

COINCIDENCE AND COMMON FIXED POINT THEOREMS FOR SINGLE-VALUED AND SET-VALUED MAPPINGS

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ABSTRACT. In the present paper, we prove common fixed point theorems for single-valued and set-valued occasionally weakly compatible mappings in Menger spaces. Our results improve and extend the results of Chen and Chang [Chi-Ming Chen and Tong-Huei Chang, *Common fixed point theorems in Menger spaces*, Int. J. Math. Math. Sci. **2006** (2006), Article ID 75931, Pages 1–15].

1. Introduction

The notion of statistical metric spaces, as a generalization of metric spaces, with non-deterministic distance, was introduced by Menger [34] in 1942. The study of this space was expanded rapidly with the pioneering works of Schweizer and Sklar [43] and some of their coworkers. Such a probabilistic generalization of metric spaces appears to be well adapted for the investigation of physiological thresholds and physical quantities. It is also of fundamental importance in probabilistic functional analysis, nonlinear analysis and applications (see [14], [16], [23], [48]). In 1972, Sehgal and Bharucha-Reid [44] initiated the study of contraction maps in probabilistic metric spaces (shortly, PM-spaces) which is an important step in the development of fixed point theorems.

In 1991, Mishra [36] extended the notion of compatible maps (introduced by Jungck [30] in metric spaces) to PM-spaces and proved a common fixed point theorem in presence of continuity of at least one of the maps, completeness of the underlying space and containment of the ranges amongst involved maps. Further, Singh and Jain [45] noticed these criteria for fixed points of contraction mappings and extended the notion of weak compatibility (introduced by Jungck and Rhoades [31] in metric spaces) to PM-spaces. In 2008, Al-Thagafi and Shahzad [5] introduced the notion of occasionally weakly compatible (shortly owc) maps in metric spaces and showed that this notion is more general among all the commutativity concepts. It is worth mentioning that every pair of weak

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compatible self-maps is owc but the reverse is not always true. Many authors proved common fixed point theorems for owc maps on various spaces (see [1]-[4], [7], [8], [13], [18], [19], [20], [24], [32], [33], [38], [39], [42], [47]).

In 1976, Caristi [12] proved the Caristi's fixed point theorem without any requirement of continuity of the involved maps. Zhang et al. [49] proved set-valued Caristi's theorem in probabilistic metric spaces. Chuan [22] brought forward the concept of Caristi type hybrid fixed point in Menger PM-space. A generalization of the notion of a C-contraction for multivalued maps is given by Pap et al. [40], and they proved a fixed point theorem for (Φ, C) -multivalued contraction in Menger space. Various authors proved several fixed point theorems for multi-valued maps in different settings (see [6], [9]-[11], [15], [17], [22], [25], [26], [28], [29], [35], [41], [46], [50]). In 2006, Chen and Chang [21] proved common fixed point theorems for single and set-valued maps in Menger PM-spaces using the notion of compatibility.

The aim of this paper is to prove common fixed point theorems for single-valued and set-valued owc maps in Menger spaces. Our results do not require conditions on completeness (or closedness) of the underlying space (or subspaces), containment of ranges and continuity of the involved maps.

2. Preliminaries

Definition 2.1 ([43]). A mapping $\Delta : [0, 1] \times [0, 1] \rightarrow [0, 1]$ is t-norm if Δ satisfies the following conditions:

- (1) Δ is commutative and associative,
- (2) $\Delta(a, 1) = a$ for all $a \in [0, 1]$,
- (3) $\Delta(a, b) \leq \Delta(c, d)$ whenever $a \leq c$ and $b \leq d$ and $a, b, c, d \in [0, 1]$.

Examples of t-norms are $\Delta(a, b) = \min\{a, b\}$, $\Delta(a, b) = ab$ and $\Delta(a, b) = \max\{a + b - 1, 0\}$.

Definition 2.2 ([43]). A mapping $F : \mathbb{R} \rightarrow \mathbb{R}^+$ is called a distribution function if it is non-decreasing and left continuous with $\inf\{F(t) : t \in \mathbb{R}\} = 0$ and $\sup\{F(t) : t \in \mathbb{R}\} = 1$.

We shall denote by \mathfrak{S} the set of all distribution functions defined on $[-\infty, \infty]$ while $H(t)$ will always denote the specific distribution function defined by

$$H(t) = \begin{cases} 0, & \text{if } t \leq 0; \\ 1, & \text{if } t > 0. \end{cases}$$

If X is a non-empty set, $\mathcal{F} : X \times X \rightarrow \mathfrak{S}$ is called a probabilistic distance on X and the value of \mathcal{F} at $(x, y) \in X \times X$ is represented by $F_{x,y}$.

Definition 2.3 ([43]). A PM-space is an ordered pair (X, \mathcal{F}) , where X is a nonempty set of elements and \mathcal{F} is a probabilistic distance satisfying the following conditions: for all $x, y, z \in X$ and $t, s > 0$,

- (1) $F_{x,y}(t) = H(t)$ for all $t > 0$ if and only if $x = y$,
- (2) $F_{x,y}(t) = F_{y,x}(t)$,

(3) if $F_{x,y}(t) = 1$ and $F_{y,z}(s) = 1$, then $F_{x,z}(t + s) = 1$.

The ordered triple (X, \mathcal{F}, Δ) is called a Menger space if (X, \mathcal{F}) is a PM-space, Δ is a t-norm and the following inequality holds:

$$F_{x,y}(t + s) \geq \Delta(F_{x,z}(t), F_{z,y}(s))$$

for all $x, y, z \in X$ and $t, s > 0$.

Every metric space (X, d) can always be realized as a PM-space by considering $\mathcal{F} : X \times X \rightarrow \mathfrak{S}$ defined by $F_{x,y}(t) = H(t - d(x, y))$ for all $x, y \in X$. So PM-spaces offer a wider framework than that of the metric spaces and are better suited to cover even wider statistical situations.

Throughout this paper, $\mathcal{B}(X)$ will denote the family of non-empty bounded subsets of a Menger space (X, \mathcal{F}, Δ) . For all $A, B \in \mathcal{B}(X)$ and for every $t > 0$, we define

$${}_D F_{A,B}(t) = \sup\{F_{a,b}(t); a \in A, b \in B\}$$

and

$${}_\delta F_{A,B}(t) = \inf\{F_{a,b}(t); a \in A, b \in B\}.$$

If set A consists of a single point a , we write

$${}_D F_{A,B}(t) = {}_\delta F_{a,B}(t).$$

If set B also consists of a single point b , we write

$${}_\delta F_{A,B}(t) = F_{a,b}(t).$$

It follows immediately from the definition that ${}_D F_{A,B}(t) = 1$ for all $t > 0$ if and only if $A = B = \{a\}$ for some $a \in X$.

Lemma 2.4 ([37]). *If a Menger space (X, \mathcal{F}, Δ) satisfies the condition $F_{x,y}(t) = C$ for all $t > 0$ with fixed $x, y \in X$. Then we have $C = 1$ and $x = y$.*

Lemma 2.5 ([27]). *Let the function $\phi(t)$ satisfy the following condition $(\Phi) : \phi(t) : [0, \infty) \rightarrow [0, \infty)$ is non-decreasing and $\sum_{n=1}^{\infty} \phi^n(t) < \infty$ for all $t > 0$, when $\phi^n(t)$ denotes the n^{th} iterative function of $\phi(t)$. Then $\phi(t) < t$ for all $t > 0$.*

The following definition is on the lines of Jungck and Rhoades [31].

Definition 2.6. Maps $f : X \rightarrow X$ and $g : X \rightarrow \mathcal{B}(X)$ of a Menger space (X, \mathcal{F}, Δ) are said to be weakly compatible (or coincidentally commuting) if they commute at their coincidence points, that is $gx = \{fx\}$ for some $x \in X$ then $fgx = gfx$ (Note that the term $gx = \{fx\}$ implies that gx is a singleton).

Recently, Abbas and Rhoades [2] generalized the concept of weak compatibility in the setting of single-valued and multi-valued maps by introducing the notion of owc maps due to [5].

Definition 2.7. Maps $f : X \rightarrow X$ and $g : X \rightarrow \mathcal{B}(X)$ of a Menger space (X, \mathcal{F}, Δ) are said to be owc if and only if there is a point $x \in X$ which is a coincidence point of f and g at which f and g commute.

From the following example it is clear that the notion of owc is more general than weak compatibility.

Example 2.8. Let (X, \mathcal{F}, Δ) be a Menger space, where $X = [0, \infty)$ and

$$F_{x,y}(t) = \begin{cases} \frac{t}{t+|x-y|}, & \text{if } t > 0; \\ 0, & \text{if } t = 0. \end{cases}$$

Let $A : X \rightarrow X$ and $B : X \rightarrow \mathcal{B}(X)$ be single-valued and set-valued maps defined by

$$A(x) = \begin{cases} 0, & \text{if } x = 0; \\ x^2, & \text{if } x \in (0, \infty). \end{cases} \quad B(x) = \begin{cases} \{0\}, & \text{if } x = 0; \\ \{3x\}, & \text{if } x \in (0, \infty). \end{cases}$$

Here, 0 and 3 are two coincidence points of A and B . That is $A(0) = \{0\} \in B(0)$, $A(3) = \{9\} \in B(3)$, but $AB(0) = \{0\} = BA(0)$, $AB(3) \neq BA(3)$. Thus A and B are owc but not weakly compatible.

Lemma 2.9 ([2, 32]). *Let (X, \mathcal{F}, Δ) be a Menger space, $f : X \rightarrow X$ and $g : X \rightarrow \mathcal{B}(X)$ are owc self maps of X . If f and g have a unique point of coincidence, $\{w\} = \{fx\} = gx$, then w is the unique common fixed point of A and B .*

3. Results

In 2006, Chen and Chang [21] proved the following common fixed point theorem for single-valued and set-valued maps in Menger spaces (X, \mathcal{F}, Δ) where $\Delta(x, y) = \min\{x, y\}$.

Theorem 3.1 ([21, Theorem 2.9]). *Let (X, \mathcal{F}, \min) be a complete Menger space. Let $f, g, \eta, \xi : X \rightarrow X$ be four single-valued functions and let $S, T : X \rightarrow \mathcal{B}(X)$ two set-valued functions. If the following conditions are satisfied:*

- (1) $S(X) \subset \xi g(X), T(X) \subset \eta f(X)$,
- (2) $\eta f = f\eta, \xi g = g\xi, Sf = fS, Tg = gT$,
- (3) ηf or ξg is continuous,
- (4) $(S, \eta f)$ and $(T, \xi g)$ are compatible and
- (5) for $t > 0$,

$$\delta F_{Sx, Ty}(\phi(t)) \geq \min \left\{ \begin{array}{l} F_{\eta f x, \xi g y}(t), \delta F_{\eta f x, Sx}(t), \delta F_{\xi g y, Ty}(t), \\ \delta F_{\xi g y, Sx}(\beta t), \delta F_{\eta f x, Ty}(2t - \beta t) \end{array} \right\}$$

for all $x, y \in X, \beta \in (0, 2)$, where $\phi \in \Phi$. Then f, g, η, ξ, S and T have a unique common fixed point z in X .

Our improvements in this theorem are four-fold which include:

- (1) relaxing the condition of the completeness of the whole space,
- (2) relaxing the condition (1) in Theorem 3.1,

- (3) relaxing the condition of continuity of involved maps,
- (4) replacing the concept of compatibility by owc maps which is the most general concept among all the commutativity concepts.

First we prove a common fixed point theorem for any even number of single-valued and two set-valued maps in Menger space.

Theorem 3.2. *Let (X, \mathcal{F}, \min) be a Menger space. Let $P_1, P_2, \dots, P_{2n} : X \rightarrow X$ be single-valued and $A, B : X \rightarrow \mathcal{B}(X)$ two set-valued maps satisfying the following conditions:*

- (1) for $t > 0$,

$$\delta F_{Ax,By}(\phi(t)) \geq \min \left\{ \begin{array}{l} F_{P_1 P_3 \dots P_{2n-1} x, P_2 P_4 \dots P_{2n} y}(t), \delta F_{Ax, P_1 P_3 \dots P_{2n-1} x}(t), \\ \delta F_{By, P_2 P_4 \dots P_{2n} y}(t), \delta F_{Ax, P_2 P_4 \dots P_{2n} y}(\beta t), \\ \delta F_{By, P_1 P_3 \dots P_{2n-1} x}(2t - \beta t) \end{array} \right\}$$

for all $x, y \in X, \beta \in (0, 2)$ and $\phi \in \Phi$.

- (2)

$$\left\{ \begin{array}{l} P_1(P_3 \dots P_{2n-1}) = (P_3 \dots P_{2n-1})P_1, \\ P_1 P_3(P_5 \dots P_{2n-1}) = (P_5 \dots P_{2n-1})P_1 P_3, \\ \vdots \\ P_1 \dots P_{2n-3}(P_{2n-1}) = (P_{2n-1})P_1 \dots P_{2n-3}, \\ A(P_3 \dots P_{2n-1}) = (P_3 \dots P_{2n-1})A, \\ A(P_5 \dots P_{2n-1}) = (P_5 \dots P_{2n-1})A, \\ \vdots \\ AP_{2n-1} = P_{2n-1}A, \\ P_2(P_4 \dots P_{2n}) = (P_4 \dots P_{2n})P_2, \\ P_2 P_4(P_6 \dots P_{2n}) = (P_6 \dots P_{2n})P_2 P_4, \\ \vdots \\ P_2 \dots P_{2n-2}(P_{2n}) = (P_{2n})P_2 \dots P_{2n-2}, \\ B(P_4 \dots P_{2n}) = (P_4 \dots P_{2n})B, \\ B(P_6 \dots P_{2n}) = (P_6 \dots P_{2n})B, \\ \vdots \\ BP_{2n} = P_{2n}B. \end{array} \right.$$

If the pairs $(A, P_1 P_3 \dots P_{2n-1})$ and $(B, P_2 P_4 \dots P_{2n})$ are each owc, then there exists a point $z \in X$ such that $\{z\} = \{P_1 z\} = \{P_2 z\} = \dots = \{P_{2n} z\} = Az = Bz$.

Proof. Since the pairs $(A, P_1 P_3 \dots P_{2n-1})$ and $(B, P_2 P_4 \dots P_{2n})$ are each owc, there exist points $u, v \in X$ such that $P_1 P_3 \dots P_{2n-1} u \in Au$, $(P_1 P_3 \dots P_{2n-1})Au \subseteq A(P_1 P_3 \dots P_{2n-1})u$ and $P_2 P_4 \dots P_{2n} v \in Bv$, $(P_2 P_4 \dots P_{2n})Bv \subseteq B(P_2 P_4 \dots P_{2n})v$. Now we show that $Au = Bv$. Putting $x = u, y = v$ and $\beta = 1$ in (1),

we have

$$\begin{aligned} \delta F_{Au,Bv}(\phi(t)) &\geq \min \left\{ \begin{array}{l} F_{P_1 P_3 \dots P_{2n-1} u, P_2 P_4 \dots P_{2n} v}(t), \delta F_{Au, P_1 P_3 \dots P_{2n-1} u}(t), \\ \delta F_{Bv, P_2 P_4 \dots P_{2n} v}(t), \delta F_{Au, P_2 P_4 \dots P_{2n} v}(t), \\ \delta F_{Bv, P_1 P_3 \dots P_{2n-1} u}(t) \end{array} \right\}, \\ \delta F_{Au,Bv}(\phi(t)) &\geq \min \left\{ \begin{array}{l} \delta F_{Au,Bv}(t), \delta F_{Au,Au}(t), \delta F_{Bv,Bv}(t), \\ \delta F_{Au,Bv}(t), \delta F_{Bv,Au}(t) \end{array} \right\}, \\ \delta F_{Au,Bv}(\phi(t)) &\geq \min \{ \delta F_{Au,Bv}(t), 1, 1, \delta F_{Au,Bv}(t), \delta F_{Bv,Au}(t) \}, \\ \delta F_{Au,Bv}(\phi(t)) &\geq \delta F_{Au,Bv}(t). \end{aligned}$$

On the other hand, since F is non-decreasing, we get

$$\delta F_{Au,Bv}(\phi(t)) \leq \delta F_{Au,Bv}(t).$$

Hence $\delta F_{Au,Bv}(t) = C$ for all $t > 0$. From Lemma 2.4 we conclude that $C = 1$, that is $Au = Bv$. Moreover, if there is another point z such that $P_1 P_3 \dots P_{2n-1} z \in Az$. From (1), it follows that $Az = \{P_1 P_3 \dots P_{2n-1} z\} = Bv = \{P_2 P_4 \dots P_{2n} v\}$, or $Au = Az$. Hence $\{w\} = Au = \{P_1 P_3 \dots P_{2n-1} u\}$ is the unique point of coincidence of A and $P_1 P_3 \dots P_{2n-1}$. By Lemma 2.9, it follows that w is the unique common fixed point of A and $P_1 P_3 \dots P_{2n-1}$. By symmetry, $\{q\} = Bv = \{P_2 P_4 \dots P_{2n} v\}$ is the unique common fixed point of B and $P_2 P_4 \dots P_{2n}$. Since $w = q$, we obtain that w is the unique common fixed point of B and $P_2 P_4 \dots P_{2n}$. Now we show that w is the fixed point of all the component mappings, by putting $x = P_3 \dots P_{2n-1} w$, $y = w$, $P_1' = P_1 P_3 \dots P_{2n-1}$ and $P_2' = P_2 P_4 \dots P_{2n}$ with $\beta = 1$ in (1), we have

$$\begin{aligned} \delta F_{AP_3 \dots P_{2n-1} w, Bw}(\phi(t)) &\geq \min \left\{ \begin{array}{l} F_{P_1' P_3 \dots P_{2n-1} w, P_2' w}(t), \\ \delta F_{AP_3 \dots P_{2n-1} w, P_1' P_3 \dots P_{2n-1} w}(t), \\ \delta F_{Bw, P_2' w}(t), \delta F_{AP_3 \dots P_{2n-1} w, P_2' w}(t), \\ \delta F_{Bw, P_1' P_3 \dots P_{2n-1} w}(t) \end{array} \right\}, \\ F_{P_3 \dots P_{2n-1} w, w}(\phi(t)) &\geq \min \left\{ \begin{array}{l} F_{P_3 \dots P_{2n-1} w, w}(t), \\ F_{P_3 \dots P_{2n-1} w, P_3 \dots P_{2n-1} w}(t), F_{w, w}(t), \\ F_{P_3 \dots P_{2n-1} w, w}(t), F_{w, P_3 \dots P_{2n-1} w}(t) \end{array} \right\}, \\ F_{P_3 \dots P_{2n-1} w, w}(\phi(t)) &\geq \min \left\{ \begin{array}{l} F_{P_3 \dots P_{2n-1} w, w}(t), 1, 1, \\ F_{P_3 \dots P_{2n-1} w, w}(t), F_{w, P_3 \dots P_{2n-1} w}(t) \end{array} \right\}, \\ F_{P_3 \dots P_{2n-1} w, w}(\phi(t)) &\geq F_{P_3 \dots P_{2n-1} w, w}(t). \end{aligned}$$

On the other hand, since F is non-decreasing, we get

$$F_{P_3 \dots P_{2n-1} w, w}(\phi(t)) \leq F_{P_3 \dots P_{2n-1} w, w}(t).$$

Hence $F_{P_3 \dots P_{2n-1} w, w}(t) = C$ for all $t > 0$. From Lemma 2.4 we conclude that $C = 1$, i.e., $P_3 \dots P_{2n-1} w = w$. Hence, $P_1 w = w$. Continuing this procedure, we have

$$Aw = \{P_1 w\} = \{P_3 w\} = \dots = \{P_{2n-1} w\} = \{w\}.$$

So,

$$Bw = \{P_2w\} = \{P_4w\} = \dots = \{P_{2n}w\} = \{w\},$$

i.e., w is the unique common fixed point of $P_1, P_2, \dots, P_{2n}, A$ and B . □

Corollary 3.3. *Let (X, \mathcal{F}, \min) be a Menger space. Let $S, T : X \rightarrow X$ be single-valued and $A, B : X \rightarrow \mathcal{B}(X)$ two set-valued maps satisfying:*

$$\delta F_{Ax,By}(\phi(t)) \geq \min \left\{ \begin{array}{l} F_{Sx,Ty}(t), \delta F_{Ax,Sx}(t), \delta F_{By,Ty}(t), \\ \delta F_{Ax,Ty}(\beta t), \delta F_{By,Sx}(2t - \beta t) \end{array} \right\}$$

for all $x, y \in X, \beta \in (0, 2), t > 0$ and $\phi \in \Phi$. If the pairs (A, S) and (B, T) are each owc, then there exists a point $z \in X$ such that $\{z\} = \{Sz\} = \{Tz\} = Az = Bz$.

Proof. If we set $P_1P_3 \dots P_{2n-1} = S$ and $P_2P_4 \dots P_{2n} = T$ in Theorem 3.2, then the result follows. □

Now, we give an example which illustrates Corollary 3.3.

Example 3.4. Let $X = [0, 4]$ with the metric d defined by $d(x, y) = |x - y|$ and for each $t \in [0, 1]$, define

$$F_{x,y}(t) = \begin{cases} \frac{t}{t+|x-y|}, & \text{if } t > 0; \\ 0, & \text{if } t = 0 \end{cases}$$

for all $x, y \in X$. Clearly (X, \mathcal{F}, \min) is a Menger space. Define the single-valued maps $S, T : X \rightarrow X$ and set-valued maps $A, B : X \rightarrow \mathcal{B}(X)$ by

$$\begin{aligned} A(x) &= \begin{cases} \{2\}, & \text{if } 0 \leq x \leq 2; \\ \{0\}, & \text{if } 2 < x \leq 4. \end{cases} & S(x) &= \begin{cases} x, & \text{if } 0 \leq x \leq 2; \\ 3, & \text{if } 2 < x \leq 4. \end{cases} \\ B(x) &= \begin{cases} \{2\}, & \text{if } 0 \leq x \leq 2; \\ \{4\}, & \text{if } 2 < x \leq 4. \end{cases} & T(x) &= \begin{cases} 2, & \text{if } 0 \leq x \leq 2; \\ \frac{x}{4}, & \text{if } 2 < x \leq 4. \end{cases} \end{aligned}$$

Clearly all the conditions of Corollary 3.3 are satisfied with respect to the distribution function $F_{x,y}$.

That is,

$$S(2) = \{2\} \in A(2) \text{ and } SA(2) = \{2\} = AS(2),$$

and

$$T(2) = \{2\} \in B(2) \text{ and } TB(2) = \{2\} = BT(2).$$

So, A and S as well as B and T are owc maps. Also 2 is the unique common fixed point of A, B, S and T . On the other hand, it is clear to see that the maps A, B, S and T are discontinuous at 2.

Further, we have

$$A(X) = \{0, 2\} \text{ is not a subset of } T(X) = \left(\frac{1}{2}, 1\right] \cup \{2\}$$

and

$$B(X) = \{2, 4\} \text{ is not a subset of } S(X) = [0, 2] \cup \{3\},$$

which generalizes our result.

The following results are slight generalizations of Theorem 3.2 and Corollary 3.3.

Corollary 3.5. *Let (X, \mathcal{F}, \min) be a Menger space. Let $P_1, P_2, \dots, P_{2n} : X \rightarrow X$ be single-valued and $A, B : X \rightarrow \mathcal{B}(X)$ two set-valued maps satisfying condition (2) of Theorem 3.2 such that*

$$\delta F_{Ax, By}(\phi(t)) \geq \min \left\{ \begin{array}{l} F_{P_1 P_3 \dots P_{2n-1} x, P_2 P_4 \dots P_{2n} y}(t), \delta F_{Ax, P_1 P_3 \dots P_{2n-1} x}(t), \\ \delta F_{By, P_2 P_4 \dots P_{2n} y}(t), \\ D F_{Ax, P_2 P_4 \dots P_{2n} y}(t) + D F_{By, P_1 P_3 \dots P_{2n-1} x}(t) \end{array} \right\}$$

holds for all $x, y \in X$, $t > 0$ and $\phi \in \Phi$. If the pairs $(A, P_1 P_3 \dots P_{2n-1})$ and $(B, P_2 P_4 \dots P_{2n})$ are each owc, then there exists a point $z \in X$ such that $\{z\} = \{P_1 z\} = \{P_2 z\} = \dots = \{P_{2n} z\} = Az = Bz$.

Corollary 3.6. *Let (X, \mathcal{F}, \min) be a Menger space. Let $S, T : X \rightarrow X$ be single-valued and $A, B : X \rightarrow \mathcal{B}(X)$ two set-valued maps satisfying:*

$$\delta F_{Ax, By}(\phi(t)) \geq \min \left\{ \begin{array}{l} F_{Sx, Ty}(t), \delta F_{Ax, Sx}(t), \delta F_{By, Ty}(t), \\ D F_{Ax, Ty}(t) + D F_{By, Sx}(t) \end{array} \right\}$$

for all $x, y \in X$, $t > 0$ and $\phi \in \Phi$. If the pairs (A, S) and (B, T) are each owc, then there exists a point $z \in X$ such that $\{z\} = \{Sz\} = \{Tz\} = Az = Bz$.

Remark 3.7. The conclusions of Theorem 3.2, Corollary 3.3, Corollary 3.5 and Corollary 3.6 remain true for $\phi(t) = kt$, where $k \in (0, 1)$.

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