Advanced Static Over-modulation Scheme using Offset Voltages Injection for Simple Implementation and Less Harmonics

Dong-Myung Lee†

Abstract – In this paper, a novel static overmodulation scheme (OVM) for space-vector PWM (SVPWM) is proposed. The proposed static OVM scheme uses the concept of adding offset voltages in linear region as well as overmodulation region to fully utilize DC-link voltage. By employing zero sequence voltage injection, the proposed scheme reduces procedures for achieving SVPWM such as complicated gating time calculation. In addition, this paper proposes a stepwise discontinuous angle movement in high modulation region in order to reduce Total Harmonic Distortion (THD). The validity of the proposed scheme is verified through theoretical analysis and experimental results.

Keywords: Static overmodulation scheme, 2-level inverter, Offset voltage injection, Total harmonic distortion

1. Introduction

Space vector PWM (SVPWM) technique is widely used due to its high linear region of output voltage and the easiness of implementation using microprocessors or micro-controllers. The fundamental frequency component of output voltage by SVPWM matches the magnitude of the voltage command in the linear region. Whereas, in the overmodulation region, because the reference voltage vector partially or totally moves along the outside of the hexagon, the fundamental voltage magnitude is not linearly proportional to the magnitude of the reference vector. In some applications such as V/F control, it is important to ensure linearity of the output voltage corresponding to modulation index in the overmodulation (hereinafter OVM) region including six-step operation, so that PWM scheme for OVM is required.

OVM techniques can be divided into two. The first one is called as dynamic OVM [1, 2]. Dynamic OVM, which is usually used in vector control, focuses on the improvement of dynamic performance in OVM region. When the reference vector exists outside the hexagon, the voltage vector is projected on the side of hexagon according to specific control purpose. In contrast, for the V/F control of induction machines, more important control aspect is to have the linearity in output voltage and smooth transition from linear region to six-step operation. Hence, so-called the static OVM technique is applied [3].

This paper proposes a new static OVM that retains the linearity of the output voltage, as well as reduces harmonics. Especially in the proposed scheme, the static OVM is realized by offset voltage adding method instead of through calculation of switching times done in conventional methods.

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Hence, in order to take into account the linearity of the fundamental voltage component, THD reduction, and simple implementation, this paper proposes an advanced SVPWM done by offset voltages injection method applied in the linear as well as OVM region. The proposed OVM utilizes FSE to get approximate equations for voltage magnitude and phase angle, and applies the concept of adding offset voltages to realize the static OVM scheme by a simple procedure and lower THD.

As the concept of adding zero sequence voltages [13] is used for the proposed scheme, gating times of switches can be determined without complicated calculation process such as decision of voltage vector location and duty time calculation for right and left side vertex voltage vectors. In conventional methods, the phase-angle reference is commended in a manner that jumping into predetermined location after staying a vertex vector in the OVM region II [4-12]. Unlike conventional methods, to reduce THD the voltage vector moves not just in one-step but with several steps toward predetermined location in this proposed scheme. Through simulation and experimental results, the effectiveness of the proposed method is verified.

2. Proposed Static Over-modulation Method

In the proposed method, OVM region is divided into two corresponding to modulation index \( m \) same as the calculation based SVPWM. Modulation index, which ranges from 0 to 1, defined as the ratio of fundamental frequency component of voltage output \( (V_p) \) to that from six-step operation is expressed as follows

\[
m = \frac{V_p}{\frac{2}{\pi} V_{dc}}
\]

(1)

2.1 OVM region I (0.9069 ≤ \( m \) < 0.9514)

OVM region I begins at \( m=0.9069 \) [6]. In OVM region I, the reference vector partially exists outside of the hexagon. Dotted circle in Fig. 1 (a) shows the trajectory of the original vector \( V_{old}^{*} \), and that of new voltage reference \( V_{new}^{*} \) after modification is illustrated in Fig. 1 (b). As shown in Fig. 1, the magnitude of \( V_{new}^{*} \) is boosted to compensate the reduction of the magnitude of \( V_{old}^{*} \). When the voltage vector is located outside of the hexagon, for the implementation of SVPWM the voltage vector is projected on the side of the hexagon, and as the result of the projection the magnitude of the developed vector is reduced. Therefore, \( V_{new}^{*} \) has larger magnitude than \( V_{old}^{*} \) to maintain the linearity of voltage magnitude in OVM region corresponding to \( m \) value.

The approximate expression corresponding to \( m \) is summarized in Table 1. How to get Table 1 is explained in [6]. \( m_b \) means boosted \( m \) and defined as (2). In (2), \( V_{inst}^{*} \) represents the instantaneous peak value of the voltage vector. Using instantaneous value rather than fundamental frequency value is easy to implement the static OVM, so that the term \( m_b \) (boosted modulation index) is introduced.

\[
m_b = \frac{V_{new}^{*}}{\frac{2}{\pi} V_{dc}}
\]

(2)

In Fig. 1, the voltage references before \( (V_{old}^{*}) \) and after modification \( (V_{new}^{*}) \) are shown in the span of 60°, and they can be expressed as (3). When \( V^{*} \) rotates within the hexagon, it can be expressed as (3)-a. This trajectory resides between (1) \( \text{①} \) or (3) \( \text{③} \)-\( \text{④} \). On the other hand, when \( V^{*} \) is positioned outside the hexagon, \( V^{*} \) can be denoted as (3)-b, which represents the vector projected on the side (between (2) \( \text{②} \)\( \cdots \) (3)) of the hexagon. It should be mentioned that Eq. (3) is not used in implementation for SVPWM. Eq. (3) is just shown to help the understanding of the proposed method in the viewpoint of getting \( m_b \).

- Original voltage reference
  \[
  V^{*} = V_{inst}^{*} e^{j \theta}
  \]

- Modified (new) voltage reference in OVM region I
  \[
  V^{*} = \begin{cases} 
  \frac{kV_{inst}^{*} e^{j \theta}}{V_{dc}} & \text{inside hexagon} \\
  \sqrt{3} \cos \left( \frac{\pi}{6} - \theta \right) e^{j \theta} & \text{outside hexagon}
  \end{cases}
  \]

\( kV_{inst}^{*} \) in (3)-a can be rephrased as \( V_{b} \) (magnitude of boosted reference vector), and it has the magnitude of \( m_b \times 2V_{dc} / \pi \) obtained using (2).

Whereas, in the linear region, as the fundamental
voltage component is linearly proportional to the magnitude of voltage reference, \( m_b \) is equal to \( m \). In OVM region, \( m \) is different from \( m_b \). OVM I ends when \( m = 0.954 \) and it is equivalent to \( kV = V_b = 2/3V_d \) that is maximum instantaneous value of the phase voltage. The magnitude of fundamental voltage component for \( m = 0.954 \) is \( 1.908V_d/\pi \) \((=4/\pi V_d/2 * 0.954)\). For \( m = 0.954 \), corresponding \( m_b \) is \( \pi/3 \) \((=1.0472)\). In OVM region I, the magnitude of voltage vector is boosted while maintaining the phase angle reference, i.e. there is no modification in the phase angle reference.

In this paper, the concept of the offset voltage injection is utilized for achieving static OVM. The proposed OVM is applied to three-phase load in the same manner for each phase, so that the algorithm is described only with respect to a-phase. Voltage references of phase \( (V'_a) \), pole \( (V'_m) \), and offset \( (V''_m) \) for SVPWM is expressed as (4) and (5) [13].

\[
V'_m = V'_a + V''_m \\
V''_m = \frac{V''_{\max} + V''_{\min}}{2}
\]

where, \( V''_{\max} = \max(V''_a, V''_b, V''_c) \), \( V''_{\min} = \min(V''_a, V''_b, V''_c) \).

The relationship between \( m \) and \( m_b \) listed in Table 1 is used to ensure output voltage linearity of OVM. First \( m_b \) is obtained from \( m \) using Table 1 and \( k \) is determined as (6). New phase voltage, offset voltage, and pole voltage corresponding to \( m \) are expressed as (7) and (8).

\[
k = m_b \times 2V_d / \pi \\
V_{\text{new}} = kV'_a, V_{\text{new}} = kV''_a, V_{\text{new}} = V_{\text{new}} + V_{\text{new}}
\]

When the boosted (or modified) voltage vector is located outside the hexagon, the voltage vector is projected on the side of the hexagon while remaining the phase angle reference, and new voltage reference in OVM I can be expressed as (9).

\[
V_{\text{new}} = \begin{cases} 
V_{\text{max}} + V_{\text{min}} & V_{\text{new}} = V_{\text{new}} < V_d \\
V_{\text{max}} - V_{\text{min}} & V_{\text{new}} = V_{\text{new}} > V_d \\
\end{cases}
\]

Where, \( V_{\text{new}} = \max(V_{\text{new}}, V_{\text{new}}, V_{\text{new}}) \), and \( V_{\text{new}} = \min(V_{\text{new}}, V_{\text{new}}, V_{\text{new}}) \),

\[
V_{\text{new}} = \frac{V_{\text{new}} + V_{\text{min}}}{2}
\]

When the reference vector is located inside the hexagon, i.e. for the condition of \( V_{\text{new}} - V_{\text{new}} < V_d \), there is no modification in \( V_{\text{new}} \). But when the vector is positioned outside the hexagon, \( V_{\text{new}} = V_{\text{new}} - V_{\text{new}} \) is multiplied to \( V_{\text{new}} + V_{\text{new}} \) to project the voltage vector on the side of hexagon. Scaling \( V_{\text{new}} = V_{\text{max}} - V_{\text{min}} \) resembles the form of switching time modification presented as \( T_1 \) (modified) is \( T_1/(T_1 + T_3 + T_2) \times T_1 \) (original) in the minimum phase dynamic OVM method [14]. Where \( T_1, T_2, \) and \( T_3 \) represent the switching time for right vector, that for left vector, and sampling period, respectively. Fig. 2 illustrates the block diagram for generating voltage reference in OVM I region with the proposed method. First, boosted phase voltage reference is calculated according to \( m \) (equivalently \( V''_a \)), and then using (10) the offset voltage \( (V''_{\text{new}}) \) is determined. After adding \( V''_{\text{new}} \) to the voltage reference \( (V''_{\text{new}}) \), pole voltage reference \( (V''_{\text{new}}) \) for PWM is obtained finally. Fig. 2 clearly shows that the proposed method realizes the static OVM just adding \( k \) calculation block and multiplication routine if \( V_{\text{new}} - V_{\text{new}} > V_d \) hence it results in a simple procedure.

2.2 OVM scheme in region II (0.954 ≤ \( m \) ≤ 1)

Eq. (11) represents voltage reference of conventional method in OVM II region.

\[
V' = \begin{cases} 
\frac{2}{3}V_d & 0 < \theta < \alpha_b \\
\frac{1}{3}V_d e^{j\theta} & \alpha_b < \theta < \frac{\pi}{3} - \alpha_b \\
\frac{2}{3}V_d e^{j\pi/3} & \alpha_b < \theta < \frac{\pi}{3} \\
\end{cases}
\]

Where, \( \alpha_b \) represents a holding angle.

In conventional static OVMs for region II, the reference vector stays at the vertex of the hexagon until angle reference \( \theta \) reaches holding angle \( \alpha_b \) and when \( \theta \) arrives at \( \alpha_b \) position \( V' \) jumps to that point. In the period of \( \alpha_b < \theta < \pi/3 - \alpha_b \), \( V' \) moves continuously toward \( \pi/3 - \alpha_b \) position, and jumps to \( \pi/3 \) when \( \theta \) is positioned in between \( \pi/3 - \alpha_b, \alpha_b \). Thus, \( V' \) moves discontinuously in the periods of \( 0 < \theta < \alpha_b \) and \( \pi/3 - \alpha_b < \theta < \pi/3 \). In
addition, determination of $\theta$ is implemented by time calculation-based SVPWM.

On the other hand, in the proposed OVM in order to reduce THD, $\theta$ command of $V^*$ moves partially continuous in the period of phase jumping and it is realized by offset voltage adding. In the proposed scheme, $\theta$ approaches the specific position such as $\alpha_0$ not by just one step as shown in Fig. 3 (a) but with several steps as illustrated in Fig. 3 (b). Where, $\alpha$ means total span of $\theta$ segment, and $n$ is the number of intervals (or steps). It should be noticed that $\alpha$ is not $\alpha_0$, how to get $\alpha_0$ is explained in the following.

Mathematical representation of $V^*$ in OVM II by the proposed method having $n$ intervals in the range of $60^0$ can be expressed as (12).

$$V^* = \begin{cases} \frac{2}{3}V_{dc} & 0 \leq \theta < \frac{1}{n} \alpha \\ V_e^{\frac{\alpha}{\alpha + \theta}} & \frac{1}{n} \alpha \leq \theta < \alpha \\ V_e^{\frac{\pi}{3} - \theta} & \pi/3 - \theta < \pi/3 - \frac{n-1}{n} \alpha \\ \vdots & \vdots \\ \frac{2}{3}V_{dc}e^{\frac{\pi}{3} } & \frac{\pi}{3} - \frac{1}{n} \alpha \leq \theta < \pi/3 \\ \end{cases}$$  \hspace{1cm} (12)

Where, $V = V_{dc}/\sqrt{3}\cos(\pi/6 - \theta)$.

It should be emphasized again that in the conventional method such as [6] the SVPWM is implemented by using time based formulas rather than the offset voltages injection. To get the relationship among $m, \alpha$, and $n$, FSE is used in the proposed scheme. By equalizing the coefficient of FSE of the original vector ($V_{in}e^{j\theta}$) and that of the modified vectors of (12), Eqs. (13) and (14) can be obtained. Where, $V_{in}$ denotes the magnitude of fundamental voltage component.

$$\int_{0}^{\pi}V_{in}e^{\theta}e^{-j\theta}d\theta = \int_{0}^{\pi}2V_{dc}e^{-j\theta}d\theta + $$

$$\int_{\pi}^{2\pi}V_{e}^{\frac{\theta-\pi}{\theta}}e^{-j\theta}d\theta + \cdots + \int_{\pi}^{\frac{\pi}{3}}2V_{dc}e^{j\frac{\pi}{3}}e^{-j\theta}d\theta$$  \hspace{1cm} (13)

$$V_{in} = \frac{V_{dc}}{\sqrt{3}}\sum_{i=0}^{n}\frac{1}{\cos\left(\frac{\pi}{6} - \frac{i}{n}\alpha\right)} \times 2\sin\left(\frac{\alpha}{n}\right)$$  \hspace{1cm} (14)

Fig. 4 shows the relationship of $\alpha$, $n$, and $m$ (equivalently $V_{in}$) obtained by using (14). The amount of calculation increases as $n$ becomes greater. In this paper, $n$ is set as 1, 2, or 3. $\alpha$ value corresponding to $n$ for satisfying the linearity in voltage magnitude with less THD is summarized in Table 2. Curve fitting tool in MATLAB is used to get the approximate equation of $\alpha$. In Table 2, $m$ is divided into 5 sections, and the number of sections is chosen with consideration of the complexity of PWM routine implementation.

In OVM II, the magnitude of boosted voltage is fixed as $2V_{dc}$, which is the maximum magnitude of phase voltage. In OVM II, $k$ in (6) is $2V_{dc}$ (equivalently $m_b = \pi/3 = 1.0472$), and the pole voltage reference is expressed as (15).

$$V_{new} = \frac{V_{dc}}{V_{new} + V_{new}}(V_{new} + V_{new})$$  \hspace{1cm} (15)

As shown in (15), the minimum phase error method is
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![Block Diagram](image)

Fig. 5. The overall block diagram for generating PWM in OVM II region by the proposed method.

applied to limit the magnitude of pole voltage reference. Fig. 5 shows the block diagram for generating pole voltages with offset voltages in OVM II region by the proposed method. Regardless of \(m, k\) is set as \(2/3V_{dc}\). In OVM II region, the proposed scheme determines holding span \(\alpha\) corresponding to \(m\) with the information of Table 2. Magnitude of \(V^*\) is fixed as \(2/3V_{dc}\) and the phase angle is commended by using the predetermined \(\alpha\) and \(n\) steps.

### 2.3 Reduction of total harmonic distortion

THD of output voltage is defined as (16). It is the ratio of RMS value of total harmonic components to that of fundamental frequency.

\[
THD = \frac{\sqrt{2}V_{on}^2 - V_1^2}{V_1} \tag{16}
\]

In OVM II region, stepwise movement of \(V^*\) is proposed to reduce THD. Fig. 6 illustrates (a) \(V^*\) movement in conventional method, and (b) that in the proposed method with for \(n=2\). In Fig. 6, the solid line represents \(V_1^*\) and is trajectory of the magnitude of fundamental frequency in \(V^*\). Eq. (16) says that the smaller difference between \(V^*\) and \(V_1^*\) results in smaller THD. Hence comparing Figs. 6 (a) and (b), it can be known that Fig. 6 (b) showing smaller difference between \(V^*\) and \(V_1^*\) will result in lower THD.

![Reference Voltage Shapes](image)

Fig. 6. Reference voltage shape in conventional method (top) and the proposed method (bottom) with \(n = 2\) (solid line \(-V^*\), dotted line \(-V_1^*\), fundamental frequency component of \(V^*\)).

![Comparison Graph](image)

Fig. 7. Comparison graph of n-th order harmonic in conventional method [6] and the proposed method for \(m = 0.96\) and 0.97.

![Comparison of THD Value](image)

Fig. 8. Comparison of THD value in OVM II region.

Fig. 7 illustrates \(n\)th order harmonic existed in conventional method and that in the proposed method for \(m=0.96\) and 0.97. From Fig. 7, it can be seen that the proposed method has less harmonic component compared with the conventional method that has one step \(\theta\) movement in OVM II region. Fig. 8 compares amount of the THD for various \(m\) values in the conventional method and that in the proposed method. THD is reduced by the proposed method, especially \(m\) ranging from 0.9514 to 0.97 thanks to step-wise \(\theta\) movement. Fig. 8 shows that the step-wise phase angle movement by the proposed method in OVM II has lower THD.

### 3. Verifications

Fig. 9 is the flowchart of the proposed algorithm for the
PWM generation in OVM regions. First, magnitude and phase angle references of voltage vectors are decided. In OVM I region, Eqs. (6)-(10) are used for generating pole voltages for SVPWM. In OVM II region, \( m_b \) is fixed as 1.047 (equivalently \( k = 2V_{dc}/3 \)), and new phase reference for less harmonics is generated using Table 2.

Fig. 10 shows simulation waveforms of \( V_{as} \) (a-phase voltage reference), \( V_{sn} \) (offset voltage), and \( V_{an} \) (pole voltage) for various \( m \) and \( n \). The \( m \) value for Fig. 10 (a) corresponds to OVM region I, so that there is no change in phase angle of the voltage reference and offset voltage (\( V_{as} \)) has a triangular shape.

When \( m \) is in OVM II region, the phase angle moves discontinuously, therefore the \( V_{sn} \) does not appear as a triangular shape. In Fig. 10 (c) the number of steps in OVM II region is selected as \( n = 3 \). Comparing with Fig. 10 (b) for \( n = 1 \) and (c) for \( n = 3 \), it is clear that the pole voltage generated by the proposed method with \( n = 3 \) has more sinusoidal phase voltage waveform than that with \( n = 1 \). The proposed method that utilizes the offset voltage injection for generating SVPWM works well for six-step operation as illustrated in Fig. 10 (d).

To verify the validity of the proposed scheme, experiments have been conducted. SVPWM for an induction motor with \( V/F \) control was carried out. DC link voltage is 300 \( V \), and switching frequency \( (f_s) \) is 10 kHz. Fig. 11 (a) illustrates experimental waveforms of pole, offset, and phase voltage references in linear region \( (m = 0.6) \) from top to bottom. Fig. 11 (b) shows voltage waveforms in OVM I region \( (m = 0.93) \). As the phase angle of voltage reference in OVM I region changes continuously, the shape of the phase voltage is sinusoidal.

Figs. 12 (a) and (b) show experimental waveform of pole voltage, phase voltage, and phase angle in OVM II with \( n = 1 \) and 3, respectively. It is clearly shown in Fig. 12 that step-wise change of the phase angle in OVM II has been achieved by the proposed method. It should be
mentioned that the experimental waveform are generated by the proposed offset voltage injection scheme for static OVM regardless on $n$ value.

### 4. Conclusion

This paper proposed a simple static OVM utilizing offset voltages injection. The magnitudes of stepping up and phase angle were determined by modulation index and span of holding angle with $n$-steps. Those values were implemented by approximate equations predetermined by FSE to ensure linearity of the output voltage corresponding to $m$. The proposed OVM method was not based on the SVPWM by formula but that using injection of offset voltages. In addition, reference voltage vector has stepwise movement in OVM II region, and as the result THD reduction was achieved. Simulation and experimental results verified the performance of the proposed method. This scheme is expected to contribute the improvement of control performance in static OVM scheme.

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### References


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