

## RADIATION EFFECTS ON MHD BOUNDARY LAYER FLOW OF LIQUID METAL OVER A POROUS STRETCHING SURFACE IN POROUS MEDIUM WITH HEAT GENERATION

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**ABSTRACT.** The present paper analyses the radiation effects of mass transfer on steady non-linear MHD boundary layer flow of a viscous incompressible fluid over a nonlinear porous stretching surface in a porous medium in presence of heat generation. The liquid metal is assumed to be gray, emitting, and absorbing but non-scattering medium. Governing nonlinear partial differential equations are transformed to nonlinear ordinary differential equations by utilizing suitable similarity transformation. The resulting nonlinear ordinary differential equations are solved numerically using Runge–Kutta fourth order method along with shooting technique. Comparison with previously published work is obtained and good agreement is found. The effects of various governing parameters on the liquid metal fluid dimensionless velocity, dimensionless temperature, dimensionless concentration, skin-friction coefficient, Nusselt number and Sherwood number are discussed with the aid of graphs.

### 1. INTRODUCTION

In recent years, the flows of fluid through porous media are of principal interest because these are quite prevalent in nature. Such flows have attracted the attention of a number of scholars due to their application in many branches of science and technology, viz., in the field of agriculture engineering to study the underground water resources, seepage of water in riverbeds, in petroleum technology to study the movement of natural gas, oil and water through oil reservoirs, in chemical engineering for filtration and purification processes. The convection problem in porous medium has also important applications in geothermal reservoirs and

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geothermal energy extractions. The heat transfer enhancement is one of the most important technical aims for engineering systems due to its wide applications in electronics cooling systems, next-generation solar film collectors, heat exchangers technology, and various thermal systems. Liquid metals are a specific class of coolants. Their basic advantage is a high molecular thermal conductivity for which identical flow parameters, enhances heat transfer coefficients. Another distinguishing feature of liquid metals is the low pressure of their vapors, which allows their use in power engineering equipment at high temperature and low pressure, thus alleviating solution of mechanical strength problems. The most widespread liquid metals used in engineering are alkali metals. Among them sodium is first and foremost, used as a coolant of fast reactors and a working fluid of high-temperature heat pipes. Liquid metal heat transfer plays an important role in modern life, since the liquid metals are used as coolant in nuclear reactor and as working fluids in space power plants. Therefore it is essential to consider the variable thermal properties of liquid metals heat transfer problem (Arunachalam and Rajappa [1], Lubarsky and Kaufman [2], Lyon [3]).

In recent years, a great deal of interest has been generated in the area of heat and mass transfer of the boundary layer flow over a nonlinear stretching sheet, in view of its numerous and wide-ranging applications in various fields like polymer processing industry in particular in manufacturing process of artificial film and artificial fibers and in some applications of dilute polymer solution. Sakiadis [4] initiated the study of boundary layer flow over a continuous solid surface moving with constant speed as result of ambient fluid movement; this boundary flow is generally different from boundary layer flow over a semi-infinite flat plate. Erickson [5] studied this problem to the case in which the transverse velocity at the moving surface is nonzero with the effects of heat and mass transfer being taken in to account. Danberg and Fansler [6], by using non-similar solution method, studied the flow inside the boundary layer past a wall that is stretched with a velocity proportional to the distance along the wall. Gupta and Gupta [7] by using the similar solution method analysed heat and mass transfer in the boundary layer over a stretching sheet subject to suction or blowing.

The laminar boundary layer on an inextensible continuous flat surface moving with a constant velocity in a non-Newtonian fluid characterized by a power-law model is studied by Fox *et al.* [8], using both exact and approximate methods. Raja Gopal *et al.* [9] studied the flow behaviour of viscoelastic fluid over stretching sheet and gave an approximate solution to the flow field. Troy *et al.* [10] presented an exact solution for Raja Gopal problem. Vajravelu and Roper [11] studied the flow and heat transfer in a viscoelastic fluid over a continuous stretching sheet with power law surface temperature, including the effects of viscous dissipation, internal heat generation or absorption, and work due to deformation in the energy equation. Vajravelu [12] studied the flow and heat transfer characteristics in a viscous fluid over a nonlinearly stretching sheet without heat dissipation effect. Cortell [13], [14] has worked on viscous flow and heat transfer over a nonlinearly stretching sheet. Cortell [15] investigated the influence of similarity solution for flow and heat transfer of a quiescent fluid over a nonlinear stretching surface.

The deep interest in the porous medium is easily understandable since porous medium is used in vast applications, which covers many engineering disciplines. For instance, applications of the porous media includes, thermal insulations of buildings, heat exchangers, solar

energy collectors, geophysical applications, solidification of alloys, nuclear waste disposals, drying processes, chemical reactors, energy recovery of petroleum resources etc., More applications and good understanding of the subject is given in the recent books by Nield and Bejan [16], Vafai [17], Pop and Ingham [18].

The radiation effects have important applications in physics and engineering particularly in space technology and high temperature processes. But very little is known about the effects of radiation on the boundary layer. Thermal radiation effects on the boundary layer may play important role in controlling heat transfer in polymer processing industry where the quality of the final product depends on the heat controlling factors to some extent. High temperature plasmas, cooling of nuclear reactors, liquid metal fluids, and power generation systems are some important applications of radiative heat transfer. Nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering areas. Moreover, when radiative heat transfer takes place, the fluid involved can be electrically conducting since it is ionized due to the high operating temperature. Accordingly, it is of interest to examine the effect of the magnetic field on the flow. Studying such effect has great importance in the application fields where thermal radiation and MHD are correlative.

The process of fusing of metals in an electrical furnace by applying a magnetic field and the process of cooling of the first wall inside a nuclear reactor containment vessel where the hot plasma is isolated from the wall by applying a magnetic field are examples of such fields. Elbashbeshy [19] free convection flow with variable viscosity and thermal diffusivity along a vertical plate in the presence of magnetic field. Raptis and Perdakis [20] studied viscous flow over a nonlinear stretching sheet in the presence of a chemical reaction and magnetic field. Awang and Hashim [21] obtained the series solution for flow over a nonlinearly stretching sheet with chemical reaction and magnetic field. Abbas and Hayat [22] addressed the radiation effects on MHD flow due to a stretching sheet in porous space. Effect of radiation on MHD steady asymmetric flow of an electrically conducting fluid past a stretching porous sheet has been analysed analytically by Ouaf [23], Mukhopadhyay and Layek [24] investigated the effects of thermal radiation and variable fluid viscosity on free convection flow and heat transfer past a porous stretching surface. Gururaj and Pavithra [25] investigated nonlinear MHD boundary layer flow of a liquid metal over a porous stretching surface in presence of radiation. The effects of variable viscosity and nonlinear radiation on MHD flow over a stretching surface with power-law velocity were reported by Anjali Devi and Gururaj [26].

A study on MHD heat and mass transfer free convection flow along a vertical stretching sheet in the presence of magnetic field with heat generation was carried out by Samad and Mohebujjaman [27]. Singh [28] analyzed the MHD free convection and mass transfer flow with heat source and thermal diffusion. Kesavaiah *et al.* [29] reported that the effects of the chemical reaction and radiation on MHD convective heat and mass transfer flow past a semi-infinite vertical permeable moving plate embedded in porous medium with heat source and suction. Mohammed Ibrahim and Bhaskar Reddy [30] proposed the effects of thermal radiation on steady MHD free convective flow past along a stretching surface in presence of viscous dissipation and heat source. MHD heat and mass transfer free convection flow along

a vertical stretching sheet in presence of magnetic field with heat generation was studied by Samad and Mobebujjaman [31].

However the interaction of nonlinear radiation with mass transfer of an electrically conducting and diffusing fluid past a nonlinear stretching surface has received little attention. Hence an attempt is made to investigate the nonlinear radiation effects on a steady convective flow over nonlinear stretching surface in presence of magnetic field, porous medium and heat generation. The governing equations are transformed by using similarity transformation and the resultant dimensionless equations are solved numerically using the Runge–Kutta fourth order method with shooting technique. The effects of various governing parameters on the velocity, temperature, concentration, skin-friction coefficient, Nusselt number and Sherwood number are shown in Figures.

## 2. FORMULATION OF THE PROBLEM

Consider the coupled radiation and forced convection along a horizontal porous stretching surface which is kept at uniform temperature  $T_w$  and moving with velocity  $u_w = u_0 x^m$  through and stationary liquid metal. The liquid metal is assumed to be a gray, emitting, absorbing and electrically conducting, but non-scattering medium at temperature  $T_\infty$ . A variable magnetic field is applied normal to the horizontal surface  $B(x) = B_0 x^{\frac{m-1}{2}}$  in accordance with Chiam [32].

The following assumptions are made

- (1) Flow is two-dimensional, steady and laminar.
- (2) The fluid has constant physical properties.
- (3) The usual boundary layer assumptions are made [Ali [33]].
- (4) The  $x$ - axis runs along the continuous surface in the direction of motion and  $y$ - axis perpendicular to it. and
- (5) The radiation dissipation in the  $x$ -axis is negligible in comparison with that in the  $y$ -axis following the lines of Michael F. Modest (Radiative Heat Transfer. Page 696) [34].

The continuity, momentum, energy conservation and mass conservation equations under the above assumption are written as follows:

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2.1)$$

Momentum conservation equation:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \left( \frac{\sigma B^2(x)}{\rho} \right) u - \frac{\nu}{K} u \quad (2.2)$$

Energy conservation equation:

$$\rho c_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} + Q_0 T \quad (2.3)$$

Species conservation equation:

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} \quad (2.4)$$

The boundary conditions for velocity, temperature and concentration fields are

$$\begin{aligned} u = U(x) = u_0 x^m, \quad v = v_0(x), \quad T = T_w, \quad C = C_w \quad \text{at } y = 0 \\ u \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \quad \text{as } y \rightarrow \infty \end{aligned} \quad (2.5)$$

where  $u$  and  $v$  are the velocity components of fluid in  $x$  and  $y$  directions respectively,  $\rho$  is the density of the fluid,  $\nu$  is the kinematic viscosity,  $K$  is the permeability coefficient of porous medium,  $\sigma$  is the electric conductivity,  $k$  is the thermal conductivity,  $B_0$  is constant applied magnetic induction,  $c_p$  is specific heat at constant pressure,  $B(x)$  is the variable applied magnetic induction,  $v_0(x)$  is the variable injection velocity ( $v_0(x) = c\sqrt{\frac{\nu u_0}{x}}$ ),  $T$  is temperature of the fluid,  $C$  is the concentration of the fluid,  $T_w$  is the temperature of the heated surface,  $T_\infty$  is the temperature of the ambient fluid,  $C_w$  is the concentration of the heated surface,  $C_\infty$  is the concentration of the ambient fluid,  $D$  is the mass diffusivity,  $Q_o$  is the dimensional heat generation/absorption coefficient,  $q_r$  is the component of radiative heat flux,  $c$  is a stretching constant,  $u_0$  is a constant,  $m$  is the velocity exponent parameter.

The radiative heat flux term is simplified by using the Roseland diffusion approximation (Hossian *et al.* [35]) and accordingly

$$q_r = -\frac{4\sigma^*}{3\alpha^*} \frac{\partial T^4}{\partial y} \quad (2.6)$$

where  $\sigma^*$  is Stefan-Boltzmann constant,  $\alpha^*$  is the Rosseland mean absorption coefficient. We assume that the temperature difference within the flow is sufficiently small such that  $T^4$  may be expressed as a linear function of the temperature. This is accomplished by expanding in a Taylor series about the free stream temperature  $T_\infty$  and neglecting higher order terms, thus

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \quad (2.7)$$

By using equations (2.6) and (2.7), in equation (2.3) we get

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \left[ 1 + \frac{16\sigma^* T_\infty^3}{3\alpha^* k} \right] \frac{\partial^2 T}{\partial y^2} + \frac{Q_o T}{\rho c_p} \quad (2.8)$$

The equation of continuity is satisfied if we choose a stream function  $\psi(x, y)$  such that

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x} \quad (2.9)$$

Now we introduce the following usual similarity transformations [Ali [33]].

$$\begin{aligned} \eta(x, y) = y \sqrt{\frac{m+1}{2}} \sqrt{\frac{u_0 x^{m-1}}{\nu}}, \quad \psi(x, y) = \sqrt{\frac{2}{m+1}} \sqrt{\nu u_0 x^{m+1}} f(\eta), \\ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \quad \theta_w = \frac{T_w}{T_\infty} \end{aligned} \quad (2.10)$$

where  $\eta$  is the similarity variable,  $\psi$  is the stream function,  $f$  is the non-dimensional stream function,  $\theta$  is the non-dimensional temperature and  $\phi$  is the non-dimensional concentration.

With these changes of variables equation (2.1) is identically satisfied and equations (2.2), (2.4) and (2.8) are transformed into nonlinear ordinary differential equations as follows

$$f''' + ff'' - \left(\frac{2m}{m+1}\right) f'^2 - \left(M^2 + \frac{1}{K}\right) f' = 0 \quad (2.11)$$

$$\left[1 + \frac{4}{3R}(1 + (\theta_w - 1)\theta)^3\right] \theta'' + \frac{4}{R} \left([1 + (\theta_w - 1)\theta]^2\right) \theta'^2 + \text{Pr} f\theta' + \text{Pr} Q\theta = 0 \quad (2.12)$$

$$\phi'' + Scf\phi' - Scf'\phi = 0 \quad (2.13)$$

where primes denote the differentiation with respect to  $\eta$ ,  $m-$  is the velocity exponent parameter,  $M = \sqrt{\frac{2\sigma B_0^2}{\rho u_0(m+1)}}$  is the magnetic interaction parameter,  $K = \frac{K'u_0}{\nu}$  is the permeability parameter,  $R = \frac{K\alpha^*}{4\sigma^* T_\infty^3}$  is the radiation parameter,  $\theta_w = \frac{T_w}{T_\infty}$  is the surface temperature parameter,  $\text{Pr} = \frac{\nu\rho C_p}{\kappa}$  Prandtl number,  $Q = \frac{Q_0}{u_0}$  is the heat generation parameter,  $Sc = \frac{\nu}{D}$  is the Schmidt number.

The corresponding boundary conditions can take the form.

$$\begin{aligned} f(0) = -S, \quad f'(0) = 1, \quad \theta(0) = 1, \quad \phi(0) = 1 & \quad \text{at} \quad \eta = 0 \\ f'(\infty) \rightarrow 0, \quad \theta(\infty) \rightarrow 0, \quad \phi(\infty) \rightarrow 0 & \quad \text{as} \quad \eta \rightarrow \infty \end{aligned} \quad (2.14)$$

where  $S = \sqrt{\frac{2}{m+1}}c$  is the porosity parameter (For injection  $S > 0$  and for suction  $S < 0$ ) and  $c$  is non-dimensional constant.

The important physical quantities of our interest are:

The local skin-friction

$$C_f = \frac{\tau_w}{\rho U^2/2} \quad (2.15)$$

The local Nusselt number

$$Nu_x = \frac{xq_w}{k(T_w - T_\infty)} \quad (2.16)$$

The local Sherwood number

$$Sh_x = \frac{xm_w}{k(C_w - C_\infty)} \quad (2.17)$$

Where, the surface shear stress

$$\tau_w = \mu \left(\frac{\partial u}{\partial y}\right)_{y=0} \quad (2.18)$$

The surface heat flux

$$q_w = -k \left(\frac{\partial T}{\partial y}\right)_{y=0} \quad (2.19)$$

The mass flux

$$m_w = -k \left(\frac{\partial C}{\partial y}\right)_{y=0} \quad (2.20)$$

with  $\mu$  and  $k$  being the dynamic viscosity and thermal conductivity respectively.

Using the non-dimensional variables in equation (2.10) we obtain

$$\frac{1}{2}C_f\sqrt{Re_x} = f''(0), \quad \frac{Nu_x}{\sqrt{Re_x}} = -\theta'(0), \quad \frac{Sh_x}{\sqrt{Re_x}} = -\phi'(0) \quad (2.21)$$

where  $Re_x = \frac{Ux}{\nu}$  is the local Reynolds number.

### 3. SOLUTION OF THE PROBLEM

The set of coupled non-linear governing boundary layer equations (2.11) – (2.13) together with the boundary conditions (2.14) are solved numerically by using Runge–Kutta fourth order technique along with shooting method. First of all, higher order non-linear differential equations (2.11) – (2.13) are converted into simultaneous linear differential equations of first order and they are further transformed into initial value problem by applying the shooting technique (Jain *et al.* [36]). The resultant initial value problem is solved by employing Runge–Kutta fourth order technique. The step size  $\Delta\eta = 0.05$  is used to obtain the numerical solution with five decimal place accuracy as the criterion of convergence. From the process of numerical computation, the skin-friction coefficient, the Nusselt number and the Sherwood number, which are respectively proportional to  $f''(0)$ ,  $\theta'(0)$  and  $\phi'(0)$ , are also displayed in the graphs.

### 4. RESULTS AND DISCUSSION

As a result of the numerical calculations, the dimensionless velocity  $f'(\eta)$ , dimensionless temperature  $\theta(\eta)$  and dimensionless concentration  $\phi(\eta)$  distributions for the flow under consideration are obtained and their behaviour have been discussed for variations in the governing parameters viz., the Magnetic interaction parameter  $M$ , Velocity exponent parameter  $m$ , Permeability parameter  $K$ , Porosity parameter  $S$ , Radiation parameter  $R$ , Prandtl number  $Pr$ , Surface temperature parameter  $\theta_w$ , Heat generation parameter  $Q$  and Schmidt number  $Sc$ . In the present study, the following default parametric values are adopted.  $M = 1.0$ ,  $m = 1.0$ ,  $K = 1.0$ ,  $S = 0.1$ ,  $Pr = 0.71$ ,  $R = 1.0$ ,  $\theta_w = 1.1$ ,  $Q = 0.05$ ,  $Sc = 0.6$ .

In order to ascertain the accuracy of our numerical results, the present study is compared with the previous study. The temperature profiles are compared with available theoretical solution of Elbashbeshy [19], free convection flow with variable viscosity and thermal diffusivity along a vertical plate in the presence of magnetic field in Figure 1 and Figure 2. It is observed that the present results are in good agreement with that of Elbashbeshy [19]. Figure 3(a) displays the plot of dimensionless velocity  $f'(\eta)$  for different values of  $M$ . It is noted that as magnetic interaction parameter  $M$  increases, transverse velocity  $f'(\eta)$  decreases elucidating the fact that the effect of magnetic field is to decelerate the velocity. The effect of magnetic interaction parameter  $M$  over the dimensionless temperature  $\theta(\eta)$  and dimensionless concentration  $\phi(\eta)$  is shown with the help of Figure 3(b) and 3(c). It is observed that temperature and concentration increases with an increase in the magnetic interaction parameter  $M$ .

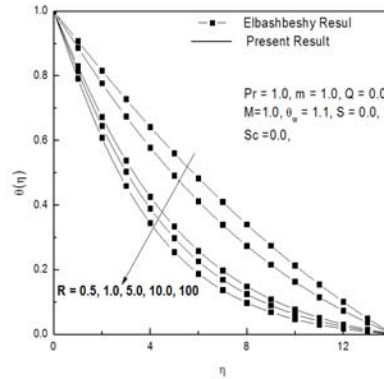


FIGURE 1. Temperature profiles for different Values of Radiation parameter  $R$ .

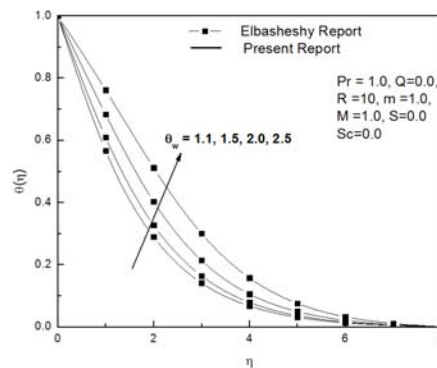


FIGURE 2. Temperature profiles for different values of Surface temperature parameter  $\theta_w$ .

The effect of velocity exponent parameter  $m$  over the dimensionless velocity field  $f'(\eta)$  is shown in the graph of Figure 4(a). It is observed that the effect of velocity exponent parameter is to increase the velocity. Figure 4(b) and 4(c) show the dimensionless temperature profiles and dimensionless concentration profiles for different values of exponent parameter  $m$ . It is noticed that the temperature and concentration reduce with an increase of exponent parameter  $m$ . Figure 5(a) shows the effect of permeability parameter  $K$  on the velocity. It is noticed that as the permeability parameter  $K$  increases, the velocity is also increases. Figure 5(b) displays



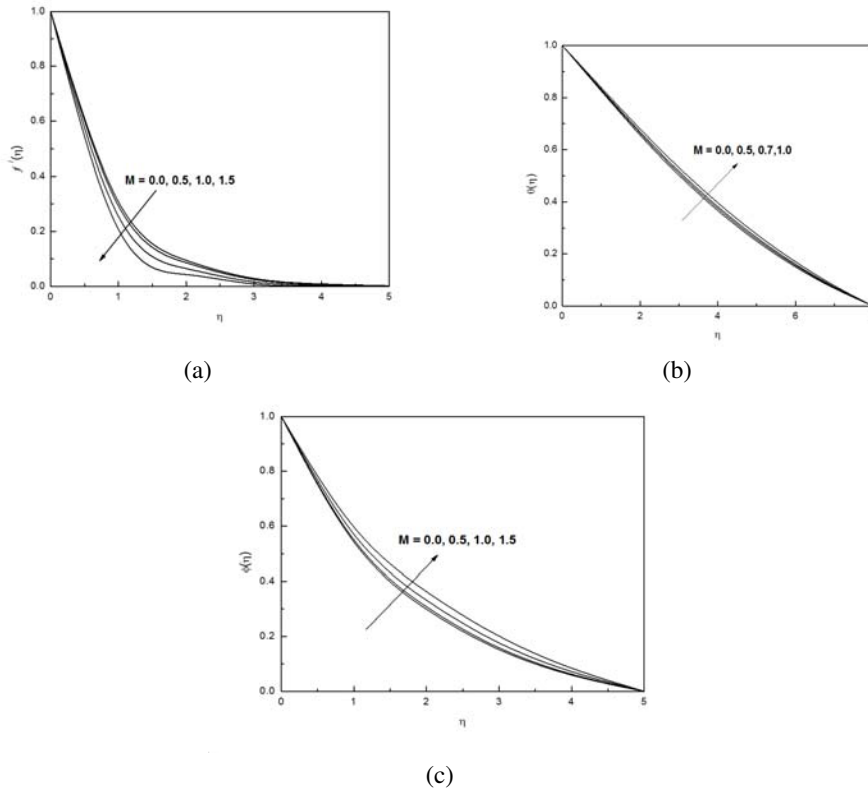


FIGURE 3. (a) Velocity Profiles for different values of Magnetic Parameter  $M$ . (b) Temperature profiles for different values of Magnetic Parameter  $M$ . (c) Concentration profiles for different values of Magnetic Parameter  $M$ .

the variation of the thermal boundary-layer with permeability parameter  $K$ . It is found that the thermal boundary layer thickness decreases with an increase in the permeability parameter  $K$ . The influence of the permeability parameter  $K$  on the concentration field is shown in Figure 5(c). It is observed that the concentration reduces with an increase of the permeability parameter  $K$ . Prandtl number variation over the dimensionless temperature profile is elucidated through Figure 6. As Prandtl number  $Pr$  increases, the temperature  $\theta(\eta)$  decreases, illustrates the fact that the effect of Prandtl number  $Pr$  is to decrease the temperature in the magnetic field. Furthermore, the effect of Prandtl number  $Pr$  is to reduce the thickness of thermal boundary layer.

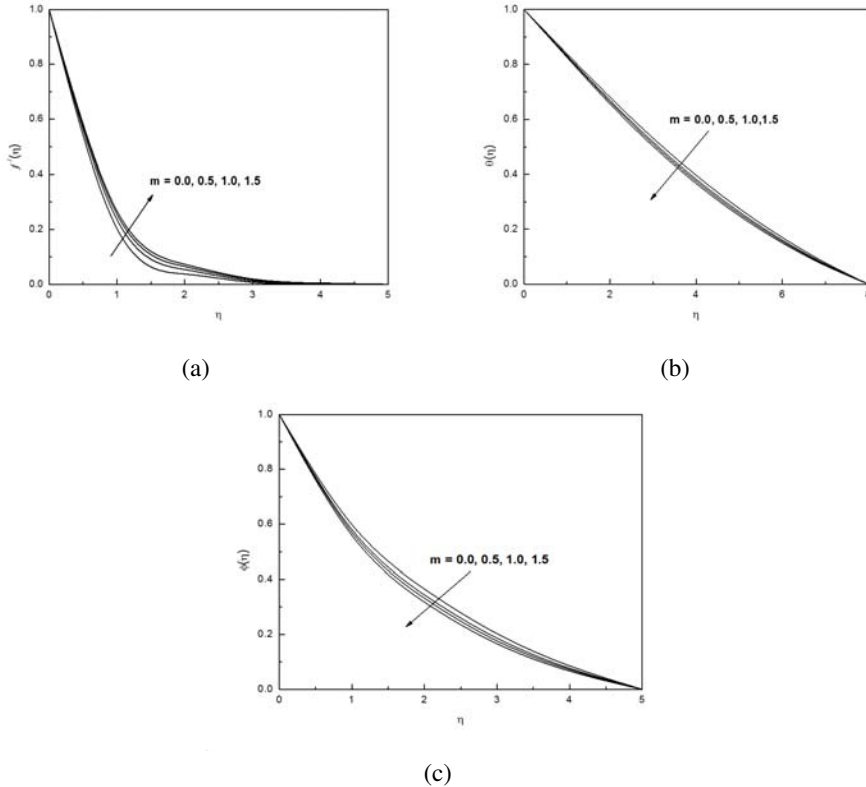


FIGURE 4. (a) Velocity Profiles for different values Velocity exponent parameter  $m$ . (b) Temperature Profiles for different values Velocity exponent parameter  $m$ . (c) Concentration profiles for different values Velocity exponent parameter  $m$ .

Figure 7 illustrates the effects of radiation parameter  $R$  over the dimensionless temperature  $\theta(\eta)$ . It is observed that the effect radiation parameter is to reduce the temperature, elucidating the fact that the thermal boundary layer thickness decreases as  $R$  increases. The effect of surface temperature parameter  $\theta_w$  over the dimensionless temperature  $\theta(\eta)$  is shown in Figure 8. Increasing surface temperature parameter  $\theta_w$  is to increase the temperature.

Figure 9 depict the dimensionless temperature profiles for different values of the heat generation parameter  $Q$ . It is observed that the temperature increase with an increase in  $Q$ . The influence of the Schmidt number  $Sc$  on the dimensionless concentration profiles is plotted in Figure 10. As the Schmidt number increases the concentration decreases.

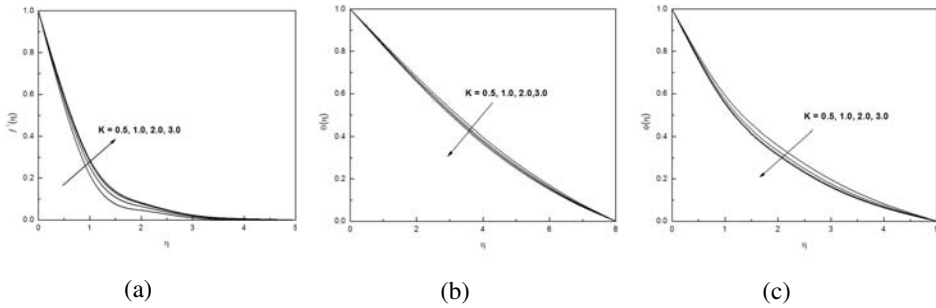


FIGURE 5. (a) Velocity Profiles for different values of Permeability parameter  $K$ . (b) Temperature profiles for different values of Permeability parameter  $K$ . (c) Concentration profiles for different values of Permeability parameter  $K$ .

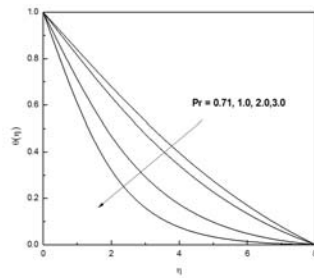


FIGURE 6. Temperature profiles for different values of Prandtl number  $Pr$ .

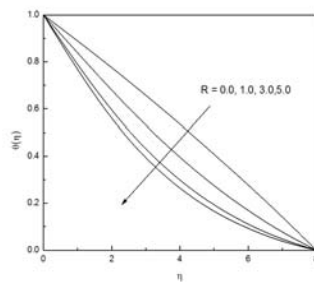


FIGURE 7. Temperature profiles for different values of Radiation parameter  $R$ .

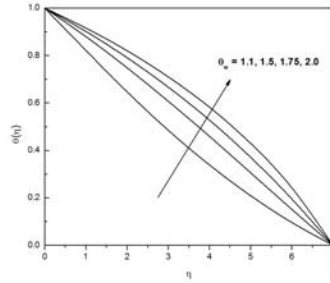


FIGURE 8. Temperature profiles for different values of Surface temperature parameter  $\theta_w$

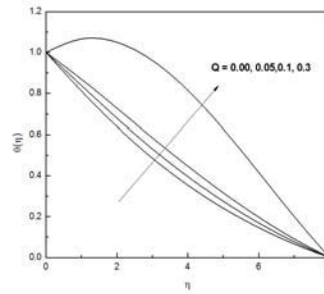


FIGURE 9. Temperature profiles for different values Heat generation parameter  $Q$ .

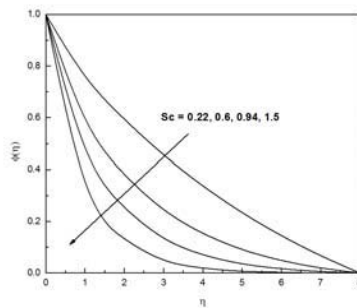


FIGURE 10. Concentration profiles for different values of Schmidt number  $Sc$ .

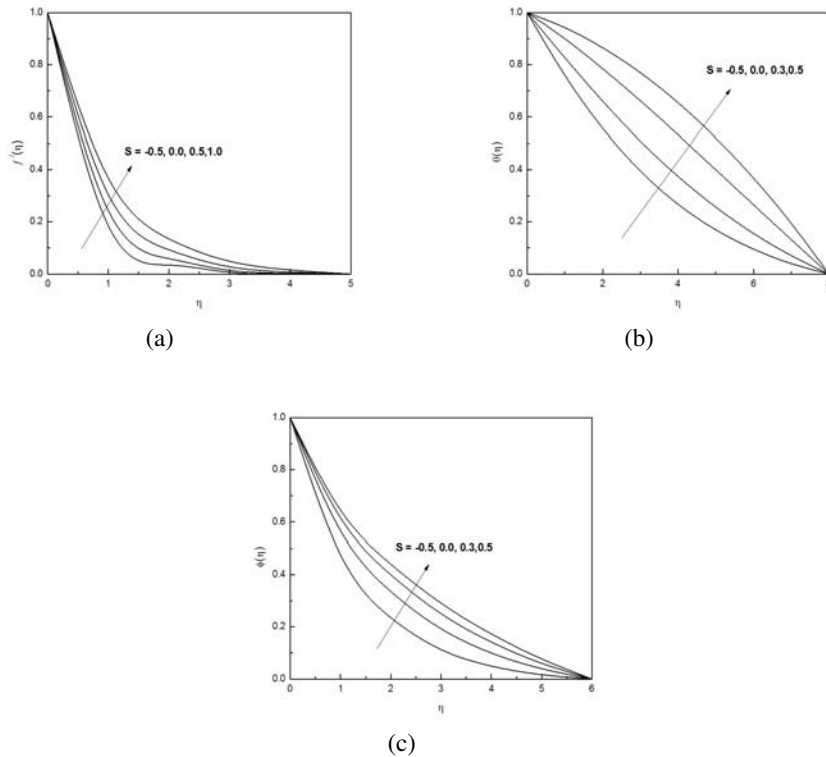


FIGURE 11. (a) Velocity Profiles for different values of Suction parameter  $S$ . (b) Temperature Profiles for different values of Suction parameter  $S$ . (c) Concentration Profiles for different values of Suction parameter  $S$ .

The non-dimensional velocity, temperature and concentration profiles for the different values of suction  $S$  are shown through Figure 11(a), 11(b) and 11(c) respectively. From these Figures it is observed that velocity, temperature and concentration profiles are increases with an increase of suction parameter  $S$ . Figure 12 displays the variation of skin-friction coefficient  $f''(0)$  against the magnetic interaction parameter  $M$  for different values of velocity exponent parameter  $m$ . It is seen that the skin-friction coefficient  $f''(0)$  decreases with increase of velocity exponent parameter  $m$  and it decreases for increasing magnetic interaction parameter  $M$ . It is observed from Figure 13 that the dimensionless rate of heat transfer  $\theta'(0)$  increases with increase of velocity exponent parameter  $m$ . further, it is noted that the dimensionless rate of heat transfer  $\theta'(0)$  increases in magnitude for increasing magnetic interaction parameter  $M$ .

Figure 14 shows the dimensionless rate of mass transfer  $\phi'(0)$  increases with increase of velocity exponent parameter  $m$ . also it is observed that the dimensionless rate of mass transfer  $\phi'(0)$  increases in magnitude for increasing magnetic interaction parameter  $M$ .

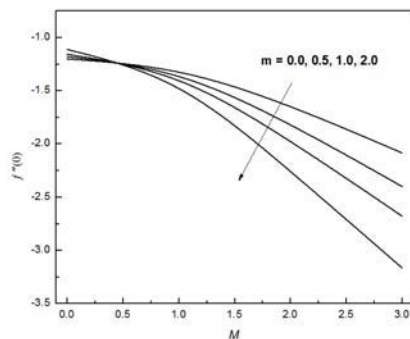


FIGURE 12. Skin friction coefficient for different values of velocity exponent parameter  $m$ .

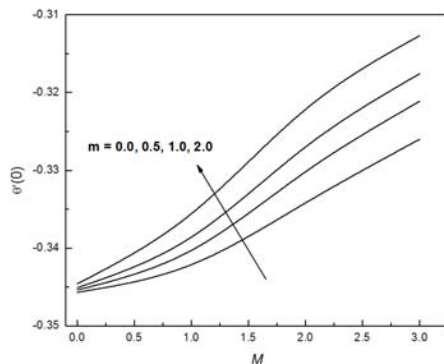


FIGURE 13. Dimensionless rate of heat transfer for different values Velocity exponent parameter  $m$ .

Figure 15 illustrates the effect of magnetic interaction parameter  $M$  over the dimensionless rate of heat transfer  $\theta'(0)$  for different values of radiation parameter  $R$ . It is seen that the

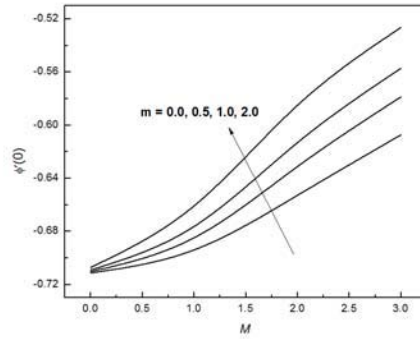


FIGURE 14. Dimensionless rate of mass transfer for different values Velocity exponent parameter  $m$ .

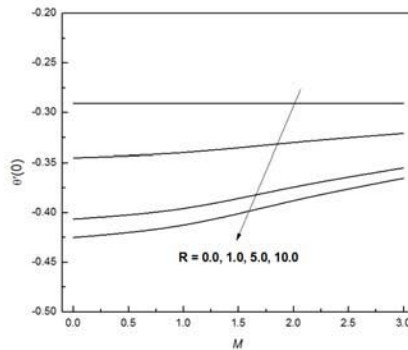


FIGURE 15. Dimensionless rate of heat transfer for different values of Radiation parameter  $R$ .

dimensionless rate of heat transfer  $\theta'(0)$  decreases with increase of radiation parameter  $R$  and increases with respect to magnetic interaction parameter  $M$ . Figure 16. portrays the variation of dimensionless rate of heat transfer  $\theta'(0)$  against the magnetic interaction parameter  $M$  for different values of heat generation parameter  $Q$ . it is apparent that increasing the dimensionless rate of heat transfer  $\theta'(0)$  decreases with increase of heat generation parameter  $Q$  and increases

with respect to magnetic interaction parameter  $M$ . Figure 17 displays the variation of skin-friction coefficient  $f''(0)$  against the magnetic interaction parameter  $M$  for different values of permeability parameter  $K$ . It is seen that the skin-friction coefficient  $f''(0)$  decreases with increase of velocity exponent parameter  $m$  and it decreases for increasing magnetic interaction parameter  $M$ .

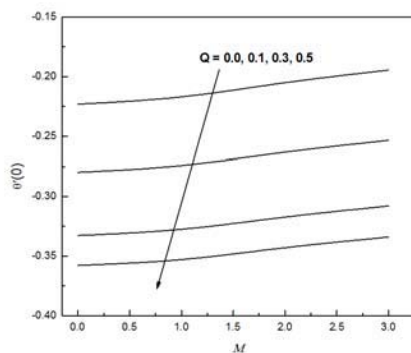


FIGURE 16. Dimensionless rate of heat transfer for different values of Heat generation parameter  $Q$ .

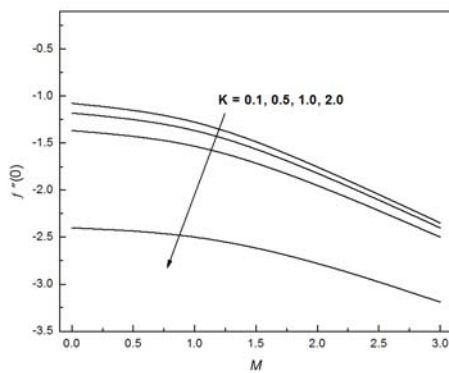


FIGURE 17. Skin friction coefficient for different values of Permeability parameter  $K$ .



It is observed that from Figure 18 the dimensionless rate of heat transfer  $\theta'(0)$  increases with increase of permeability parameter  $K$ . further, it is noted that the dimensionless rate of heat transfer  $\theta'(0)$  increases in magnitude for increasing magnetic interaction parameter  $M$ . Figure 19 shows the dimensionless rate of mass transfer  $\phi'(0)$  increases with increase of permeability parameter  $K$ . also it is observed that the dimensionless rate of mass transfer  $\phi'(0)$  increases in magnitude for increasing magnetic interaction parameter  $M$ . Figure 20 depicts the effect of radiation parameter  $R$  on the velocity  $f'(\eta)$  of the flow field. Here the velocity profiles are drawn against  $\eta$  for four different values of  $R$ . The radiation parameter is found to decelerate the velocity of the flow field at all points.

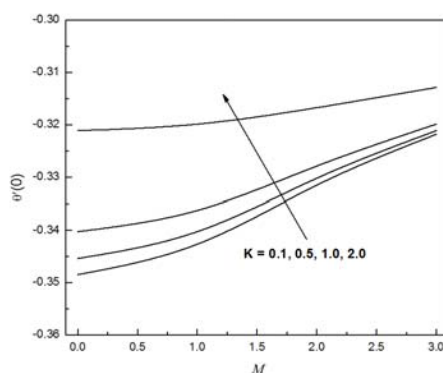


FIGURE 18. Dimensionless rate of heat transfer for different values of Permeability parameter  $K$ .

## 5. CONCLUSIONS

In this paper numerically investigated the radiation effects on MHD boundary layer flow of liquid metal over a porous stretching surface in porous medium with heat generation. The important findings of the paper are:

- (1) The fluid temperature profiles are increases if the values of the surface temperature parameter, magnetic parameter, heat generation parameter and suction parameter are increases. The fluid temperature profiles are decreases if the values of the radiation parameter, velocity exponent parameter, permeability parameter, prandtl number are increases. At any point in the flow field, the cooling of the plate is faster as prandtl number becomes larger. Thus prandtl number leads to faster cooling of the plate.
- (2) The fluid velocity profiles are increases if the values of the velocity exponent parameter, permeability parameter and suction parameter are increases. The fluid velocity is decreases if the value of magnetic parameter and radiation parameter increases.

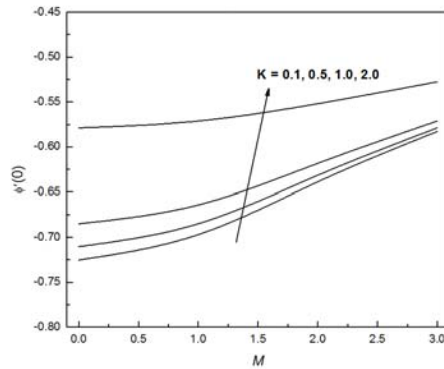


FIGURE 19. Dimensionless rate of mass transfer for different values of Permeability parameter  $K$ .

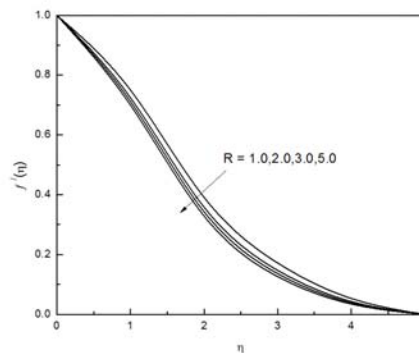


FIGURE 20. Velocity profiles for different values of radiation parameter  $R$ .

- (3) The fluid concentration profiles are increases if the values of the magnetic parameter and suction parameter are increases. The fluid concentration profiles are decreases if the values of the velocity exponent parameter, permeability parameter, and Schmidt number are increases.
- (4) The Skin-friction coefficient is decreases if the values of the velocity exponent parameter and permeability parameter are increases.

- (5) The Nusselt number is increases if the values of the velocity exponent parameter and permeability parameter are increases and it is decreases if the values of the Radiation parameter and heat generation parameter are increases. The Sherwood number is increases if the values of the velocity exponent parameter and permeability parameter are increases.

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