Performance Improvement of Model Predictive Control Using Control Error Compensation for Power Electronic Converters Based on the Lyapunov Function

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Abstract

This paper proposes a model predictive control based on the discrete Lyapunov function to improve the performance of power electronic converters. The proposed control technique, based on the finite control set model predictive control (FCS-MPC), defines a cost function for the control law which is determined under the Lyapunov stability theorem with a control error compensation. The steady state and dynamic performance of the proposed control strategy has been tested under a single phase AC/DC voltage source rectifier (S-VSR). Experimental results demonstrate that the proposed control strategy not only offers global stability and good robustness but also leads to a high quality sinusoidal current with a reasonably low total harmonic distortion (THD) and a fast dynamic response under linear loads.

Key words: Discrete Lyapunov function, Lyapunov stability theorem, Model predictive control, Power electronic converters, Robustness

I. INTRODUCTION

Power electronic converters provide many benefits to the economy and in people's livelihoods. The control strategies of converters have gradually become a research beacon in terms of the requirements of power quality [1]-[3].

With the development of high speed and powerful digital signal processors (DSPs) and microprocessors, growing attention and interest have been paid to the use of model predictive control (MPC) in power electronics. Generally, the MPC techniques applied to power electronics have been classified into two main categories: continuous control set MPC (CCS-MPC) and finite control set MPC (FCS-MPC) [4], [5]. In CCS-MPC, a modulator using sinusoidal pulse width modulation (SPWM) or space vector pulse width modulation (SVPWM) generates switching states starting from the continuous output of a predictive controller [6], [7]. On the other hand, FCS-MPC takes advantage of the discrete nature of power converters for solving optimization problems [8], [9]. Without the modulation stage, FCS-MPC applies direct control action to the converter.

The conventional FCS-MPC employs one voltage vector during one sampling period, and needs a high sampling frequency to achieve a better performance. In one sampling period, FCS-MPC consists of two main steps. The first step is prediction of the behavior for the next sampling instant for all possible voltage vectors and evaluation of the cost function for each prediction. The second step is to find the optimal voltage vector based on the traversal algorithm. This fact increases the computation burden [10]-[12]. Furthermore, due to the limited number of voltage vectors in the converter, the performance improvement caused by the conventional FCS-MPC is limited, and the THD of the controlled variable is higher than that of the conventional control based on a modulator [13], [14].

The Lyapunov function based control strategy is powerful for considering global stability and robustness. Several studies of this strategy have been published in the literature [15]-[21]. Using the discrete energy function to achieve
superior performance and global asymptotic stability for the boost PFC converters in electric vehicles was presented in [15]-[17]. In [18]-[19], a Lyapunov function based control approach was applied for a single phase inverter with a LCL filter and a single phase inverter with a LC filter, respectively. In [20-21], a three phase AC-DC voltage source rectifier achieved a fast dynamic performance by adopting the Lyapunov function based control strategy. In particular, this control approach was modified with a model predictive control in [21]. In this paper, a model predictive control based on the discrete Lyapunov function is proposed to improve control performance by adding an error term of the controlled variable and the reference variable to the control law. The discrete model of the S-VSR and the principle of the conventional FCS-MPC are elaborately described in Section II. The control law is calculated using the Lyapunov stability theorem based on the discrete Lyapunov function and the proposed control strategy is given in Section III. In Section IV, the control coefficient, α, of the error term in the control law is selected by analyzing its influence on the steady state and the dynamic performance in terms of stability and robustness. In Section V, the performance of the proposed method for the S-VSR is investigated with an experimental system, and the experimental results are presented and compared with those obtained with the conventional FCS-MPC. Finally, some conclusions are drawn in Section VI.

II. CONVENTIONAL FCS-MPC

Fig. 1 shows an S-VSR. The equation describing the operation of the converter can be written as:

$$L_s \frac{di}{dt} = e - Ri - V_r$$

where:
- $e$ the grid voltage.
- $V_r$ the rectifier voltage.
- $i$ the grid current.
- $R$ the equivalent series resistance.
- $L_s$ the inductance of the line filter.

The discrete model of the converter is obtained to approximate the derivative $di/dt$ in (1) by:

$$\frac{di}{dt} = \frac{i(k+1) - i(k)}{T}$$

where:
- $T$ the sampling time.

By substituting (2) into (1), the following expression is obtained for the future current at the $(k+1)$th instant. From (1), the equivalent eddy currents are straight-forwardly derived as follows:

$$i(k+1) = (1 - \frac{RT}{L_s})i(k) + \frac{T}{L_s}[e(k) - V_r(k+1)]$$

where:
- $V_r(k+1)$ is the future rectifier voltage of the S-VSR, and it is a continuous vector.
- $\Delta i(k)$ is the change in the current from the previous step.

III. PROPOSED CONTROL STRATEGY BASED ON THE DISCRETE LYAPUNOV FUNCTION

An effective control algorithm is essential for the S-VSR so that the current, $i(k)$, tracks the reference value, $i^*(k)$. Therefore, it is necessary to find a control function where the current tracking error, $\Delta i(k)$, asymptotically converges to zero. The Lyapunov direct method is used for specific applications. In addition, the error $\Delta i(k)$ is taken as:

$$\Delta i(k) = i(k) - i^*(k)$$

According to the Lyapunov stability theorem, the discrete Lyapunov function, $L(x(k))$, satisfies the following properties:

1) $L(0)=0$
2) $L(x(k))>0$ for all $x(k)\neq 0$
3) $L(x(k)) \to \infty$ as $|x(k)| \to \infty$
4) $\Delta L(x(k))<0$ for all $x(k)\neq 0$

Thus, the discrete Lyapunov function $L(\Delta i(k))$ of the S-VSR can be taken as:
\[ L(\Delta i(k)) = \frac{1}{2} \Delta^2 (k) \quad (7) \]

From (6) and (7), the rate of change of the Lyapunov function, \(L(\Delta i(k))\), can be expressed for the rectifier mode as:
\[
\Delta L(\Delta i(k)) = L(\Delta i(k+1)) - L(\Delta i(k)) = \frac{1}{2} \left[ i(k+1) - i^*(k+1) \right]^2 - \frac{1}{2} \left[ i(k) - i^*(k) \right]^2 \quad (8)
\]

To satisfy the Lyapunov stability theorem the following expression is defined as:
\[ i(k+1) - i^*(k+1) = \alpha \left[ i(k) - i^*(k) \right] \quad (9) \]

where \(\alpha\) is a control coefficient with a constant value.

Substitute (9) into (8), and obtain the following expression for \(\Delta L(\Delta i(k))\):
\[
\Delta L(\Delta i(k)) = L(\Delta i(k+1)) - L(\Delta i(k)) = \frac{1}{2} (\alpha^2 - 1) \left[ i(k) - i^*(k) \right]^2 \quad (10)
\]

It is apparent that \(\Delta L(\Delta i(k)) < 0\), if \(\alpha\) is chosen as:
\[ 0 < \alpha < 1 \quad (11) \]

The future control law \(V_r(k+1)\) at the \((k+1)\)th instant of the proposed control strategy can be determined with (3) and (9), and it can be written as:
\[
V_r(k+1) = e(k) + \left( \frac{L_1}{T} - R \right) i(k) - \frac{L_2}{T} i^*(k+1)
- \alpha \frac{L_2}{T} \left[ i(k) - i^*(k) \right] \quad (12)
\]

It is clearly shown that (12) is related to the controlled variable \(i(k)\) at the \(k\)th instant and the reference variable \(i^*(k+1)\) at the \((k+1)\)th instant. It is also related to the error term of the controlled variable and the reference variable at the \(k\)th instant. Therefore, the proposed control law has a feed forward and feedback structure which is the same as the model predictive control. In addition, when \(\alpha = 0\), by solving (12), the control law can be expressed as:
\[
\dot{V}_s(k+1) = e(k) + \left( \frac{L_1}{T} - R \right) i(k) - \frac{L_2}{T} i^*(k+1)
- \frac{L_2}{T} \left[ i(k) - i^*(k) \right] \quad (13)
\]

which is the same as the control law for the deadbeat control.

In the proposed strategy, the control law in (12) is used as the continuous future reference voltage vector to choose one of the three future voltage vectors of the S-VSR in a finite set. If the future voltage vector of the S-VSR closest to the future reference voltage vector obtained from (12) is applied to the S-VSR, the current at the next sampling instant can track the future reference current. Since the S-VSR only generates three voltage vectors in their finite set in contrast to the continuous reference voltage vector in (12), the cost function defined as (14) allows one proper future voltage vector to be selected among the three possible vectors.

The direct Lyapunov method gives the following stability criteria for a function \(L(\Delta i(k))\) and it is uniformly and ultimately bounded [22], i.e.:
\[
\|\Delta v(k+1)\| \leq 0.5 V_{dc} \quad (16)
\]
where $c_1$, $c_2$, $c_3$, and $c_4$ are positive constants, $l \geq 1$, $G \subseteq \mathbb{R}_+$ is a positive control invariant set, and $\Gamma \subseteq G$ is a compact set.

By applying the value of the future voltage vector (15), $V(k+1)$, for the rectifier, the rate of change of the Lyapunov function, $\Delta L^p(\Delta i(k))$, can be written as:

$$\Delta L^p(\Delta i(k)) = \frac{1}{2} \left[ i^p(k+1) - i^*(k+1) \right]^2$$

By substituting (4) and (12) into (18), $\Delta L^p(\Delta i(k))$ can be written as:

$$\Delta L^p(\Delta i(k)) = \frac{1}{2} \left( \alpha^2 - 1 \right) \Delta i^2(k) + \frac{1}{2} \left[ \frac{T}{L_s} \Delta v(k+1) \right]^2 - \alpha \frac{T}{L_s} \Delta v(k+1) \Delta i(k)$$

Solving (19), it can be expressed as:

$$\Delta L^p(\Delta i(k)) = \left( \frac{1}{2} - b \right) \left( \alpha^2 - 1 \right) \Delta i^2(k) + p(\Delta i(k))$$

where $b$ is a positive constant within $0 < b < 1/2$.

It is clear that $p(\Delta i(k))$ has a maximum, shown as (21), based on (11):

$$p(\Delta i(k))_{\text{max}} = \left[ \frac{T}{L_s} \Delta v(k+1) \right]^2 \frac{(1-2b)\alpha^2 + 2b}{4b(1-\alpha^2)}$$

As a result, by considering (16), the rate of change of the Lyapunov function in (19) is:

$$\Delta L^p(\Delta i(k))$$

$$\leq \left( \frac{1}{2} - b \right) \left( \alpha^2 - 1 \right) \Delta i^2(k) + p(\Delta i(k))_{\text{max}}$$

$$\leq \left( \frac{1}{2} - b \right) \left( \alpha^2 - 1 \right) \Delta i^2(k) + \frac{1}{4} \left[ \frac{T}{L_s} V_{\infty} \right]^2 \frac{(1-2b)\alpha^2 + 2b}{4b(1-\alpha^2)}$$

$\text{Fig. 3. The relationship between } \rho \text{ and } \alpha.$

Fig. 3 shows $\rho$ as a function of $\alpha$. It is clear that when $\alpha^2$ is gradually increased from -1 to 1, $\rho$ is decreased and the convergence speed is also decreased. It is well known that the convergence speed is closely related to dynamic performance.

Therefore, the stability condition (17) is satisfied by the constant values as:

$$c_1 = c_2 = 1; c_3 = \left( \frac{1}{2} - b \right)(1-\alpha^2);$$

$$c_4 = \frac{1}{4} \left[ \frac{T}{L_s} V_{\infty} \right]^2 \frac{(1-2b)\alpha^2 + 2b}{4b(1-\alpha^2)}$$

In addition, (22) can be expressed as:

$$\Delta L^p(\Delta i(k)) \leq -2c_3 L^p(\Delta i(k)) + c_4$$

This inequality implies that, as time increases, the current control error converges to the compact set as:

$$A = \left\{ \Delta i(k) \right\} \left\| \Delta i(k) \right\| \leq \frac{c_4}{c_3}$$

It is clear that when $\alpha^2$ is larger, the convergence domain is larger and the convergence domain is closely related to robustness.

$\text{B. Influence of Convergence Speed}$

From (8), the relationship between the Lyapunov function of the $(k+1)$th instant and the Lyapunov function of the $k$th instant can be described as:

$$L(\Delta i(k+1)) = \alpha^2 L(\Delta i(k))$$

The convergence speed of the Lyapunov function can be studied using $\rho$, which is given by:

$$\rho = \frac{L(k)}{L(k+1)} = 1/\alpha^2$$

$\text{C. Influence of Steady-State Performance}$

To study the effect of $\alpha$ on steady state performance, it is necessary to neglect the equivalent series resistance $R$ and suppose that the current reference value does not change considerably in one sampling interval. The future current value can be expressed by:

$$i(k+1) = i(k) + \frac{T}{L_s} [e(k) - V_r(k+1)]$$
In addition, the continuous future reference voltage vector can be expressed by:

\[ V_r(k+1) = e(k) + (1-\alpha) \frac{L}{T} [i(k) - i(k)] \] (29)

In the positive period of \( e(k) \), it is hoped that \( i(k+1) \) is close to \( i(k+1) \). When \( i(k) = i(k) \) at the \( k \)th instant, the optimal voltage vector should be \( V_1 \). To increase the possibility of \( V_1 \) by increasing \( V_r(k+1) \), the improved range of \( \alpha \) is \(-1<\alpha<0\). When \( i(k) = i(k) \) at the \( k \)th instant, the optimal voltage vector should be \( V_0 \). To increase the possibility of \( V_0 \) by decreasing \( V_r(k+1) \), the improved range of \( \alpha \) is also \(-1<\alpha<0\).

Therefore, the improved value range of \( \alpha \) is given as:

\[-1 < \alpha < 0 \] (30)

D. Selection of \( \alpha \)

According to the above analyses, when \( \alpha^2 \) is selected to be larger within the value range, the steady-state performance and robustness are improved. However, the convergence speed and dynamic performance are worse. Therefore, a compromise should be made during the selection of \( \alpha \). A typical range of \( \alpha \) for the rectifier in this study is found to be:

\[-0.6 < \alpha < 0.2 \] (31)

V. SIMULATION AND EXPERIMENTAL RESULTS

The proposed control strategy has been verified by simulation and experimental results. The simulations were carried out by MATLAB/Simulink. The experimental test was performed using a single phase PWM AC/DC voltage source rectifier prototype in a DSP system based on a TMS320F28069. The system parameters are given in Table I.

Fig.4 shows simulation results of the steady-state error of \( i(k) \) based on different values of \( \alpha \). It is clear that the smaller the value of \( \alpha \), the better the steady-state performance. In this paper, \( \alpha \) is selected as \(-0.45\) in the experimental test.

A. Simulation Results

Fig. 4. Simulated error \( \Delta i(k) \) in the steady state based on different values of \( \alpha \) (0.2A/div).

B. Steady-state Response Tests

Fig. 5 shows the steady-state test results of the proposed control strategy when the reference current peaks at 6.8 A. Fig.5(a) depicts the input current and voltage waveforms, where current follows the voltage to achieve a unit power factor. Fig.5(b) is the harmonic spectrum of the current. These test results demonstrate the improved steady-state response of the proposed control strategy. The converter
input current is highly sinusoidal with a measured total harmonic distortion (THD) of 2.16%.

Fig. 6 shows test results of the conventional FCS-MPC. As observed from the current waveforms, the fluctuation range of the current is larger, and the THD is 3.38%.

Fig. 7 shows test result of the online parameter estimation control method referred in [23]. The THD of the current waveform is 2.51%. From the comparison above, it can be seen that the proposed control method has the best steady-state response, followed by the online parameter estimation FCS-MPC control method. The steady-state test results of the conventional FCS-MPC are the worst.

C. Dynamic Response Tests
An important aspect of any control system is the dynamic response to changes in the reference. Fig. 8 depicts the current behavior with the proposed control strategy ($\alpha=−0.45$) when the reference step changes from 4- to 6.8- A peak and vice versa. The current reached a steady-state level in Fig.8(a) within 126μs and requires 268μs to reach a steady state in Fig. 8(b). Compared with the proposed control strategy, the conventional FCS-MPC has a faster dynamic response as shown in the analysis in section IV. Fig. 9 shows the
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current behavior with the conventional FCS-MPC when the reference step changes from 4- to 6.8- A peak and from 6.8- to 4- A peak. The dynamic response time in Fig. 9(a) is 107μs and that in Fig.9(b) is 231μs. These experiment results are consistent with the analysis in Section IV.

D. Robustness Tests

System parameters, such as the inductance and equivalent resistance, vary with temperature, core saturation, and other environmental conditions. In addition, parameter errors influence the whole control performance. The robustness of the proposed control strategy is tested when the actual inductance is mismatched by -25% and +25%.

Fig. 10 shows steady-state test results of the proposed control strategy when the inductance has a mismatch of -25% and +25%. It can be seen that when the inductance has a mismatch of -25%, there are higher input current ripples. However, inductance mismatch does not influence the system stability in the proposed control strategy.

VI. CONCLUSIONS

A model predictive control based on the discrete Lyapunov function with control error compensation of power electronic converters is proposed in this paper. The criterion for selecting the control coefficient, α, is described. Furthermore, the influence of changing α is also studied.

The proposed control strategy, based on the discrete direct Lyapunov method, leads to a globally asymptotically stable system. In addition, it shows improved steady-state performance and has a fast dynamic response that is just a little slower than the conventional FCS-MPC. The results associated in this investigation are very encouraging and will continue to play a strategic role in the improvement of modern digital control systems.

REFERENCES


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