{+\infty} f(t) \psi^* \left( \frac{t - \tau}{a} \right) dt \tag{19}
\]

Where \( WT_f(a, \tau) \) is the coefficient after wavelet transformation which represents the change of frequency on the time scale. While \( a \) is the scale factor, \( t \) is the shift factor, and \( \psi(t) \) is the mother wavelet. Morlet wavelet is defined as Eq. (20) (Grossmann et al., 1984).

\[
\psi(t) = Ce^{-t^2/2} \cos(5t) \tag{20}
\]

Comparisons of VIV response at midpoint of the riser (\( z = 2 \text{ m} \)) between predicted and test results are shown in Fig. 10 and Fig. 11 for high and small KC cases respectively. The left columns give the predicted results based on the adopted model, while the right ones are the test results originated from Fu et al. (2014). The top rows are VIV response displacement time histories and the bottom ones are the wavelet contour plots of VIV strain time history, where horizontal axis represents time and vertical axis represents frequency, while the color shows the concentration level of the response energy. The first two natural frequencies of riser model in still water, \( f_{l1}, f_{l2} \), are also plotted in the same coordinate. The time interval between the two black vertical dashed lines is one complete oscillation period marked as \( T \) in the middle.

### 4.2. High KC case

For high KC case i.e. Case 1 (\( KC = 178, V_{r,m} = 4 \)), intermittent and periodic VIV can be captured evidently in the predicted and test time histories of VIV displacement, and the vibration amplitude is close to zero within visible time intervals of each oscillation period. Another notable phenomenon observed in both the predicted and test results is amplitude modulation. In general, response amplitude would enlarge with the increase of the current velocity and reduce with the decrease of current velocity. Therefore, during each oscillation period, the response amplitude would modulate slightly like a shape of mountain peak twice correspondingly since there are two time segments for current velocity to increase and decrease. Frequency transition can also be found in the predicted and test wavelet contour plots. The frequency is basically proportional to current velocity, and there are also two similar transition processes during each oscillation period. The predicted time-varying process and maximum value of response amplitude as well as frequency both show good agreement with test results. From the wavelet contour plots, although it doesn’t match well with test results, the higher harmonics, approximately three times the dominant frequency, could also be observed in predicted results.

### 4.3. Small KC case

For small KC case i.e. Case 2 (\( KC = 31, V_{r,m} = 6.5 \)), slight amplitude modulation phenomenon can also be observed in predicted and test VIV displacement, but it should be distinguished from that at high KC number. Amplitude fluctuates in time domain without evident regularity, it is more likely to be caused by the unstable vibration of the riser rather than the periodic pattern of current velocity. In general, the response amplitude keeps relatively constant and time history is quite like the sinusoidal shape, similar with uniform flow cases. Both predicted and test results show that VIV dominant frequency keeps continuous and basically unchanged in time domain, and the comparable values are approximate as well. The higher harmonics corresponding to three times the fundamental frequency can also be observed in both predicted and test wavelet contour plots. So far, there still seems no widely accepted numerical simulation method able to predict this high frequency component precisely. This paper at present only proves the higher harmonics do exist. To deepen the understanding of the relevant mechanism and predict it under steady even unsteady flow situations, more and further research work needs to be done in the future.

---

**Table 2**

Test oscillatory flow cases simulated for comparison.

<table>
<thead>
<tr>
<th>Case Number</th>
<th>( A_{ref}(m) )</th>
<th>( T(s) )</th>
<th>( KC )</th>
<th>( V_{r,m} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.68</td>
<td>16.5</td>
<td>178</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>0.12</td>
<td>1.8</td>
<td>31</td>
<td>6.5</td>
</tr>
</tbody>
</table>
As demonstrated above, the availability of adopted time domain model has been validated well against laboratory experiments under oscillatory flow situations. It means the existing hydrodynamic force coefficient database, which was previously used to predict VIV under steady flows, is indeed equally applicable for unsteady flow cases. As the prediction accuracy for sheared flow cases has been proven satisfactory in Section 3, it is reasonable to believe the proposed numerical model can be used to investigate the VIV response characteristics for TTRs under sheared-oscillatory flow.

5. VIV response characteristics under sheared and sheared-oscillatory flows

The equivalent sheared-oscillatory flows is induced by the surge motion of top-end platform, whereas it is the instantaneous current velocity that affects VIV response essentially. Therefore, this paper describes a certain sheared-oscillatory flow case by Eq. (21) with two controlled parameters, i.e. top maximum current velocity $V_{m\text{,top}}$ and oscillation period $T$.

$$V(z, t) = V_{m\text{,top}} \sin \left( \frac{2\pi t}{T} \right) \times \frac{z}{L}$$  (21)

$K_C$ number is usually an important controlled parameter used to describe an oscillatory flow, and its effects on VIV response and fatigue damage are investigated by Fu et al. (2014) and Wang et al. (2015). However, when an oscillatory flow is defined by its instantaneous velocity, i.e. $V(t) = V_m \sin(2\pi t/T)$, $K_C$ number is actually not an independent controlled parameter, but determined by both oscillation period and maximum oscillation velocity ($K_C = V_m T/D$). In Fu et al. (2014) and Wang et al. (2015), when discussing the effect of $K_C$ number on VIV, the maximum oscillation velocity is kept fixed. So the different VIV response characteristics with different $K_C$ numbers are in fact owing to the difference of the oscillation period $T$. Thus, a more independent controlled parameter $T$ is selected to design the simulated cases in this paper, instead of $K_C$ number.

20 designed cases of the large-scale riser model in Lie et al. (2006) are simulated to investigate the VIV response characteristics for flexible risers under sheared-oscillatory flow. 5 oscillation periods from short to long (i.e. 2s, 4s, 8s, 16s and 32s) are arranged. They can also be deemed as high and small $K_C$ cases. 4 top maximum current velocities (i.e. 0.27 m/s, 0.54 m/s, 0.81 m/s and 1.08 m/s) cover low and high velocity situations. The parameter combinations of 20 designed cases are listed in Table 3. Each column can be defined as a group, which can be utilized to discuss the oscillation period effect with a constant $V_{m\text{,top}}$. For the top maximum current velocity effect, the series of cases with Case No. equal to $m + 5n$ ($n = 0, 1, 2, 3$; $m$ ranges from 1 to 5) is available, each series has the same $T$ but different $V_{m\text{,top}}$. Note that, in Lie et al. (2006) and Section 3 of this paper, pretension is acted at riser’s bottom end. However, for most TTRs in the actual marine environment, axial pretension is always acted at the top, which means local axial tension turns smaller from top to bottom. In order to make it closer to the realistic situation, pretension will be acted at riser’s top for the numerical investigations in Section 5 and Section 6.
5.1. Sheared flow case

Before discussing sheared-oscillatory flow cases, it is necessary to know the VIV response characteristics under sheared flow comprehensively. Since sheared flow is a typical non-uniform flow and the local current velocity at different axial locations of the riser is unequal, the dynamic response of different riser sections inevitably presents quite different response patterns. This paper selects three representative monitoring points to investigate the VIV response at different locations along the riser. Fig. 12 is the predicted VIV response at specific monitoring points of the 90 m riser for sheared flow case with \( V_{\text{top}} = 0.54 \) m/s. Left and middle are time histories of response displacement and frequency, and right is the corresponding response frequency spectra. \( z/L \) of row 1–3 is 0.833, 0.500 and 0.167 respectively. Time-frequency distribution plots are obtained by CWT technique with mother wavelet of Morlet wavelet. Response displacement amplitude spectra are based on fast fourier transform (FFT) method. FFT is a mature frequency spectrum analysis method, which has been widely used for data processing of time-varying signal (Welch, 1967).

The shapes of displacement time histories at different locations along the riser are obviously different. During the same time interval, the displacement time history of riser middle segment \( (z/L = 0.500) \) has smaller half wave number than the top segment \( (z/L = 0.833) \) and bottom segment \( (z/L = 0.167) \). Displacement amplitude slightly modulates in time domain, while VIV is still persistent. From wavelet analysis plots, there are stable response energy concentrating at multiple frequency components, and every frequency is excited discontinuously. The strongest frequency component also changes with the variation of time, and mode jump can be evidently captured during VIV progress, as marked with black dotted ellipses. Although response energy all appear within \([1.5 \text{ Hz}, 3 \text{ Hz}]\), the time-frequency distributions of riser's different segments still present some mutual differences, e.g. respective response energy transfer between excited modes. Associated with response frequency spectra, different riser segments have identical excited frequency bandwidth but different energy strength allocation and unequal response peak frequency. Respective peak frequencies are marked by pink dotted squares. The strongest frequency component of riser middle segment \( (1.88 \text{ Hz}) \) is lower than the corresponding value of the top and bottom segments \( (2.78 \text{ Hz}) \).

As far as sheared flow cases are concerned, the local current velocity is relatively high for the top segment of the riser, and VIV excited frequency is basically proportional to current velocity. Therefore it is easier for the higher frequency components to be excited than riser middle segment. While for bottom segment, the local bending stiffness reduces to the smallest value since local axial tension decreases linearly from top to bottom. Although local current velocity is low, the vibrational energy will transfer from top to bottom in the manner of travelling wave (which will be described in detail as follows), making higher frequency components also easier to be excited at the bottom of the riser than the middle segment. Column 1 of Fig. 13 contains the temporal-spatial distributions of response displacement along the 90 m riser and time histories of main excited modal weights for the sheared flow case with \( V_{\text{top}} = 0.54 \) m/s. The most significant VIV response characteristic under sheared flow is travelling wave, as shown by inclined black arrows, response energy transfers from top to bottom with time varying. Similar phenomenon was also captured by VIV experimental research of a 152.4 m long riser model in Vandiver et al. (2009). The current velocity was designed to increase from riser’s top to bottom, and the experimental measurements indicated that travelling wave propagates along the riser upwards. Thus, it can be concluded that response energy tends to transfer along the riser’s axial direction from high to low velocity region. Above conclusion can be evidently captured during VIV progress, as marked with black dotted ellipses. Although response energy all appear within \([1.5 \text{ Hz}, 3 \text{ Hz}]\), the time-frequency distributions of riser's different segments still present some mutual differences, e.g. respective response energy transfer between excited modes. Associated with response frequency spectra, different riser segments have identical excited frequency bandwidth but different energy strength allocation and unequal response peak frequency. Respective peak frequencies are marked by pink dotted squares. The strongest frequency component of riser middle segment \( (1.88 \text{ Hz}) \) is lower than the corresponding value of the top and bottom segments \( (2.78 \text{ Hz}) \).

As far as sheared flow cases are concerned, the local current velocity is relatively high for the top segment of the riser, and VIV excited frequency is basically proportional to current velocity. Therefore it is easier for the higher frequency components to be excited than riser middle segment. While for bottom segment, the local bending stiffness reduces to the smallest value since local axial tension decreases linearly from top to bottom. Although local current velocity is low, the vibrational energy will transfer from top to bottom in the manner of travelling wave (which will be described in detail as follows), making higher frequency components also easier to be excited at the bottom of the riser than the middle segment. Column 1 of Fig. 13 contains the temporal-spatial distributions of response displacement along the 90 m riser and time histories of main excited modal weights for the sheared flow case with \( V_{\text{top}} = 0.54 \) m/s. The most significant VIV response characteristic under sheared flow is travelling wave, as shown by inclined black arrows, response energy transfers from top to bottom with time varying. Similar phenomenon was also captured by VIV experimental research of a 152.4 m long riser model in Vandiver et al. (2009). The current velocity was designed to increase from riser’s top to bottom, and the experimental measurements indicated that travelling wave propagates along the riser upwards. Thus, it can be concluded that response energy tends to transfer along the riser’s axial direction from high to low velocity region. Above conclusion
could be proved reliable again with some newly published investigations on the travelling wave propagating along flexible cylinders under sheared flows (Chen et al., 2019; Gao et al., 2019).

The time histories of main excited modal weights also prove the existence of mode jumps, as the maximum modal weight corresponds to different mode at different time points. Mode 12–mode 14 dominate riser’s VIV by turn and modal weight allocation follows a certain periodicity. Each 4s is approximately a repeated cycle, and the maximum modal weight of every mode basically keeps constant. According to the modal weight time histories, VIV exactly happens persistently.

5.2. Long oscillation period case

The VIV response at specific monitoring points of the 90 m riser for long oscillation period case with \( V_{m,\text{top}} = 0.54 \) m/s and \( T = 32s \) is shown in Fig. 14, and the description of each sub-picture is in accordance with Fig. 12. From response displacement time histories, there are obvious amplitude increase and decrease processes, and the similar modulation repeats twice during each complete oscillation period. Intermittent VIV is captured by green dotted ellipses, and response displacement is near zero within several time intervals marked with dotted squares in green. Wavelet plots also present evident periodicity, response frequencies being excited appears four times during 60s (nearly two oscillation periods). Above response characteristics are quite like the pure oscillatory flow cases with long oscillation period. However, since local current velocity and VIV dominant frequency are unequal, the dynamic response at different monitoring points looks different, especially in wavelet plots. For \( z/L = 0.833 \) and 0.167, strongest response energy may concentrate at 2.8 Hz, while the corresponding value is generally approximate to 2.0 Hz for \( z/L = 0.500 \), which means higher frequency components of riser top and bottom segments are easier to be excited than the middle segment. Associated with response amplitude spectra, the excited frequency components are much denser and response energy of each frequency is weaker than sheared flow case. Although the response energy allocation is different at three axial locations, the excited frequencies all mainly locate between 1 Hz and 3 Hz. These response characteristics are owing to the sheared flow profile at each moment.

As numerous free vibration tests (e.g. Blevins et al., 2009) indicated, VIV will not occur in low reduced velocity region until current velocity increases to a critical value (well-accepted as \( V_r = 4 \)). Such critical value is called VIV trigger floor in this paper. The trigger floor also exists for VIV under sheared flows. Time histories of instantaneous current velocity at riser’s top during each oscillation period are presented in Fig. 15, and the red dotted line denotes the VIV trigger floor. When instantaneous current velocity is above the trigger floor, the excitation force, which plays a role to aggravate structural vibration, will act on the riser. While when below the trigger floor, excitation force transfers into hydrodynamic damping, weakening structural vibration. Thus, VIV trigger floor can be deemed as the important boundary between damping and excitation regions. During each half oscillation period, the riser will enter the excitation region from damping region firstly and then go back to damping region again. The observed intermittent VIV exactly occurs within the excitation region. The transitions between excitation and damping regions can also explain the amplitude increase and decrease processes. Since the variation trend of current velocity is repeated for the first and second half periods, similar structural response happens twice during each complete oscillation period.

The temporal-spatial distributions of response displacement along the riser and time histories of main excited modal weights are shown in Column 2 of Fig. 13. Consistent with Fig. 14, intermittent VIV appears periodically. There is very little response displacement near 64s, 80s and 96s. Another significant feature is travelling wave, which has been described in Section 5.1. Because VIV response under long period sheared-oscillatory flow has obvious periodicity and temporal-spatial distributions change twice following similar regularity during each oscillation period, here just gives the time histories of main excited modal weights within the selected half period (i.e. [64s, 80s]) for clearer presentation. Compared with the sheared flow case, the dominant mode at different moments is time-varying as well. Whereas, the main excited modes become mode 11–mode 13, rather than mode 12–mode 14. Modal weight of every mainly excited mode no longer keeps constant but generally presents a shape of mountain peak, basically in accordance with the variation trend of current velocity.

5.3. Short oscillation period case

For short oscillation period case with \( V_{m,\text{top}} = 0.54 \) m/s and
The VIV response at different monitoring points of the 90 m riser is listed in Fig. 16, and the description of each sub-picture is identical with Fig. 12. Time histories of response displacement show that VIV is dominated by low frequency and high frequency response participates in simultaneously. There is no obvious periodicity or regularity during 20s (nearly 10 oscillation periods). According to wavelet plots, response energy mainly concentrates below 1 Hz, and time-frequency distribution keeps continuous with the variation of time. From the response amplitude spectra, different axial locations also have the same excited frequency bandwidth but unequal peak frequency. Compared with the top and bottom of the riser, the response energy concentrated at higher frequency component is evidently weaker for riser middle segment. Since the excited frequencies mainly distribute within low frequency region, the frequency spectra are not as dense as long oscillation period case and the response energy at the peak frequency is much stronger.

VIV under sheared-oscillatory flow with short oscillation period presents quite different response characteristics from long oscillation period case. In fact, when oscillation period is short, current velocity changes intensively in time domain. VIV excited modal order and frequency are both proportional to current velocity. As shown by the velocity time history of the high velocity case in Fig. 15, during each half period, there are two times for the riser to enter lower mode excitation region from damping region and higher mode excitation region respectively. When a structure is acted on certain higher-frequency load, it tends to give corresponding higher mode response, but such a formation of mode switch usually needs some necessary time. The absolute time interval for the riser locating in higher mode excitation region is too short to form complete mode transitions. Low frequency component will fleetly dominate the structural response again before the high frequency response forms stably enough to be easily captured. Therefore, for short oscillation period cases, it is the time interval within lower mode excitation region that dominates riser’s VIV and the excited frequencies are always restricted within low frequency region.

Column 3 of Fig. 13 gives the temporal-spatial distributions of response displacement along the riser and time histories of main excited modal weights. Compared with sheared flow and long oscillation period cases, the displacement temporal-spatial distributions with short oscillation period are relatively unstable. The vibration shape along the riser changes in time domain without obvious regularity. Whereas, it can still be found VIV is dominated by low modes and high-frequency response apparently exists in the structural response. Vibrational energy propagates along the riser from top to bottom in manner of travelling wave. Although the excited modes are different for three representative cases in this section, the maximum displacements of these three cases are approximately equal, which once again proves VIV is self-limited in response amplitude. Time histories of main excited modal weights have no visible periodicity neither, and the mainly excited mode orders become much lower than sheared flow and long oscillation period cases, being the lowest mode 1~mode 3. Mode 1 generally dominates the structural response, while mode 2 and mode 3 may turn the strongest during several short time intervals randomly. The unstable displacement temporal-spatial distributions and low modes dominant VIV can be explained with the intensively varying current velocity as well.
6. Effects of oscillation period and top maximum current velocity

VIV under sheared-oscillatory flow with different controlled parameters may present quite different response characteristics. This section will compare the numerical results of 20 simulated cases systematically, and respectively investigate the effects of oscillation period T and top maximum current velocity \( V_{\text{m,top}} \) on VIV response.

6.1. Oscillation period effect

As demonstrated in Section 5, the oscillation period of sheared-oscillatory flow has significant influence on riser’s VIV response. The five cases with \( V_{\text{m,top}} = 0.81 \text{ m/s} \) in Group 3 of Table 3 are selected to discuss how the response characteristics change with the variation of T. Fig. 17 gives the VIV response comparisons between these five cases. Left is the envelopes of RMS displacement along the 90 m riser, and right parts are response amplitude spectra at the riser’s midpoint. T of row 1–row 5 is 2s, 4s, 8s, 16s and 32s respectively.

From the envelopes along the riser, VIV RMS displacement gradually decreases with the increase of T. The maximum RMS displacement for the short oscillation period case with \( T = 2s \) is about 0.34D, while the corresponding value for the long oscillation period case with \( T = 32s \) becomes only 0.20D. In fact, if T is short, the boundary layer separation at the rear of the riser will not be obvious. When the riser is still located in the disturbance generated by the first half period flow, the current has reversed during the latter half period. In that case, the effect of fluid viscosity on structural response, which can also be deemed as the hydrodynamic damping, is not as important as long oscillation period situations. That is also the reason why intermittent VIV only appears under long oscillation period cases rather than short oscillation period ones. The reduction effect of hydrodynamic damping on structural response will gradually fade with the increase of T, so shorter T tends to cause larger RMS VIV response displacement.

Besides, although \( \Delta T \) turns larger for T from 2s to 32s, the variation span of RMS displacement between adjacent cases actually becomes smaller especially for the cases with \( T = 16s \) and 32s. That is because when T is long enough, vortices around the riser are able to shed completely as they do under steady flows. The further increase of T will not enhance the reduction effect of hydrodynamic damping any more. Another important effect of T is that RMS displacement envelop along the riser presents higher mode response with T increasing from short to long. When \( T = 2s \), the 3rd modal vibration shape can be captured, while the main excited modal order has been up to about 17 when \( T = 32s \).

Since the incoming flow is sheared-oscillatory i.e. the minimum current velocity is zero, the excited frequency bandwidth approximately starts from 0.2 Hz, corresponding to the first and lowest structural natural frequency. Response amplitude spectra at riser’s midpoint under sheared-oscillatory flow indicate that the highest excited frequency and the strongest response frequency both increase with the increase of T. The reasonable explanation is that intensively varying current velocity under short oscillation period case restricts the stable formation of high mode response, while longer T is easier to excite high frequency components. The essential analysis has been mentioned in Section 5.3. With T increasing from short to long, the excited frequency bandwidth broadens, and the distributions of response energy tend to be more dispersed, making the strength of peak frequency weaken evidently.

6.2. Top maximum current velocity effect

Current velocity is always one of the most important influence factors of VIV response, which is also true for sheared-oscillatory flow cases. This section compares the predicted results with identical T but different \( V_{\text{m,top}} \) to investigate the top maximum current velocity effect on VIV response under long and short oscillation period cases respectively. VIV RMS displacement envelopes along the 90 m riser and corresponding response amplitude spectra at riser’s midpoint for four cases with \( T = 32s \) are shown in Fig. 18, and

Fig. 16. VIV response at different monitor points of the 90 m riser for sheared-oscillatory flow case with short oscillation period \( (T_{\text{m,top}} = 0.54 \text{ m/s}; \ T = 2s) \), left: time histories of response displacement; middle: time-frequency distributions; right: corresponding response frequency spectra, \( z/L \) of row 1–3: 0.833, 0.500 and 0.167 respectively.
Fig. 19 contains the results of another four cases with $T = 2s$. $V_{m,\text{top}}$ of row 1–row 4 at right side is 0.27 m/s, 0.54 m/s, 0.81 m/s and 1.08 m/s.

For the long oscillation period cases (i.e. $T = 32s$), since VIV excited modal order is proportional to current velocity, the increasing $V_{m,\text{top}}$ makes higher mode excited by VIV. For example, mode 7 dominates the structural response when $V_{m,\text{top}} = 0.27$ m/s while the main excited mode reaches 21 when $V_{m,\text{top}} = 1.08$ m/s. Different from sheared flow cases, the RMS displacement no longer keeps approximate with different $V_{m,\text{top}}$. The maximum RMS displacement enlarges when $V_{m,\text{top}}$ increases from 0.27 m/s to 1.08 m/s. It can be explained with the help of comparisons between current velocity time histories for low and high velocity cases, as shown in Fig. 15. During the same half period, with the current velocity increasing, the time interval for the riser locating within excitation region (above VIV trigger floor) turns longer, which is marked by the blue dash dotted lines. That means the excitation force has more time to aggravate VIV. The reduction effect of hydrodynamic damping on structural response becomes weaker simultaneously due to the shorter time interval within damping region.

From the response amplitude spectra at riser’s midpoint, the excited frequency bandwidth consists of two parts, i.e. low frequency and high frequency components. Because the time interval for the riser transiting from lower to higher mode excitation region is relatively short, the middle frequencies have no enough time to be excited stably. With the increase of $V_{m,\text{top}}$, the high frequency components become more and gradually move to higher frequency region, making the whole excited frequency bandwidth broader. While the low frequency component is always fixed at the first natural frequency and turns stronger with higher $V_{m,\text{top}}$.

For the short oscillation period cases (i.e. $T = 2s$), the magnitude of RMS displacement also enlarges due to longer excitation time intervals from low to high velocity case. But the effect of $V_{m,\text{top}}$ on VIV response has a little difference from long oscillation period cases. The envelops of RMS displacement along the riser all present low mode response, mainly dominated by mode 1. The high-frequency response is still not very obvious even when $V_{m,\text{top}}$ is
up to 1.08 m/s. As short $T$ prevents the high-frequency response from forming stably, the response amplitude spectra only have low frequency components. The increasing $\dot{V}_{m,top}$ does not change the peak frequency (always the first natural frequency) but gradually strengthens its response energy. The excited frequency bandwidth broadens with the increase of $\dot{V}_{m,top}$ as well, but the variation span between adjacent cases is much smaller than long oscillation period situation.

7. Conclusions

Based on a time domain force-decomposition model, the VIV response characteristics for flexible risers under sheared and shear-oscillatory flows are investigated comprehensively. The adopted hydrodynamic force coefficient database completely originates from the experimental data of cylinder forced vibration test. By validating against published laboratory experiments, the VIV numerical model is proved available for both sheared and oscillatory flow cases. 20 sheared-oscillatory flow cases with different combinations of oscillation period $T$ and top maximum current velocity $V_{m,top}$ for a large-scale 90 m riser model are designed and simulated to analyze the respective VIV response characteristics under long and short oscillation period cases. Then, the effects of $T$ and $V_{m,top}$ on VIV response are systematically discussed. Some conclusions are drawn as follows:

(1) By adapting the hydrodynamic force model in the traditional frequency domain prediction tools and improving the lock-in judgement criterion for time domain analysis, the present VIV numerical model is applicable for much more extensive cases. With qualitative comparison and quantitative analysis, the adopted time domain numerical model shows better performance than traditional frequency domain tool.

(2) Owing to the sheared flow profile at each moment, VIV response under sheared-oscillatory flow has a feature of travelling wave, and vibrational energy propagates along the riser from high to low velocity region. The excited frequency bandwidth at different axial locations of the riser is identical but the strongest response frequency may be unequal. Compared with the middle segment of the riser, high-frequency response is easier to be excited for the top and bottom segments.

(3) For long oscillation period cases, intermittent VIV presents obvious regularity, and similar response repeats twice during each complete oscillation period. The displacement amplitude and main excited modal weight generally modulate following the similar variation trend of current velocity. While for short oscillation period cases, structural response no longer has visible periodicity. VIV is dominated by low mode with some high frequency response participating in. Multi-frequency superposed dynamic response is relatively unstable in time domain.

(4) With the increase of $T$, the excited frequency bandwidth broadens and dominant frequency turns higher, while the response energy at peak frequency becomes weaker. Longer $T$ results in smaller VIV RMS displacement. When $\dot{V}_{m,top}$ increases, RMS displacement turns larger. Higher $\dot{V}_{m,top}$ makes VIV dominated by higher modes under long oscillation period cases. While for short oscillation period cases, the peak frequency is always fixed at low order natural frequency with different $\dot{V}_{m,top}$, but its response energy strengthens with $\dot{V}_{m,top}$ increasing.

VIV prediction of flexible risers under unsteady flows is still a very challenging research issue. This paper could be deemed as a preliminary attempt to investigate the VIV under the relatively complicated sheared-oscillatory flows, and to discuss the response characteristics differences from steady flow or pure oscillatory flow cases. The obtained conclusions could make efforts to achieve a deeper insight of VIV combined with the influence of top-end platform surge motions. In fact, the proposed numerical model by this paper is not optimal yet, and its performance needs to be improved continuously, especially for the unsteady flow cases with ultra-small $KC$ number under which the VIV hydrodynamic characteristics are quite special and have not been understood clearly enough. Besides, the proposed numerical model cannot predict the higher harmonics precisely at the current stage. Therefore, the further research work should be carried out with the help of more experimental investigations in the future.

Acknowledgement

This paper is based on the projects supported by the National Natural Science Foundation of China (Grant No. 51579146, 51490674) and State Key Laboratory of Ocean Engineering, Shanghai jiao Tong University (GKZD010074).
References


