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Group Decision Making Using Intuitionistic Hesitant Fuzzy Sets

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Abstract

Dealing with uncertainty is always a challenging problem. Intuitionistic fuzzy sets was presented to manage situations in which experts have some membership and non-membership value to assess an alternative. Hesitant fuzzy sets was used to handle such situations in which experts hesitate between several possible membership values to assess an alternative. In this paper, the concept of intuitionistic hesitant fuzzy set is introduced to provide computational basis to manage the situations in which experts assess an alternative in possible membership values and non-membership values. Distance measure is defined between any two intuitionistic hesitant fuzzy elements. Fuzzy technique for order preference by similarity to ideal solution is developed for intuitionistic hesitant fuzzy set to solve multi-criteria decision making problem in group decision environment. An example is given to illustrate this technique.

Keywords: Hesitant fuzzy set, Intuitionistic fuzzy set, Multiple attribute group decision making, Technique for order preference by similarity to ideal solution

1. Introduction

Group decision making is the process of finding the best option among a set of feasible alternatives. Basic problem is how to aggregate several inputs into a single representative output [1–3]. Problems that are defined under uncertain situations are common in real world decision making. Zadeh [4] in his seminal paper introduced the notion of fuzzy sets to handle uncertainty. Bellman and Zadeh [5] used fuzzy sets in decision making for the solution of ambiguity in information obtained from human preferences. Recently, Dubois [6] presented a beautiful comparison about old and new techniques for fuzzy decision analysis. Atanassov [7, 8] introduced the concept of intuitionistic fuzzy sets (IFS) characterized by a membership function and a non-membership function, which is more suitable for dealing with fuzziness and uncertainty than the fuzzy set. The IFS is highly useful in depicting uncertainty and vagueness of an object, and thus can be used as a powerful tool to express data information under various different fuzzy environments which has attracted great attentions. Recently, the intuitionistic fuzzy set has been widely applied to decision making problems [9–12]. IFSs have been found to be a particularly useful tool to deal with vagueness. Torra [13] extended the concept of fuzzy sets to hesitant fuzzy sets. Hesitant fuzzy set theory tries to manage those situations where a set of values are possible in the definition process of the membership of an element. Group decision making problems are solved by using hesitant fuzzy sets and with aggregation operators in [1–3, 14, 15].

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Hwang and Yoon [16] developed technique for order preference by similarity to ideal solution (TOPSIS) for multi-attribute/multi-criteria decision making (MADM/MCDM) problems. Shih et al. [17] addressed four advantages of TOPSIS: first is that a sound logic represents the rationale of human choice; secondly a scalar value considers the best and worst alternative simultaneously; third advantage is that it has a simple computation process and can be easily programmed and the last but not the least advantage is that it has ability of the performance measures of all alternatives on attributes to be visualized on a polyhedron, at least for any two dimensions. Fuzzy numbers are applied to establish a fuzzy TOPSIS [18].

The aim of this paper is to introduce the concept of intuitionistic hesitant fuzzy sets (IHFS) by merging the concept of IFS and HFS. It helps to manage those situations of uncertainty in which some values are possible as membership values of element as well as non-membership values of the same element. Additionally, we also develop fuzzy TOPSIS for IHFS. This article is organized as follows: In Section 2, we introduce the concept of IHFS and notion of distance between any two elements of IHFS. In Section 3, TOPSIS is constructed for IHFS. Then in Section 4, this fuzzy TOPSIS method is applied for the ranking of alternatives in an example, to demonstrate its feasibility. Conclusion is given in the last section.

2. Intuitionistic Hesitant Fuzzy Set

In this section, we introduce an extension of IFS to manage those situations in which several values are possible for the definition of a membership function and non-membership function. We propose the concept of IHFS by keeping in view the importance of IFS and HFS. IHFS is defined in terms of a function that returns a set of membership values and a set of non-membership values for each element in the domain.

Definition 2.1. An IHFS on X are functions h and h' that when applied to X return subsets of $[0,1]$, which can be represented as the following mathematical symbol:

$$E = \{(x, h(x), h'(x))|x \in X\},$$

where $h(x)$ and $h'(x)$ are sets of some values in $[0,1]$, denoting the possible membership degrees and non-membership degrees of the element $x \in X$ to the set E with the conditions that $\max(h(x))+\min(h'(x)) \leq 1$ and $\min(h(x))+\max(h'(x)) \leq 1$. For convenience, $(h(x),\ h'(x))$ an intuitionistic hesitant fuzzy element (IHFE).

Examples of IHFS are given below where $h(x)$ and $h'(x)$ represent the possible membership and non-membership values of the set at x, respectively.

It is noted that the number of values in different IHFEs may be different, let $l_{h(x)}$ and $l_{h'(x)}$ be the number of values in $h(x)$ and $h'(x)$. In case values in an IHFE are out of order; we can arrange them in such a order, that IHFE (h, h'), let $\sigma : (1,2,...,n) \to (1,2,...,n)$ and $\varsigma : (1,2,...,m) \to (1,2,...,m)$ be two permutations satisfying $h_{\sigma(i)} \leq h_{\sigma(i+1)}$, $i = 1,2,...,l_h - 1$ and $h'_{\sigma(i)} \leq h'_{\sigma(i+1)}$, $j = 1,2,...,l_{h'} - 1$. We proposed that two IHFEs (h_1, h'_1) and (h_2, h'_2) have $l_{h_1} = l_{h_2}$, $l_{h'_1} = l_{h'_2}$, $h_{1_{\sigma(i)}} = h_{2_{\sigma(i)}}$ and $h'_{1_{\sigma(j)}} = h'_{2_{\sigma(j)}}$ if and only if $(h_1, h'_1) = (h_2, h'_2)$ for $i = 1,2,...,l_{h_1}$ and $j = 1,2,...,l_{h'_1}$.

Example 2.2. Consider an IHFS A given by

$A = \{(x_1, (0.2, 0.3, 0.5, 0.6, 0.9), (0.01, 0.05, 0.1)), (x_2, (0.1, 0.4, 0.7), (0.1, 0.15, 0.18, 0.2, 0.25))\}$.

Then

$\max h_A(x_1) = \max(0.2, 0.3, 0.5, 0.6, 0.9)$=0.9;
$\max h'_A(x_1) = \max(0.01, 0.05, 0.1)$=0.1;
$\max h_A(x_2) = \max(0.1, 0.4, 0.7)$=0.7;
$\max h'_A(x_2) = \max(0.1, 0.15, 0.18, 0.2, 0.25)$=0.25;
$\min h_A(x_1) = \min(0.2, 0.3, 0.5, 0.6, 0.9)$=0.2;
$\min h'_A(x_1) = \min(0.01, 0.05, 0.1)$=0.01;
$\min h_A(x_2) = \min(0.1, 0.4, 0.7)$=0.1;
$\min h'_A(x_2) = \min(0.1, 0.15, 0.18, 0.2, 0.25)$=0.1.

It is clear that $\max h_A(x_1) + \min h'_A(x_1) = 0.9 + 0.01 = 0.91 \leq 1$ and $\min h_A(x_1) + \max h'_A(x_1) = 0.2 + 0.1 = 0.3 \leq 1$, so $(h_A(x_1), h'_A(x_1))$ is an IHFE. Similarly, $(h_A(x_2), h'_A(x_2))$ is an IHFE. Thus A is an IHFS.

Definition 2.3. Let x and y be two IHFEs, such that $x = (h_x, h'_x) = ((a_1, a_2, \ldots, a_n), (a'_1, a'_2, \ldots, a'_{n'}))$ and $y = (h_y, h'_y) = ((b_1, b_2, \ldots, b_m), (b'_1, b'_2, \ldots, b'_{m'}))$ then distance 'd' between x and y is defined as

$$d(x,y) = \max \left\{ \begin{array}{c} \max\limits_{a_i \in h_x} \left\{ \min\limits_{b_i \in h_y} (|a_i - b_i|) \right\}, \\ \max\limits_{b_i \in h_y} \left\{ \min\limits_{a_i \in h_x} (|a_i - b_i|) \right\}, \\ \max\limits_{a'_i \in h'_x} \left\{ \min\limits_{b'_i \in h'_y} (|a'_i - b'_i|) \right\}, \\ \max\limits_{b'_i \in h'_y} \left\{ \min\limits_{a'_i \in h'_x} (|a'_i - b'_i|) \right\} \end{array} \right\}.$$

It is easy to show that this distance 'd' satisfies the following properties:

1) $d(x,y) = 0$ if and only if $x = y$;

2) $d(x, y) = d(y, x)$.

3. TOPSIS for IHFS

In this section, we give construction of TOPSIS for IHFS. This TOPSIS is used for multi-criteria group decision making where the opinions about the criteria values are expressed in IHFS. We suppose that in this group decision making problem, $E = \{e_1, e_2, \ldots, e_K\}$ is the set of the decision makers involved in the decision problem; $A = \{A_1, A_2, \ldots, A_m\}$ is the set of the considered alternatives and $C = \{C_1, C_2, \ldots, C_n\}$ is the set of the criteria used for evaluating the alternatives.

Step 1. Let $\tilde{X}^l = [x_{ij} = (H^l_{S_{ij}}, H'^l_{S_{ij}})]_{m\times n}$ be a fuzzy decision matrix for the MCDM problem where performance of alternative A_i with respect to decision maker e_l and criterion C_j is denoted as $H^l_{S_{ij}}$, in a group decision environment with K decision makers.

Step 2. We calculate the one decision matrix X by aggregating the opinions of DMs ($\tilde{X}^1$, $\tilde{X}^2$, $\cdots$, $\tilde{X}^K$);

$X = [x_{ij}]$, where $x_{ij} = ((x \mid x \in H^l_{S_{ij}}$ and $s_{p_{ij}} \le x \le s_{q_{ij}}$ for all l), $(x \mid x \in H'^l_{S_{ij}}$ and $s'_{p_{ij}} \le x \le s'_{q_{ij}}$ for all l)) where

$$s_{p_{ij}} = \min\left\{\min_{l=1}^{K}(\max H^l_{S_{ij}}), \max_{l=1}^{K}(\min H^l_{S_{ij}})\right\},$$

$$s_{q_{ij}} = \max\left\{\min_{l=1}^{K}(\max H^l_{S_{ij}}), \max_{l=1}^{K}(\min H^l_{S_{ij}})\right\},$$

$$s'_{p_{ij}} = \min\left\{\min_{l=1}^{K}(\max H'^l_{S_{ij}}), \max_{l=1}^{K}(\min H'^l_{S_{ij}})\right\}$$

and

$$s'_{q_{ij}} = \max\left\{\min_{l=1}^{K}(\max H'^l_{S_{ij}}), \max_{l=1}^{K}(\min H'^l_{S_{ij}})\right\}.$$

Performance of alternative A_i with respect to criterion C_j is denoted as x_{ij}, in an aggregated matrix X.

Step 3. Let Ω_b be the collection of benefit criteria (i.e., the larger C_j, the greater preference) and Ω_c be the collection of cost criteria (i.e., the smaller C_j, the greater preference). The IHFS positive-ideal solution (IHFS-PIS), denoted as $\tilde{A}^+ = (\tilde{V}_1^+ \ \tilde{V}_2^+ \ \ldots \ \tilde{V}_n^+)$, and the IHFS negative-ideal solution (IHFS-NIS), denoted as $\tilde{A}^- = (\tilde{V}_1^- \ \tilde{V}_2^- \ \ldots \ \tilde{V}_n^-)$, are defined as follows:

$$\begin{aligned}\tilde{A}^+ = \Big[&(x, x')|x \in H^l_{S_{ij}} \text{ and } x' \in H'^l_{S_{ij}} \ \forall\, i \\ &\text{and } \left(\max_{l=1}^{K}\left(\max_i(\min H^l_{S_{ij}})\right) \le x,\right. \\ &x \le \max_{l=1}^{K}\left(\max_i(\max H^l_{S_{ij}})\right), \\ &\min_{l=1}^{K}\left(\min_i(\min H'^l_{S_{ij}})\right) \le x', \\ &\left. x' \le \min_{l=1}^{K}\left(\min_i(\max H'^l_{S_{ij}})\right)\right) |j \in \Omega_b, \\ &(x, x')|x \in H^l_{S_{ij}} \text{ and } x' \in H'^l_{S_{ij}} \ \forall\, i \\ &\text{and } \left(\min_{l=1}^{K}\left(\min_i(\min H^l_{S_{ij}})\right) \le x,\right. \\ &x \le \min_{l=1}^{K}\left(\min_i(\max H^l_{S_{ij}})\right), \\ &\max_{l=1}^{K}\left(\max_i(\min H'^l_{S_{ij}})\right) \le x', \\ &\left. x' \le \max_{l=1}^{K}\left(\max_i(\max H'^l_{S_{ij}})\right)\right) |j \in \Omega_c\Big] \\ &i = 1, 2, \ldots, m, \text{ and } j = 1, 2, \ldots, n.\end{aligned}$$

$$\tilde{A}^+ = (\tilde{V}_1^+ \ \tilde{V}_2^+ \ \ldots \ \tilde{V}_n^+)$$

$$\begin{aligned}\tilde{A}^- = \Big[&(x, x')|x \in H^l_{S_{ij}} \text{ and } x' \in H'^l_{S_{ij}} \ \forall\, i \\ &\text{and } \left(\max_{l=1}^{K}\left(\max_i(\min H^l_{S_{ij}})\right) \le x,\right. \\ &x \le \max_{l=1}^{K}\left(\max_i(\max H^l_{S_{ij}})\right), \\ &\min_{l=1}^{K}\left(\min_i(\min H'^l_{S_{ij}})\right) \le x', \\ &\left. x' \le \min_{l=1}^{K}\left(\min_i(\max H'^l_{S_{ij}})\right)\right) |j \in \Omega_c, \\ &(x, x')|x \in H^l_{S_{ij}} \text{ and } x' \in H'^l_{S_{ij}} \ \forall\, i \\ &\text{and } \left(\min_{l=1}^{K}\left(\min_i(\min H^l_{S_{ij}})\right) \le x,\right. \\ &x \le \min_{l=1}^{K}\left(\min_i(\max H^l_{S_{ij}})\right), \\ &\max_{l=1}^{K}\left(\max_i(\min H'^l_{S_{ij}})\right) \le x', \\ &\left. x' \le \max_{l=1}^{K}\left(\max_i(\max H'^l_{S_{ij}})\right)\right) |j \in \Omega_b\Big] \\ &i = 1, 2, \ldots, m, \text{ and } j = 1, 2, \ldots, n.\end{aligned}$$

$$\tilde{A}^- = (\tilde{V}_1^- \ \tilde{V}_2^- \ \ldots \ \tilde{V}_n^-)$$

Step 4. Construct positive ideal separation matrix (D^+) and negative ideal separation matrix (D^-) which are defined as follows:

$$D^+ = \begin{bmatrix} d(x_{11}, \tilde{V}_1^+) & + & \cdots & + & d(x_{1n}, \tilde{V}_n^+) \\ d(x_{21}, \tilde{V}_1^+) & + & \cdots & + & d(x_{2n}, \tilde{V}_n^+) \\ \vdots & & \vdots & & \vdots \\ d(x_{m1}, \tilde{V}_1^+) & + & \cdots & + & d(x_{mn}, \tilde{V}_n^+) \end{bmatrix}$$

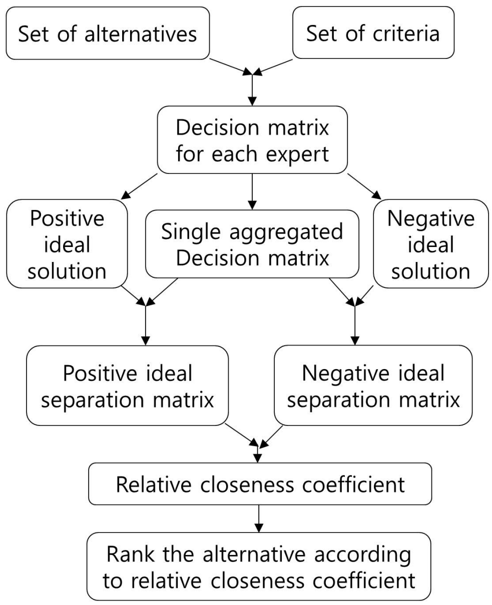

Figure 1. Graphical structure of technique for order preference by similarity to ideal solution.

and

$$D^- = \begin{bmatrix} d(x_{11}, \tilde{V}_1^-) & + & \cdots & + & d(x_{1n}, \tilde{V}_n^-) \\ d(x_{21}, \tilde{V}_1^-) & + & \cdots & + & d(x_{2n}, \tilde{V}_n^-) \\ \vdots & & \vdots & & \vdots \\ d(x_{m1}, \tilde{V}_1^-) & + & \cdots & + & d(x_{mn}, \tilde{V}_n^-) \end{bmatrix}$$

Step 5. Calculate the relative closeness (RC) coefficient of each alternative to the ideal solution as follows:

$$RC(A_i) = \frac{D_i^-}{D_i^+ + D_i^-},\ i = 1, 2, \ldots, m,$$

where $D_i^- = \sum_{j=1}^{n} d(x_{ij}, \tilde{V}_j^-)$ and $D_i^+ = \sum_{j=1}^{n} d(x_{ij}, \tilde{V}_j^+)$.

Step 6. Rank all the alternatives A_i $(i = 1, 2, \ldots, m)$ according to the $RC(A_i)$ coefficient, greater the value $RC(A_i)$, better the alternative $A_{i.}$

Graphical representation of this proposed technique is given in Figure 1.

4. Illustrative Example

In this section, we give an example. We utilized the method proposed in Section 4 to get the most desirable alternative as well as rank the alternatives from the best to worst or vice versa. Consider five schools (School of Business and Economics (A_1), School of Science and Technology (A_2), School of Social Sciences and Humanities (A_3), School of Communication and Cultural Studies (A_4), School of Textile and Design (A_5)) all in the same university. Management of the university want to manage the allocation of funds to these schools based on their performance. There are four criteria (expenses of school (C_1), students in take per year in school (C_2), publications from school (C_3), covered area of the school (C_4)) for assessing the performance of these five schools. These assessments are given by the nine members from board of directors.

Table 1. Decision matrix $(\tilde{X}^1)$ with respect to experts 1,2,3,4 (e_1, e_2, e_3, e_4)

	C_1
A_1	((0.5,0.6,0.8),(0.1,0.2))
A_2	((0.1,0.3),(0.3,0.4,0.5))
A_3	((0.5,0.7),(0.2,0.25))
A_4	((0.7,0.9),(0.05,0.1))
A_5	((1),(0))
	C_2
A_1	((0.6,0.8),(0.1,0.2))
A_2	((0.5,0.7,0.8),(0.1))
A_3	((0.5,0.6),(0.2,0.35))
A_4	((0.1,0.2),(0.6,0.7))
A_5	((0.1,0.3),(0.5,0.65))
	C_3
A_1	((0.1,0.3),(0.6,0.7))
A_2	((0.5,0.6),(0.1,0.3))
A_3	((0.7,0.9),(0.05,0.1))
A_4	((0.1,0.3),(0.6,0.7))
A_5	((0,0.2),(0.7,0.8))
	C_4
A_1	((0.1,0.3),(0.5,0.6))
A_2	((0.5,0.6),(0.2,0.3))
A_3	((0.1,0.2),(0.6,0.7))
A_4	((0.5,0.6,0.7),(0.2))
A_5	((0.4,0.7),(0.1,0.2))

Step 1. There are five possible alternatives A_i $(i = 1, 2, 3, 4, 5)$ are to be evaluated on the criteria C_j $(j = 1, 2, 3, 4)$ using the IHFS by nine experts e_K $(K = 1, 2, \ldots, 9)$, as listed in Tables 1-3.

Step 2. The decision matrix constructed in Table 4 by utilize Tables 1-3.

Step 3. For cost criteria C_1, C_4 and benefit criteria C_2, C_3 IHFS-PIS is as follows:

Table 2. Decision matrix ($\tilde{X}^2$) with respect to experts 5,6,7 (e_5, e_6, e_7)

	C_1
A_1	((0.1,0.2),(0.5,0.6))
A_2	((0,0.2),(0.6,0.7))
A_3	((0.4,0.6),(0.1,0.3))
A_4	((0.6,1),(0))
A_5	((0.5,0.7),(0.1,0.2))
	C_2
A_1	((0.4,0.9),(0,0.1))
A_2	((0.1,0.3),(0.5,0.6))
A_3	((0.1,0.2),(0.6,0.7))
A_4	((0.4,0.7),(0.1,0.2))
A_5	((0.4,0.6),(0.2,0.3))
	C_3
A_1	((0,0.2),(0.6,0.7))
A_2	((0.4,0.5),(0.3,0.4))
A_3	((0.4,0.6),(0.3,0.4))
A_4	((0,0.1),(0.6,0.8))
A_5	((0,0.1),(0.7,0.8))
	C_4
A_1	((0.4,0.6),(0.1,0.3))
A_2	((0.6,1),(0))
A_3	((0,0.2),(0.5,0.7))
A_4	((0.5,0.7),(0.2,0.3))
A_5	((0.6,1),(0))

Table 3. Decision matrix ($\tilde{X}^3$) with respect to experts 8,9 (e_8, e_9)

	C_1
A_1	((0.4,0.6),(0.2,0.3))
A_2	((0.3,0.6),(0.2,0.3))
A_3	((0.1,0.3),(0.5,0.6))
A_4	((0.6,0.9),(0,0.1))
A_5	((0.5,0.6),(0.3,0.4))
	C_2
A_1	((0.6,1),(0))
A_2	((0.1,0.3),(0.6,0.7))
A_3	((0.6,0.9),(0,0.1))
A_4	((0.5,0.7),(0.1,0.2))
A_5	((0.1,0.3),(0.6,0.7))
	C_3
A_1	((0.3,0.5),(0.3,0.4))
A_2	((0.5,0.9),(0,0.05))
A_3	((0.3,0.7),(0.1,0.2))
A_4	((0,0.2,0.4),(0.4,0.5))
A_5	((0.2,0.4),(0.4,0.5))
	C_4
A_1	((0,0.3),(0.5,0.6))
A_2	((0.3,0.5),(0.3,0.4))
A_3	((0,0.1),(0.7,0.8))
A_4	((0.5,0.6,0.8),(0))
A_5	((1),(0))

$$A^+ = \begin{bmatrix} ((0,0.1,0.2),(0.6,0.7)) & ((0.6,0.7,0.8,0.9,1),(0)) & ((0.7,0.8,0.9),(0,0.05)) & ((0,0.1),(0.7,0.8)) \end{bmatrix}$$

For cost criteria C_1, C_4 and benefit criteria C_2, C_3 IHFS-NIS is as follows:

$$A^- = \begin{bmatrix} ((0.7,0.8,0.9),(0)) & ((0.1,0.2),(0.6,0.65,0.7)) & ((0,0.1),(0.7,0.8)) & ((1),(0)) \end{bmatrix}$$

Step 4. Positive ideal separation matrix (D^+):

$$D^+ = \begin{bmatrix} 0.4 & + & 0.2 & + & 0.6 & + & 0.4 \\ 0.3 & + & 0.6 & + & 0.4 & + & 0.7 \\ 0.35 & + & 0.6 & + & 0.25 & + & 0.1 \\ 0.7 & + & 0.6 & + & 0.8 & + & 0.7 \\ 0.8 & + & 0.6 & + & 0.7 & + & 0.9 \end{bmatrix}$$

$$D^+ = \begin{bmatrix} 1.6 \\ 2 \\ 1.3 \\ 2.8 \\ 3.0 \end{bmatrix}$$

Negative ideal separation matrix (D^-):

$$D^- = \begin{bmatrix} 0.5 & + & 0.6 & + & 0.3 & + & 0.7 \\ 0.6 & + & 0.5 & + & 0.65 & + & 0.5 \\ 0.5 & + & 0.5 & + & 0.6 & + & 0.9 \\ 0.1 & + & 0.4 & + & 0.2 & + & 0.5 \\ 0.3 & + & 0.3 & + & 0.2 & + & 0.3 \end{bmatrix}$$

$$D^- = \begin{bmatrix} 2.1 \\ 2.25 \\ 2.5 \\ 1.2 \\ 1.1 \end{bmatrix}$$

Step 5. *RC* of each alternative to the ideal solutions:

$RC(A_1) = 2.1/(1.6 + 2.1) = 0.5676;$

$RC(A_2) = 2.25/(2 + 2.25) = 0.5294;$

$RC(A_3) = 2.5/(1.3 + 2.5) = 0.6579;$

$RC(A_4) = 1.2/(2.8 + 1.2) = 0.3;$

$RC(A_5) = 1.1/(3 + 1.1) = 0.2683.$

Table 4. Decision matrix (X)

	C_1
A_1	((0.2,0.4,0.5),(0.2,0.3,0.5))
A_2	((0.2,0.3),(0.3,0.4,0.5,0.6))
A_3	((0.3,0.4,0.5),(0.25,0.3,0.5))
A_4	((0.7,0.9),(0,0.05))
A_5	((0.6,0.7,1),(0,0.1,0.2,0.3))
	C_2
A_1	((0.6,0.8),(0,0.1))
A_2	((0.3,0.5),(0.1,0.5,0.6))
A_3	((0.2,0.5,0.6),(0.1,0.2,0.35,0.6))
A_4	((0.2,0.4,0.5),(0.2,0.6))
A_5	((0.3,0.4),(0.3,0.5,0.6))
	C_3
A_1	((0.2,0.3),(0.4,0.6))
A_2	((0.5),(0.05,0.1,0.3))
A_3	((0.6,0.7),(0.1,0.2,0.3))
A_4	((0.1),(0.5,0.6))
A_5	((0.1,0.2),(0.5,0.7))
	C_4
A_1	((0.3,0.4),(0.3,0.5))
A_2	((0.5,0.6),(0,0.2,0.3))
A_3	((0.1),(0.7))
A_4	((0.5,0.6,0.7),(0,0.2))
A_5	((0.7,1),(0,0.1))

Step 6. Rank all the alternatives A_i $(i = 1, 2, \ldots, 5)$ according to the closeness coefficient $RC(A_i)$:

$A_3 \succ A_1 \succ A_2 \succ A_4 \succ A_5$.

Thus the most desirable alternative is A_3. So the rector will allocate the funds according to this ranking.

5. Conclusions

IHFS is the best way to deal with uncertainty when fuzzy set theory is not able to cope with the situation. Decision makers gave their opinions about the criteria of alternatives by IHFS. Multi-criteria analysis provides an effective frame work for the evaluation of alternatives. Fuzzy TOPSIS method is proposed for IHFS to solve multi-criteria decision-making problem in group decision environment. The RC coefficient has ranked the alternatives from the best to worst by considering the smallest distance from the PIS and also the largest distance from the NIS. In future, we plan to continue to study Choquet integral based TOPSIS for IHFS. Furthermore, algebraic operations for IHFS will also be develop.

Conflict of Interest

No potential conflict of interest relevant to this article was reported.

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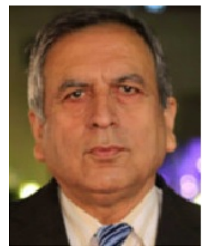

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