Stability analysis of deepwater compliant vertical access riser about parametric excitation

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A B S T R A C T
If heave motion in the platform causes horizontal parametric vibration of a Compliant Vertical Access Riser (CVAR), the riser may become unstable. A combination of riser parameters lies in the unstable region aggravates vibrational damage to the riser. Change of axial tensile stress in the riser combined with its natural frequency and mode shape change results in mode coupling. In accordance with the state transition matrices of the riser in the coupled and uncoupled states, the stable and unstable regions were obtained by Floquet theory, and the vibration response under different conditions was obtained. The parametric excitation of the CVAR is shown to occur mainly in first-order unstable regions. Mode coupling may cause parametric excitation in the least stable regions. Damping reduces the extent of unstable regions to a certain extent.

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1. Introduction

Due to the limitations of traditional riser forms and improvements in automation, information and intelligence in the petroleum industry, a new type of riser system adapted to deep water is urgently required to increase market competitiveness. The CVAR is a type of riser currently at the research stage. Its compliance compensates for the motion of the platform. Because it is vertically connected to the well, multiple operations on the well can be performed from the platform, greatly reducing operating costs. Hsu (1975) first proposed analysis of marine cable parametric resonance. Mao and Yang (2016) studied the parametric pitch instability of a deep-draft semi-submersible platform in irregular waves. The results emphasized the importance of predicting deep-draft semi-submersible platform parametric pitch in irregular waves during the design process. Friedmann et al. (2010) proposed two efficient numerical methods for dealing with the instability of linear periodic systems. Park and Jung (2002) used the finite element method to study the dynamic response of a riser subjected to the combined action of parametric excitation and wave current excitation, and calculated the parametric instability region. Fujiwara et al. (2011) used a large-scale model test to study the effect of parametric excitation on vortex-induced vibration. Patel and Seyed (1995) studied the unstable region of a tension leg platform for different parametric excitation. Suzuki et al. (2004) conducted a small-scale model test with parametric excitation at the top of the riser. They simulated a freely suspended 3000 m riser, and showed that parametric resonance had considerable impact on riser safety. Chatjigeorgiou and Mavrakos (2005) considered only the first two modes in their analysis of a simplified mathematical model of the riser’s parametric resonance, but did not consider modal coupling for dynamic analysis. Thampi and Niedzwiecki (1992) discussed the nonlinear dynamic response of marine risers using the Markov method. Haquang et al. (1987) studied the nonlinear dynamic response of marine risers under combined excitation. Kuiper et al. (2008) analyzed the parametric stability of the riser using Floquet theory, and obtained two mechanisms of parametric motion instability, one being the parametric resonance instability caused by the cyclical time-varying axial tension and the other was when the platform motion was such that the pulsating tension in the riser was being studied changed the pressure in the riser, and it became unstable. Chatjigeorgiou and Mavrakos (2002) studied the case in which the frequency of the nonlinear resonance of the riser approximated the parametric excitation frequency, and considered the influence of damping. Their study showed that the instability caused by the parametric excitation cannot be ignored. Brouwers (2011) studied the asymptotic solution of the Mathieu function with its natural frequency and mode shape change.
for random parametric excitation and nonlinear damping.

To date, CVAR studies have not involved stability analysis, but if the parametric vibration generated by the motion of the platform is neglected in CVAR design, it is likely to lead to catastrophic consequences. In this study, a theoretical model of the deep sea CVAR is established. According to Vandiver’s theory, the CVAR is equivalent to a Top Tension Riser (TTR). The linear hydrodynamic drag force is given, and nonlinear damping is considered. Taking into account the variation of the natural frequency and mode shape caused by the change of axial tension in the CVAR, the parametric excitation stability characteristics of the CVAR were analyzed using Floquet theory.

2. Theory

The vibration in the riser subjected to parametric excitation is given by the partial differential equation

\[ E I \frac{\partial^4 \alpha(z, t)}{\partial z^4} - \frac{\partial}{\partial z} \left[ T(z, t) \frac{\partial \alpha(z, t)}{\partial z} \right] + m \frac{\partial^2 \alpha(z, t)}{\partial z^2} = \tilde{f}(z, t), \]

where \( \tilde{f}(z, t) \) is the external force per unit length acting on the riser; \( \alpha(z, t) \) is the horizontal displacement; \( z \) is the vertical coordinate along the riser; \( t \) is the time; \( E \) is the elastic modulus; \( I \) is the moment of inertia; and

\[ I = \frac{\pi}{64} \left( D^4 - d^4 \right), \]

where \( D \) is the diameter for the riser and \( d \) is its inside diameter; and \( m \) is given by

\[ m = m_r + m_f + m_a, \]

where \( m_r \) is the mass per unit length of the riser, obtained from

\[ m_r = \frac{1}{4} \rho_d \pi \left( D^2 - d^2 \right), \]

in which \( \rho_d \) is the material density, and \( m_f \) is the mass per unit length of the internal fluid, obtained from

\[ m_f = \frac{1}{4} \rho_f \pi d^2, \]

in which \( \rho_f \) is the density of the internal fluid.

In Eq. (3), \( m_a \) is the added mass per unit length, given by

\[ m_a = \frac{1}{4} C_d \rho_w \pi D^2, \]

where \( \rho_w \) is the seawater density; \( C_d \) is the added mass coefficient; and \( T(z, t) \) is the effective tension, reflecting the coupling of parametric excitation and structural response:

\[ T(z, t) = T(z) + T(t), \]

where \( T(z) \) is the static component of the effective tension, calculated from

\[ T(z) = T_t - wz, \]

in which \( T_t \) is the tension at the top of the riser; \( w \) is the weight per unit length of the riser immersed in water; and \( T(t) \) is the dynamic component of the effective tension (i.e., the tension in the riser over time due to the heave motion of the platform), given by

\[ T(t) = ka \cos(\omega t), \]

where \( k \) is a tension transfer coefficient; \( a \) is the amplitude of platform heave; and \( \Omega \) is the frequency of platform heave (\( a \) and \( \Omega \) both act as excitation parameters in the riser).

Regardless of the wave and current effect, Eq. (1) reduces to a second-order ordinary differential equation by Galerkin’s method. A solution to Eq. (1) is written in the form:

\[ x(z, t) = \sum_{n=1}^{\infty} \phi_n(z) q_n(t), \]

where \( n \) is the mode order; \( \phi_n(z) \) is the mode shape function, or displacement amplitude of the \( n \)th normal mode; and \( q_n(t) \) is a time function.

In a mathematically simplified form, Eq. (1) yields

\[ \ddot{q}_m + \frac{\omega_n^2}{m} q_m - \frac{ka \cos(\omega t)}{m} \sum_{n} f_{mn} q_n = 0 (m = 1, 2, \cdots, n), \]

where \( m \) is the mode order; \( \omega_n \) is the natural frequency of the \( n \)th normal mode; and \( f_{mn} \) is the mode coupling coefficient of the riser, given by

\[ f_{mn} = \int_0^L \phi_m \phi_n dz \quad (m = 1, 2, \cdots, n), \]

in which \( L \) is the riser length, and \( \phi_n'' \) is the second derivative.

Since the drag force is nonlinear, the superposition principle cannot be applied. To avoid this problem, the linearized Morison equation is used (Krolkowski and Gay, 1980). The linearization is described by

\[ f_{\text{drag}}(z, t) = -\frac{1}{2} \rho_w \pi d \sum_{n=1}^{N} \left| \phi_n(z) \right|^2 |\dot{q}_n(z)| \phi_n(z) \phi_n''(z), \]

where \( C_d \) is the drag coefficient; \( \dot{q}_n \) is the first derivative of time; \( \phi_{\text{max}} \) is the maximum value of the displacement amplitude; and \( \alpha_n \) is a mode-dependent coefficient given by

\[ \alpha_n = \frac{1}{\phi_{\text{max}}^2} \int_{-\infty}^{\infty} \left[ \int_{-\infty}^{\infty} \left( \phi_n(z) \dot{q}_n(t) \right)^2 \phi_n(z) \dot{q}_n(t) \right] dz dt \]

\[ \int_{-\infty}^{\infty} \left( \phi_n(z) \dot{q}_n(t) \right)^2 dz dt. \]

Eq. (13) leads to the following set of differential equations for the riser:

\[ \ddot{\dot{q}}_m + \frac{\omega_n^2}{m} q_m - \frac{ka \cos(\omega t)}{m} \sum_{n} f_{mn} q_n = 0 (m = 1, 2, \cdots, n), \]

where \( \ddot{\dot{q}}_m \) is the second derivative of time, and \( C \) is the damping coefficient:

\[ C = \frac{\rho_w DC_d}{2m \phi_{\text{max}}^2}. \]

To use the Floquet theory, the differential equation for every normal mode should be rewritten to a system of first-order differential equations by defining
\[
x_n = q_n(t) \quad (n = 1, 2, 3, \cdots, N),
\]
\[
X_{n+1} = q_n(t),
\]
where \(N\) is the number of degrees of freedom that is taken into account. In the present study, \(N = 8\), so that Eq. (15) becomes, for every normal mode:

\[
\begin{bmatrix}
0 & \cdots & 0 & 1 & \cdots & 0 \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & 0 & \cdots & 1 \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
\frac{k_a \cos(\Omega t)}{m} & \cdots & \frac{k_a \cos(\Omega t)}{m} & -C_{a1} \omega_1 & \cdots & 0 \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
-\omega_8^2 & \cdots & -\omega_8^2 & \frac{k_a \cos(\Omega t)}{m} & \cdots & -C_{a8} \omega_8
\end{bmatrix}
\begin{bmatrix}
x_1 \\
\vdots \\
x_8 \\
\vdots \\
x_{16}
\end{bmatrix}
= \begin{bmatrix}
x_1 \\
\vdots \\
x_8 \\
\vdots \\
x_{16}
\end{bmatrix}
\]

Eq. (18) is expressed in matrix form as

\[
\dot{X} = A(t)X,
\]
where \(X = (x_1, x_2, \ldots, x_N)^T; A(t)\) is a \((2N \times 2N)\) periodic matrix; \(A(t) = A(t + \frac{2\pi}{T}); \) and \(X = (x_1, x_2, \ldots, x_N)^T\).

The stability of Eq. (19) can be determined using Floquet theory (Yang and Li, 2009), which provides a quantitative measure of the stability of a system of first-order differential equations. The assumption \(\Phi(t, 0)\) is the state transition matrix of Eq. (19), and at an arbitrarily chosen moment in time is given by

\[
\dot{\Phi}(t, 0) = A(t)\Phi(t, 0), \quad t_0 = 0.
\]

The state transition matrix at the end of one period, \(\Phi(T, 0)\), is computed numerically by integrating the matrix differential equations over one period, \(T = \frac{2\pi}{\Omega}\) with the initial condition \(\Phi(0, 0) = I\);. If one of the eigenvalues of \(\Phi(T, 0) > 1\), it is concluded that the riser is unstable.

When the platform heave frequency and the natural frequency of the riser satisfy a certain relationship, the riser generates a parametric resonance. The riser mode directly determines the mode-coupling coefficient and the mode-dependent coefficient; thus, the natural vibration characteristics of the riser are closely related to parametric vibration. Since the small displacement at the top of the riser has little effect on the linear stability analysis, the riser may be considered to be hinged at both ends; that is,

\[
\frac{\partial^2 x(z, t)}{\partial z^2} \Big|_{z=0} = 0, \quad \frac{\partial^2 x(z, t)}{\partial z^2} \Big|_{z=L} = 0, \quad \frac{\partial^2 x(z, t)}{\partial z^2} \Big|_{z=L} = 0.
\]

Since the analysis of the variable tension is based on the constant tension, the influence of gravity is firstly ignored in performing the mode analysis of the riser.

Ignoring the self-weight of the riser, the internal tensile stress is equal throughout, i.e., \(T(z, t) = T_0\). The natural frequency of the riser is then obtained using the separation of variables method:

\[
\omega_n = \sqrt{\frac{n^2 \pi^2 E I}{mL^2} + \frac{n^2 \pi^2 T}{mL^2}}.
\]

The mode shape of the riser is given by

\[
\phi(z) = B_1 \sin \left( \frac{\pi z}{L} \right).
\]

where the coefficient \(B_1\) is determined by the initial conditions. Substituting Eq. (23) into Eq. (12):

\[
f_{mn} = \begin{cases} \frac{n^2 \pi^2}{L^2} & m = n \\ 0 & m \neq n \end{cases}
\]

At this time, no coupling occurs between the various modes. When the riser vibrates, the various modes do not interfere, and the riser exhibits single-mode vibration under parametric excitation.

When the self-weight of the riser is taken into account, however, the internal tension changes linearly along the axis; that is, \(T(z) = T_0 + wz\). The natural frequency of the riser is then calculated (Vandiver and Li, 2005):

\[
\omega_n = \sqrt{\frac{T(z)}{2E I} + \frac{\rho g z}{2E I} - \frac{T(z)}{2E I} dz} = n\pi.
\]

For low-order modes, the mode shape of the riser is expressed by (Senjanovic et al., 2006):

\[
\phi(z) = B_1 \sin \left( \frac{z}{L} \sqrt{\frac{T(z)}{2E I} + \frac{\rho g z}{2E I}} \right).
\]

### Table 1

<table>
<thead>
<tr>
<th>CVAR parameters</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outside diameter</td>
<td>(D)</td>
<td>0.3 m</td>
</tr>
<tr>
<td>Wall thickness</td>
<td>(t)</td>
<td>0.038 m</td>
</tr>
<tr>
<td>Riser length</td>
<td>(L)</td>
<td>2601 m</td>
</tr>
<tr>
<td>Water depth</td>
<td>(H)</td>
<td>2438 m</td>
</tr>
<tr>
<td>Elastic modulus</td>
<td>(E)</td>
<td>2.07E11 Pa</td>
</tr>
<tr>
<td>Added mass coefficient</td>
<td>(C_g)</td>
<td>1</td>
</tr>
<tr>
<td>Drag coefficient</td>
<td>(C_d)</td>
<td>1</td>
</tr>
<tr>
<td>Material density</td>
<td>(\rho)</td>
<td>7850 kg/m$^3$</td>
</tr>
<tr>
<td>Seawater density</td>
<td>(\rho_w)</td>
<td>1025 kg/m$^3$</td>
</tr>
<tr>
<td>Fluid density in the riser</td>
<td>(\rho_f)</td>
<td>800 kg/m$^3$</td>
</tr>
<tr>
<td>Gravity acceleration</td>
<td>(g)</td>
<td>9.807 m/s$^2$</td>
</tr>
</tbody>
</table>
Due to the influence of self-weight, the tension in the riser gradually decreases with decreasing water depth. This is equivalent to reducing the rigidity of the riser, and results in mode coupling. When the riser vibrates, the various modes affect each other, and the riser exhibits multi-mode vibration in parametric excitation conditions.

Adopting the structural configuration of the CVAR suggested by Mungall et al. (2004), the parameters of the CVAR model analyzed in this study are listed in Table 1. This model comprises different buoyancy and gravity blocks in different sections of the CVAR. Their position and length need to be designed using detailed and reasonable parameters to meet the requirements of the specific task. Most CVARs are divided into three regions: upper, transition and lower regions. To ensure that the CVAR exits the wellhead at a subvertical angle, different buoyancy factors must be set at different positions to simulate the effect of the buoyancy and gravity blocks. The design parameters of each section of the CVAR in this study were obtained by static analysis of its initial position, as listed in Table 2.

Depending on the magnitude of the relevant parameters in Tables 1 and 2, MATLAB software was used to obtain an initial-position chart for static equilibrium of the CVAR (Fig. 1). In order to verify the rationality of the buoyancy factor design, static analysis of the buoyancy block corresponding to given buoyancy factors was conducted using OrcaFlex dynamic analysis software. The static equilibrium position of the CVAR is shown in Fig. 2. Comparing the position information in Figs. 1 and 2, it can be seen that the buoyancy factor setting of each section of the CVAR is reasonable and feasible; the initial position of the CVAR was used for subsequent analysis.

Static analysis of the CVAR in the static equilibrium state yields the axial tension curve shown in Fig. 3. Since the internal tension in the CVAR is affected by its self-weight and also by the buoyancy and gravity blocks, a plot of the variation of the internal tension along the length of riser does not result in a smooth curve. This poses the problem that the derivative of the tension is needed to obtain the coupling coefficient, but if the segmented tension function is used, the derivative is discontinuous and integration cannot be performed. It is therefore necessary to fit the tension curve to the data. The fitted curve is compared with the original curve in Fig. 4. As discussed above, Vandiver’s theory states that the CVAR is equivalent to a TTR in obtaining the natural frequency of the riser, and the internal tension is distributed along its length.

**Table 2** Parameters for section design of the CVAR.

<table>
<thead>
<tr>
<th>Section</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>The lower region</td>
<td>Riser length 490 m</td>
</tr>
<tr>
<td></td>
<td>Buoyancy block arrangement 300–490 m</td>
</tr>
<tr>
<td></td>
<td>Buoyancy factor 6</td>
</tr>
<tr>
<td>The transition region</td>
<td>Riser length 416 m</td>
</tr>
<tr>
<td></td>
<td>Buoyancy block arrangement 491–907 m</td>
</tr>
<tr>
<td></td>
<td>Buoyancy factor 2</td>
</tr>
<tr>
<td>The upper region</td>
<td>Riser length 1695 m</td>
</tr>
<tr>
<td></td>
<td>Buoyancy block arrangement 908–996 m</td>
</tr>
<tr>
<td></td>
<td>Buoyancy factor (-1.5)</td>
</tr>
</tbody>
</table>

**Fig. 1.** Configuration of the CVAR.

**Fig. 2.** OrcaFlex simulated CVAR configuration.

**Fig. 3.** Axial tension along the length of riser.
3. Stability analysis

3.1. Single-mode stability analysis

When the tension in the riser is constant, no coupling occurs between the modes. The value of the mode-dependent coefficient, $a_n = 0.7205$, was obtained from Eq. (15). A MATLAB routine was used to calculate the numerical value of $F(T, 0)$ and the corresponding eigenvalues of this matrix. The MATLAB routine applies the Runge-Kutta method to solve the initial value problem defined by Eq. (20). The results are shown as stability charts in Figs. 5 and 6. The combinations of $\alpha$ and $\Omega$ that lead to instability are indicated by red dots, which together indicate several unstable regions. Fig. 5 shows that when the platform is undergoing low-amplitude motion, it is mainly the first- and second-order unstable regions that are excited — that is, $\Omega = 2\omega_h$ and $\Omega = \omega_h$. As the amplitude of the platform motion increases, the higher-order unstable regions are gradually provoked. This is consistent with the parametric resonance law deduced by Brugmans (2005) using the small parameter method, which states that parametric resonance occurs when the platform heave frequency and the riser natural frequency satisfy the following relationship:

$$\Omega = \frac{2\omega_h}{N} \quad (N = 1, 2, 3, \cdots), \quad (27)$$

where $N$ determines the order of riser instability: $N = 1(\Omega = 2\omega_h)$ corresponds to first-order unstable regions of the riser; $N = 2(\Omega = \omega_h)$ corresponds to second-order unstable regions of the riser; and so on. If it is assumed that the range of the wave period is between 5 and 13 s (corresponding frequency 0.48–1.25 rad/s), then, when the natural frequency of CVAR = 1.25 rad/s, the seventeenth order is calculated to be the highest order; however, since the higher-order modes correspond to smaller frequencies of the unstable regions and do not conform to actual engineering conditions, this study mainly considered the effect of the first eight modes. The frequency analysis for unstable regions is shown in Table 3.

Without considering the effect of damping, the first-order ordinary differential equations for parametric excitation are

$$\begin{bmatrix} x_1 \\ \vdots \\ x_8 \\ x_9 \\ \vdots \\ x_{16} \end{bmatrix} = \begin{bmatrix} 0 & \cdots & 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 1 \\ -\omega_1^2 + \frac{ka \cos(\Omega t)}{m} f_{11} & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & -\omega_8^2 + \frac{ka \cos(\Omega t)}{m} f_{88} & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_8 \\ x_9 \\ \vdots \\ x_{16} \end{bmatrix}, \quad (28)$$

This implies that Eq. (28) contains two variables: the platform heave amplitude $\alpha$ and the platform heave frequency $\Omega$. Fig. 5 shows the combinations of $\alpha$ and $\Omega$ for which this CVAR configuration is unstable if no damping is present.

The unstable regions in Fig. 5 are mainly first- and second-order. The second-order unstable regions are not completely displayed because of they partly overlap.

In reality, the riser is surrounded by water. The resistance caused by the surrounding water is referred to as fluid drag. Considering the effect of damping, the first-order ordinary differential equations for parametric excitation are:
Fig. 6. Stability charts of the CVAR, including damping.
achieve parameter combinations when the platform is experi-
ence and amplitude of the heave motion and is also most likely to
vent parametric excitation may begin with modifying riser damp-
order unstable regions. The stable region of the riser increases as
reduced the unstable regions of the riser, especially for the higher-
1.2, using Floquet theory.

Table 3
Frequency (rad/s) corresponding to unstable regions of the CVAR.

<table>
<thead>
<tr>
<th>Mode order</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>First-order unstable regions ($\Omega - \omega_n$)</td>
<td>0.142</td>
<td>0.286</td>
<td>0.430</td>
<td>0.572</td>
<td>0.716</td>
<td>0.862</td>
<td>1.006</td>
<td>1.152</td>
</tr>
<tr>
<td>Second-order unstable regions ($\Omega - \omega_n$)</td>
<td>0.071</td>
<td>0.143</td>
<td>0.215</td>
<td>0.286</td>
<td>0.358</td>
<td>0.431</td>
<td>0.503</td>
<td>0.576</td>
</tr>
</tbody>
</table>

Table 4
Mode coupling coefficients of first eight modes ($10^{-6}$).

<table>
<thead>
<tr>
<th>$f_1$</th>
<th>$f_2$</th>
<th>$f_3$</th>
<th>$f_4$</th>
<th>$f_5$</th>
<th>$f_6$</th>
<th>$f_7$</th>
<th>$f_8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.8</td>
<td>0.2</td>
<td>1.8</td>
<td>1.2</td>
<td>0.1</td>
<td>0.5</td>
<td>0.4</td>
<td>0.1</td>
</tr>
<tr>
<td>0.1</td>
<td>0.1</td>
<td>0.4</td>
<td>3.4</td>
<td>2.9</td>
<td>0.6</td>
<td>1.1</td>
<td>0.7</td>
</tr>
<tr>
<td>1.5</td>
<td>1.7</td>
<td>3.1</td>
<td>29.5</td>
<td>4.7</td>
<td>10.1</td>
<td>8.0</td>
<td>1.6</td>
</tr>
<tr>
<td>0.9</td>
<td>4.2</td>
<td>6.4</td>
<td>4.7</td>
<td>7.3</td>
<td>5.9</td>
<td>6.5</td>
<td>14.8</td>
</tr>
<tr>
<td>0.1</td>
<td>2.7</td>
<td>2.7</td>
<td>1.1</td>
<td>0.4</td>
<td>6.6</td>
<td>66.1</td>
<td>10.8</td>
</tr>
<tr>
<td>0.5</td>
<td>0.6</td>
<td>5.1</td>
<td>10.1</td>
<td>1.1</td>
<td>14.8</td>
<td>10.7</td>
<td>20.6</td>
</tr>
<tr>
<td>0.3</td>
<td>1.0</td>
<td>1.1</td>
<td>7.9</td>
<td>1.1</td>
<td>14.8</td>
<td>10.7</td>
<td>13.4</td>
</tr>
<tr>
<td>0.1</td>
<td>0.7</td>
<td>1.8</td>
<td>1.6</td>
<td>11.5</td>
<td>20.4</td>
<td>13.4</td>
<td>116.1</td>
</tr>
</tbody>
</table>

To investigate the effect of different damping coefficients on the
stability of the CVAR, the stability charts in Fig. 6 were obtained for
a damped riser system having C values of 0.1, 0.2, 0.5, 0.8, 1.0 and
1.2, using Floquet theory.

Compared to the no-damping case, linear damping rapidly
reduced the unstable regions of the riser, especially for the higher-
order unstable regions. The stable region of the riser increases as C
is increased. For larger values of C, the minimum critical heave
amplitude of the platform increases. Therefore, measures to pre-
vent parametric excitation may begin with modifying riser damp-
ing. Parametric vibration has the greatest influence on the first-
order unstable region. Fig. 6 clearly shows that the combined fre-
quency and amplitude of the heave motion and is also most likely to
achieve parameter combinations when the platform is experi-
encing heave motion.

3.2. Multi-mode stability analysis

When the tension changes along the riser axis, Eqs. (12) and (26)
give the mode coupling coefficient of the first eight modes, as shown in Table 4. Without considering the effect of damping, the first-order ordinary differential equations under parametric excitation are:

\[
\begin{bmatrix}
    x_1 \\
    \vdots \\
    x_8 \\
    x_9 \\
    \vdots \\
    x_{16}
\end{bmatrix} = 
\begin{bmatrix}
    0 & \cdots & 0 & 1 & \cdots & 0 \\
    \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\
    0 & \cdots & 0 & 0 & \cdots & 1 \\
    \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\
    0 & \cdots & -\omega_n^2 & + \frac{\kappa k \cos(\Omega t)}{m} f_{11} & 0 & \cdots & -C \alpha_1 \omega_1 \\
    \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\
    0 & \cdots & -\omega_n^2 & + \frac{\kappa k \cos(\Omega t)}{m} f_{18} & 0 & \cdots & -C \alpha_8 \omega_8
\end{bmatrix}
\cdot 
\begin{bmatrix}
    x_1 \\
    \vdots \\
    x_8 \\
    x_9 \\
    \vdots \\
    x_{16}
\end{bmatrix}
\]

The unstable regions in the coupled case in Fig. 7 is significantly
larger than for the uncoupled case in Fig. 5. In the coupled case, the
unstable regions of the CVAR are dominated by the first-order
unstable regions. The critical line between the stable and the un-
stable regions is complex due to the resonances of combination
($\Omega = \omega_n + \omega_{n+1}$) that occur between modes.

Considering the effect of damping, the first-order ordinary
differential equations for parametric excitation are:

\[
\begin{bmatrix}
  \ddot{x}_1 \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
\end{bmatrix}
= -\omega_1^2 + \frac{ka \cos(\Omega t)}{m} f_{11} \begin{bmatrix}
  1 \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  0 \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
\end{bmatrix}
\begin{bmatrix}
  0 \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  0 \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
\end{bmatrix}
- C_{\alpha \Omega} \omega_1 \begin{bmatrix}
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
\end{bmatrix} + \frac{ka \cos(\Omega t)}{m} f_{81} \begin{bmatrix}
  0 \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
\end{bmatrix}
\begin{bmatrix}
  0 \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
\end{bmatrix}
- C_{\alpha \Omega} \omega_1 \begin{bmatrix}
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
\end{bmatrix} + \frac{ka \cos(\Omega t)}{m} f_{88} \begin{bmatrix}
  0 \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
\end{bmatrix}
\begin{bmatrix}
  0 \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
\end{bmatrix}
\end{bmatrix}
\begin{bmatrix}
  x_1 \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
  \ddots \\
\end{bmatrix}
\end{equation}
Fig. 8. Stability charts of the CVAR, including damping.

(a) $C = 0.5$

(b) $C = 0.8$

(c) $C = 1.0$

(d) $C = 1.2$
stable and the other modes are gradually attenuated. The fact that
they are not excited by the parametric excitation indicates that the
riser is also stable, but because it is within the critical area, this
combination should also be avoided.

5. Conclusion

The conclusions drawn from this study are as follows:

(1) Because a CVAR is affected by the heave motion of the plat-
form, periodic changes in the axial tension are caused, which
may easily lead to parametric resonance in the riser. Since
the length of the CVAR in this study is designed for deep
water, its natural frequency is small and the values of the
mode frequencies do not differ greatly. Therefore, within the
range of particular platform heave frequencies, modes of
other orders are excited. From the stability charts of the
CVAR derived from the analysis, it is clear that the first two
unstable regions are readily excited; of these, the main one is
the first-order unstable region. If the CVAR is subjected to a
parametric excitation combination that lies within an un-
stable region, the CVAR undergoes parametric resonance,
and therefore it is unsafe.

(2) Due to the particular structural arrangement of the CVAR in
the study, appropriate lengths of buoyancy block and gravity
block were located at different positions along the length of
the riser. Therefore, the axial tensile force within the CVAR
varies along its length, causing a coupling between different
modes. Considering the coupling effect on the parametric
excitation stability of the CVAR, the first-order instability and
the combination of resonances are both equivalent to an
increase in the magnitude of the unstable regions. Therefore,
if the axial tension changes in the CVAR, the mode coupling
effect cannot be ignored.
If damping is present, the unstable regions of the CVAR decrease significantly. The range of the stable regions of the CVAR increase with increasing damping coefficient; so, an appropriate increase in structural damping plays a role in increasing CVAR stability.

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